EC204: Networks & Systems
Solutions to Problem Set 1

1.

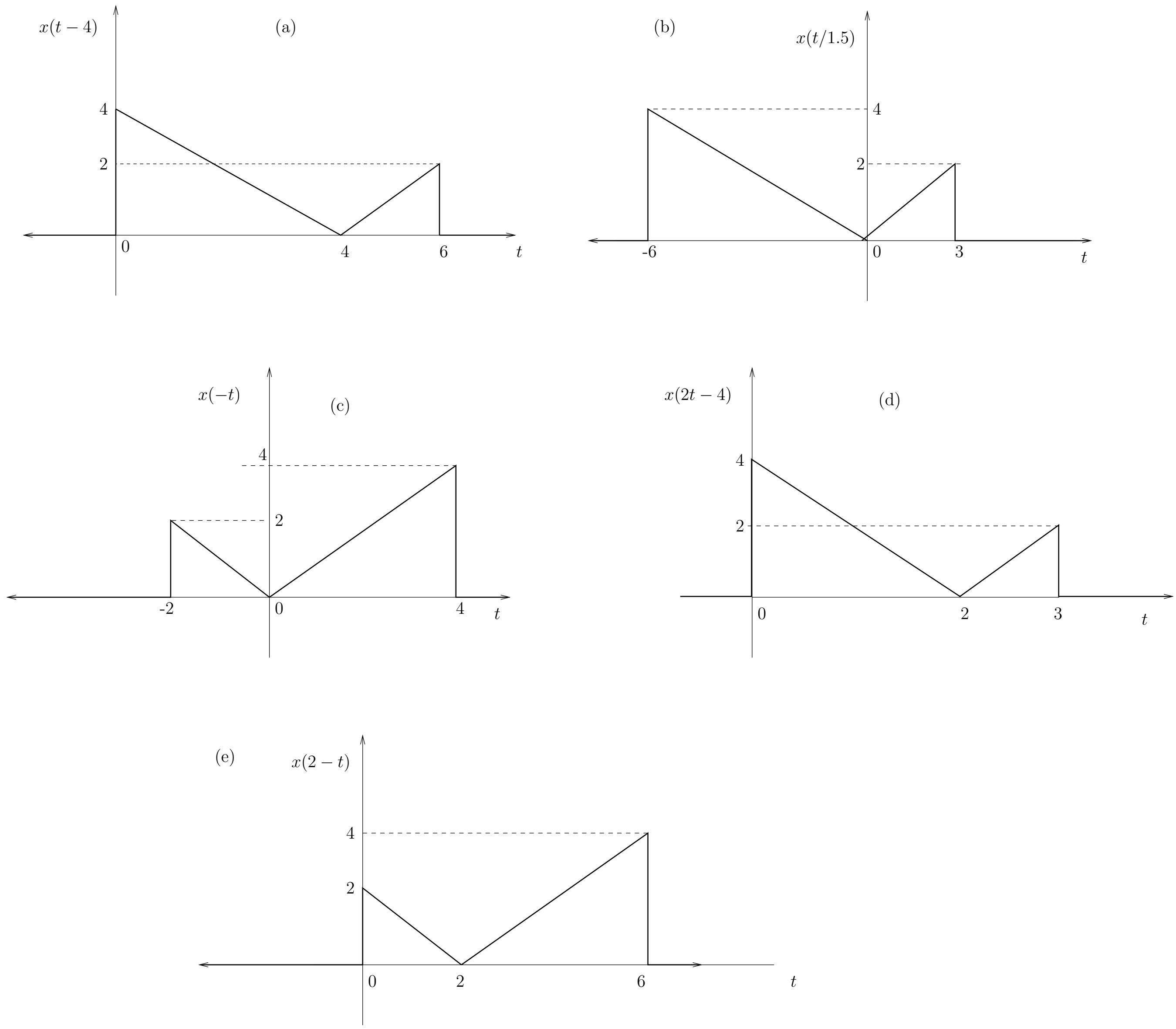

Figure 1: Solution to problems 1(a)-1(e)

2. We have,

$$y(t) = (1/5)x(-2t-3)$$

$$\therefore x(-2t-3) = 5y(t)$$

Make the substitution $-2t-3 = u$. Hence, $t = -\frac{1}{2}\big(u+3\big)$.

$$\therefore x(u) = 5y\Big(-\frac{1}{2}\big(u+3\big)\Big) \Longrightarrow x(t) = 5y\Big(-\frac{1}{2}\big(t+3\big)\Big)$$

