EC204: Networks & Systems
Solutions to Problem Set 2

1.

$$u(t) * u(t) = \int_{-\infty}^{\infty} u(\tau)u(t-\tau)d\tau = \begin{cases} 0 & \text{if } t < 0 \\ \int_0^t d\tau = t & \text{if } t \geq 0 \end{cases}$$

2.

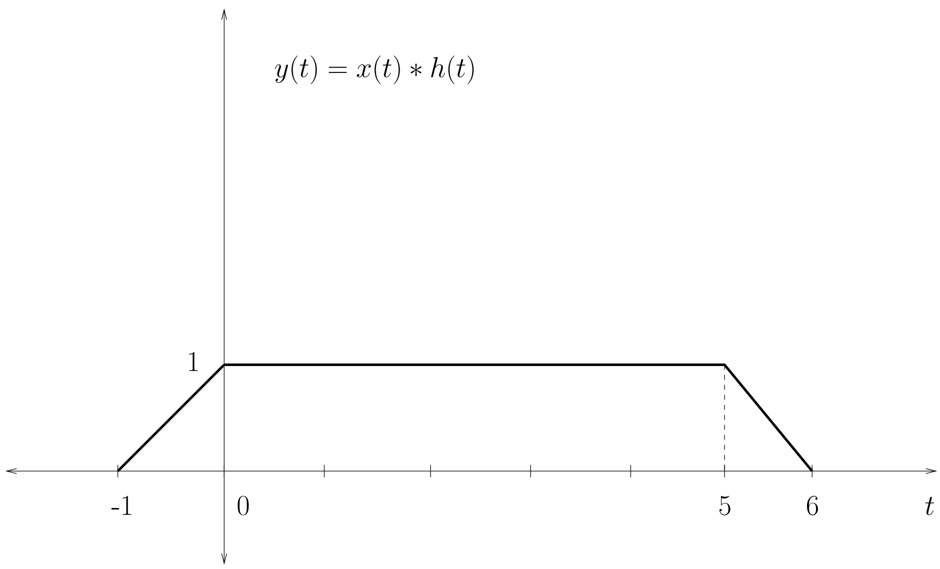

Figure 1: Solution to problem 2

3.

$$
\begin{aligned}
x(t) * h(t) &= \int_{-\infty}^{+\infty} h(\tau)x(t-\tau)d\tau \\
&= \int_{-\infty}^{+\infty} (-e^{-\tau} + 2e^{-2\tau})u(\tau)10e^{-3(t-\tau)}u(t-\tau)d\tau \\
&= \int_{0}^{t} (-e^{-\tau} + 2e^{-2\tau})10e^{-3(t-\tau)}d\tau, \qquad t \geq 0 \\
&= 10e^{-3t}\int_{0}^{t} e^{3\tau}(-e^{-\tau} + 2e^{-2\tau})d\tau, \qquad t \geq 0 \\
&= 10e^{-3t}\int_{0}^{t} (-e^{2\tau} + 2e^{\tau})d\tau \qquad t \geq 0 \\
&= 10e^{-3t}\left[-\frac{e^{2\tau}}{2}\Big|_0^t + 2e^{\tau}|_0^t \right] \qquad t \geq 0 \\
&= \begin{cases} -5e^{-t} + 20e^{-2t} - 15e^{-3t} & t \geq 0 \\ 0 & t < 0 \end{cases}
\end{aligned}
$$

4. $f_1(t) = 10r(t-1) - 10r(t-2)$ and $f_2(t) = 2r(t-1) - 5u(t-2) - 2r(t-3.5)$.

5. For finding the zero-input response, we set $x(t) = 0$, i.e., we have

$$(D^2 + 5D + 6)y_0(t) = 0 \tag{1}$$

where $y_0(t)$ is the desired zero-input response.

$y_0(t) = C_1e^{\lambda_1 t} + C_2e^{\lambda_2 t}$, where λ_1, λ_2, C_1, and C_2 are constants to be determined. From eqn. (1), we get

$$\lambda^2 + 5\lambda + 6 = 0$$

$$\Rightarrow \lambda = -2 \text{ or } \lambda = -3$$

Hence,

$$y_0(t) = C_1e^{-2t} + C_2e^{-3t}$$

where C_1 and C_2 are constants to be determined from the given initial conditions. Applying the conditions $y_0(0^-) = 2$ and $\dot{y}_0(0^-) = -1$, we get

$$2 = C_1 + C_2 \tag{2}$$

$$-1 = -2C_1 - 3C_2 \tag{3}$$

which gives $C_1 = 5$ and $C_2 = -3$. Hence, we obtain the desired zero-input response for $t \geq 0$ as

$$y_0(t) = 5e^{-2t} - 3e^{-3t} \tag{4}$$

6. We have

$$v_S = i_1 R_1 + L\frac{d}{dt}(i_1 - i_2) \tag{5}$$

$$0 = i_2 R_2 + v_C + L\frac{d}{dt}(i_2 - i_1) \tag{6}$$

Adding equations (5) and (6), differentiating with respect to t, and using $\frac{dv_c}{dt} = \frac{i_2}{C}$ gives

$$\frac{dv_S}{dt} = R_1\frac{di_1}{dt} + R_2\frac{di_2}{dt} + \frac{i_2}{C}$$

Using equation (5) to eliminate $\frac{di_2}{dt}$ gives (i.e., using $\frac{di_2}{dt} = \frac{i_1 R_1}{L} + \frac{di_1}{dt} - \frac{v_s}{L}$)

$$\frac{dv_S}{dt} = (R_1 + R_2)\frac{di_1}{dt} + \frac{R_1 R_2}{L}i_1 - \frac{v_S R_2}{L} + \frac{i_2}{C}$$

Differentiating yet again gives

$$\frac{d^2 v_S}{dt^2} + \frac{R_2}{L}\frac{dv_s}{dt} = (R_1 + R_2)\frac{d^2 i_1}{dt^2} + \frac{R_1 R_2}{L}\frac{di_1}{dt} + \frac{1}{C}\frac{di_2}{dt}$$

We eliminate $\frac{di_2}{dt}$ again using equation (5), to arrive at

$$(R_1 + R_2)\frac{d^2 i_1}{dt^2} + \left(\frac{R_1 R_2}{L} + \frac{1}{C}\right)\frac{di_1}{dt} + \frac{R_1}{LC}i_1 = \frac{d^2 v_S}{dt^2} + + \frac{R_2}{L}\frac{dv_s}{dt} + \frac{1}{LC}v_S$$

i.e.,

$$\left((R_1 + R_2)D^2 + \left(\frac{R_1 R_2}{L} + \frac{1}{C}\right)D + \frac{R_1}{LC}\right)i_1 = \left(D^2 + \frac{R_2}{L}D + \frac{1}{LC}\right)v_S$$

$$\left(6D^2 + 5D + 1\right)i_1 = \left(2D^2 + D + 1\right)v_S$$