Quiz 2 9/10/13

(a) $J(\underline{k}) = E\left[(x-\underline{k}^H\underline{y})^*(x-\underline{k}^H\underline{y})\right]$

$$= \sigma_x^2 - E\left[\underline{k}^H \underline{y}\, x^*\right] - E\left[\underline{y}^H \underline{k}\, x\right] + E\left[(\underline{k}^H \underline{y})(\underline{y}^H \underline{k})\right]$$

$$= \sigma_x^2 - \underline{k}^H R_{yx} - R_{xy}\underline{k} + \underline{k}^H R_y \underline{k}$$

$$\nabla_{\underline{k}} J(\underline{k}) = -R_{xy} + \underline{k}^H R_y$$

$$\Rightarrow \underline{k}_i = \underline{k}_{i-1} - \mu\left(-R_{yx} + R_y \underline{k}_{i-1}\right)$$

$$= \underline{k}_{i-1} - \mu\left(R_y \underline{k}_{i-1} - R_y \underline{k}_{opt}\right) \qquad \left(\underline{k}_{opt} \overset{\text{Given}}{=} R_y^{-1} R_{yx}\right)$$

$$\underline{k}_i - \underline{k}_{opt} = \underline{k}_{i-1} - \underline{k}_{opt} - \mu R_y\left(\underline{k}_{i-1} - \underline{k}_{opt}\right)$$

$$= \left(I - \mu R_y\right)\left(\underline{k}_{i-1} - \underline{k}_{opt}\right)$$

$$\underline{\tilde{k}}_i = \left(I - \mu R_y\right)\underline{\tilde{k}}_{i-1} \qquad \left(\underline{\tilde{k}}_i \triangleq \underline{k}_i - \underline{k}_{opt}\right)$$

$R_y = U\Lambda U^H$ (where Λ diag >0, U unitary)

$$\underline{\tilde{k}}_i = \left(UIU^H - \mu U\Lambda U^H\right)\underline{\tilde{k}}_{i-1}$$

$$= U\left[I - \mu\Lambda\right]U^H \underline{\tilde{k}}_{i-1}$$

Let $\underline{\tilde{x}}_i \triangleq U^H \underline{\tilde{k}}_i$.

$$\boxed{\underline{\tilde{x}}_i = (I - \mu\Lambda)\,\underline{\tilde{x}}_{i-1}} \qquad \Lambda = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \lambda_M \end{bmatrix}$$

$\underline{\tilde{x}}_i \longrightarrow 0$ as $i \longrightarrow \infty$ ($\Rightarrow$ convergence of $\underline{k}_i \rightarrow \underline{k}_{opt}$)

if $|1 - \mu\lambda_k| < 1$ for all k.

(or) $-1 < 1 - \mu\lambda_k < 1$

(or) $0 < \mu < \dfrac{2}{\lambda_k}$ for all k.

(or) $\boxed{0 < \mu < \dfrac{2}{\lambda_{max}}}$ where λ_{max} is the largest eigenvalue of R_y.

(Sufficient condition for μ that guarantees convergence)

(b) $\underline{k}_i = \underline{k}_{i-1} - \mu\left(R_y \underline{k}_{i-1} - R_{yx}\right)$

Instantaneous approximation:

$$R_y \approx \underline{y}_i \underline{y}_i^H$$

$$R_{yx} \approx \underline{y}_i x_i^*$$

$$\underline{k}_i = \underline{k}_{i-1} - \mu\left(\underline{y}_i \underline{y}_i^H \underline{k}_{i-1} - \underline{y}_i x_i^*\right)$$

$$\boxed{\underline{k}_i = \underline{k}_{i-1} - \mu \underline{y}_i\left(\underline{y}_i^H \underline{k}_{i-1} - x_i^*\right)}$$

2 (a) $\min\limits_{\underline{k}_i} \|\underline{k}_i - \underline{k}_{i-1}\|^2$

subject to $R_i = \left(1 - \dfrac{\mu\|\underline{y}_i\|^2}{\epsilon + \|\underline{y}_i\|^2}\right) E_i$

$R_i = x_i^* - \underline{y}_i^H \underline{k}_i$

$E_i = x_i^* - \underline{y}_i^H \underline{k}_{i-1}$

Let $\delta\underline{k} = \underline{k}_i - \underline{k}_{i-1}$.

$$R_i - E_i = -\frac{\mu\|\underline{y}_i\|^2}{\epsilon + \|\underline{y}_i\|^2} E_i$$

Also $R_i - E_i = \underline{y}_i^H \underline{k}_{i-1} - \underline{y}_i^H \underline{k}_i = -\underline{y}_i^H \delta\underline{k}$

$$\Rightarrow \underline{y}_i^H \delta\underline{k} = \frac{\mu\|\underline{y}_i\|^2}{\epsilon + \|\underline{y}_i\|^2} E_i \quad \text{---(1)}$$

One soln to (1) is

$$\delta\underline{k} = \frac{\mu \underline{y}_i E_i}{\epsilon + \|\underline{y}_i\|^2}.$$

Also the min. norm soln.

$\Rightarrow \min\limits_{\delta\underline{k}} \|\delta\underline{k}\|^2$ subject to $\underline{y}_i^H \delta\underline{k} = \dfrac{\mu\|\underline{y}_i\|^2 E_i}{\epsilon + \|\underline{y}_i\|^2}$

is minimized is $\delta\underline{k} = \dfrac{\mu \underline{y}_i E_i}{\epsilon + \|\underline{y}_i\|^2} \Rightarrow \boxed{\underline{k}_i = \underline{k}_{i-1} + \dfrac{\mu \underline{y}_i E_i}{\epsilon + \|\underline{y}_i\|^2}}.$

Choose μ such that $\left|1-\frac{\mu\|\underline{y}_i\|^2}{\epsilon+\|\underline{y}_i\|^2}\right|<1$

(or) $0<\frac{\mu\|\underline{y}_i\|^2}{\epsilon+\|\underline{y}_i\|^2}<2$

(or) $0<\mu<2\frac{\left(\epsilon+\|\underline{y}_i\|^2\right)}{\|\underline{y}_i\|^2}$. —(2)

(2) can be satisfied for any $\|\underline{y}_i\|^2$ by choosing $\boxed{0<\mu<2}$.

(3) $\lambda_1=4$

$\lambda_2=1$.

Need to compare $|1-\mu\lambda_k|$

(a) $\mu=0.3$

$1-\mu\lambda_1 = 1-1.2 = -0.2$

$1-\mu\lambda_2 = 1-0.3 = 0.7$.

Mode corresponding to 4 converges faster

(b) $\mu=0.45$

$1-\mu\lambda_1 = 1-1.8 = -0.8$

$1-\mu\lambda_2 = 1-0.45 = 0.55$

Mode corresponding to 1 converges faster.

(c) $|1-\mu(4)| = |1-\mu|$

$4\mu-1 = 1-\mu \Rightarrow 5\mu=2$

$\boxed{\mu=2/5}$