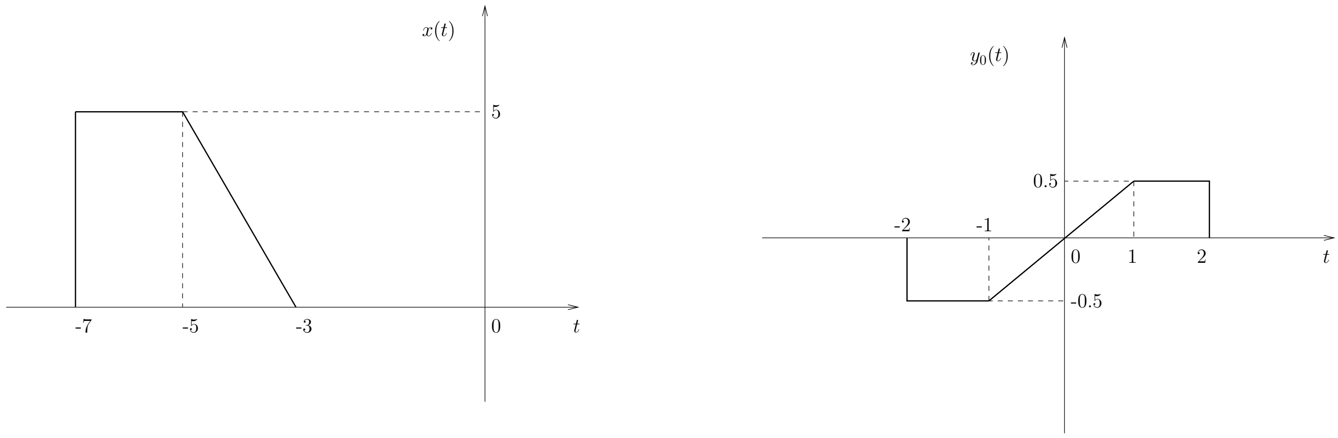

Figure 2: Solution to problem 2

3. (a) $\cos(3t) = 0.5e^{\jmath 3t} + 0.5e^{-\jmath 3t}$, Complex frequencies: $\pm\jmath 3$

   (b) $e^{-3t}\cos(3t) = 0.5e^{(-3+\jmath 3)t} + 0.5e^{(-3-\jmath 3)t}$, Complex frequencies: $-3 \pm \jmath 3$

   (c) $e^{2t}\cos(3t) = 0.5e^{(2+\jmath 3)t} + 0.5e^{(2-\jmath 3)t}$, Complex frequencies: $2 \pm \jmath 3$

   (d) $e^{-2t} = e^{(-2)t}$, Complex frequency: -2

   (e) $e^{2t} = e^{(+2)t}$, Complex frequency: $+2$

   (f) $5 = 5e^{0t}$, Complex frequency: 0

4.

$$x(t) \longrightarrow y(t)$$
$$x_1(t) \longrightarrow y_1(t)$$

- Linearity: $x(t) + x_1(t) \longrightarrow y(t) + y_1(t)$, $\alpha x(t) \longrightarrow \alpha y(t)$, $\forall \alpha \in \mathbb{C}$
- Time-invariance: $x(t-\tau) \longrightarrow y(t-\tau)$

(a) $y(t) = \int_{-5}^{5} x(\tau)d\tau$: Linear, Time-variant

(b) $y(t) = tx(t-2)$: Linear, Time-variant

(c) $3y(t) + 2 = x(t)$: Nonlinear, Time-invariant

(d) $(t^2 + 1)\frac{dy(t)}{dt} + 2y(t) = x(t)$: Linear, Time-variant

(e) $y[n] = \cos(n\omega)x[n]$: Linear, Time-variant

(f) $y[n+1] + y^2[n] = 2x[n+1] - x[n]$: Nonlinear, Time-invariant

(g) $\frac{d^2y(t)}{dt^2} + 2y(t)\frac{dy(t)}{dt} + 4y(t) = 2\frac{dx(t)}{dt} + x(t)$: Nonlinear, Time-invariant

5. Clearly, an input of $x_1(t) = \alpha x(t)$, $\alpha \in \mathbb{C}$ produces an output

$$y_1(t) = \frac{x_1^2(t)}{dx_1(t)/dt} = \frac{\alpha^2 x(t)}{d(\alpha x(t))/dt} = \alpha y(t)$$

verifying the property of homogeneity. However, the quadratic term in $x^2(t) = y(t)\frac{dx}{dt}$ violates additivity.

6. Evidently, an input of $x_1(t) + x_2(t)$ yields an output of $Re\{x_1(t) + x_2(t)\} = Re\{x_1(t)\} + Re\{x_2(t)\}$, showing that the system is additive. However, an input of $\alpha x_1(t)$, where $\alpha = \alpha_r + \jmath\alpha_i$ and $x_1(t) = x_{1r}(t) + \jmath x_{1i}(t)$ produces an output of

$$\begin{aligned} &Re\{\alpha x_1(t)\} \\ &= Re\{(\alpha_r + \jmath\alpha_i)(x_{1r}(t) + \jmath x_{1i}(t))\} \\ &= \alpha_r x_{1r}(t) - \alpha_i x_{1i}(t) \\ &\neq \alpha Re\{x_1(t)\} \end{aligned}$$

showing that the given system is not homogeneous.

7. (a) The output $y(t)$ is shown in the figure below.

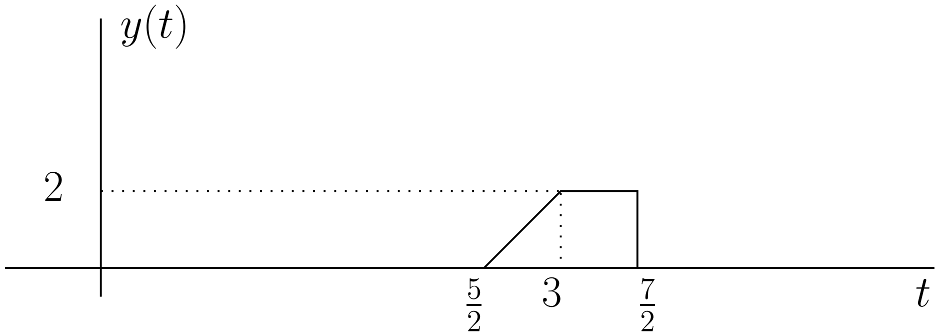

Figure 3: Solution to problem 7

(b) (i) If $x_1(t) \longrightarrow y_1(t)$, and $x_2(t) \longrightarrow y_2(t)$, then $a_1x_1(t) + a_2x_2(t) \longrightarrow a_1y_1(t) + a_2y_2(t)$ for all a_1, a_2. Therefore, the system is linear.
(ii) If $x(t) \longrightarrow y(t)$, then $x(t-\tau) \longrightarrow x(2t-4-\tau) \neq y(t-\tau)$. Therefore, the system is time-varying.
(iii) $y(5) = x(6)$. Therefore, the system is non-causal.