Understanding the DFT

1 Introduction

The Discrete Fourier Transform (DFT) is a cornerstone of Digital Signal Processing (DSP). In practice, the DFT is computed using the Fast Fourier Transform (FFT) algorithm, which is nothing but a computationally efficient means for obtaining the DFT coefficients.[1] It was the discovery of the FFT algorithm that made most DSP algorithms practical.

In the first part of the experiment we will examine the DFT coefficients for various cases to improve our understanding of its workings. In the second part, we will use the DFT to carry out circular convolution and compare it with linear convolution.

Use Scilab's `fft` command to obtain the coefficients.

2 Frequency Estimation and Effect of Zero-Padding

Let $x[n]$ represent samples of a sine wave sampled at 8 kHz, i.e., $x[n] = \sin\left(2\pi \frac{f_0}{f_s} n + \phi\right)$. Generate 16 samples assuming $f_0 = 1021$ Hz and $\phi = 0$.

1. Compute the 16-point DFT of $x[n]$.

   (a) Plot the magnitude, real part, and imaginary part of the DFT coefficients. Note down any symmetry that may be present.

   (b) At what frequency is the peak located in the magnitude spectrum? How does it compare with the true frequency? Use the `max` function to locate the peak.

2. Pad $x[n]$ with zeros to get an L-length sequence. Let L be (a) 32, (b) 64, (c) 128, (d) 256, and (e) 1024. Compute the DFT of the padded sequence and locate the frequency at which the magnitude is maximum in each case. How do these compare with the true frequency? If the maximum is not at the true frequency, also note down the magnitude at the true frequency (if there is no DFT bin corresponding to the true frequency note the magnitude of the bins on either side of it).

3. Generate 32 samples and repeat (1)–(2). What can you say about the width of the main lobe when compared with the 16-sample case?

4. Change f_0 to 1 kHz and repeat (1)–(3). Next, for this frequency what happens when $\phi = 3\pi/8$ (choose $L = 1024$)?

In the plots of the spectrum (i.e., the DFT coefficients) requested above, the x-axis should be labelled in Hz. If you want a "stem plot" use `plot2d3`.

[1] It is a misnomer to call the coefficients as "FFT coefficients" or the spectrum as the "FFT spectrum", even though these terms are fairly well entrenched in the literature.

3 Circular Convolution Using the DFT

In this part of the experiment we compare circular and linear convolution. Circular convolution is carried out using the DFT.

1. Let `x[n] = ones(6,1)` and `y[n] = ones(4,1)`. Compute the linear convolution $x[n] * y[n]$ using the `convol` command.

   (a) Plot the result of linear convolution.

   (b) What is the length of the resulting sequence?

2. Compute an N-point circular convolution using the DFT for $N = 7, 8, 9$, and 10. Compare these results with that obtained using linear convolution.