Spatial and Temporal Power Allocation for MISO Systems with Delayed Feedback

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Outline

Scope of the work

Problem considered:

- Minimize outage probability of Multiple Input Single Output (MISO) system in the presence of delayed feedback.

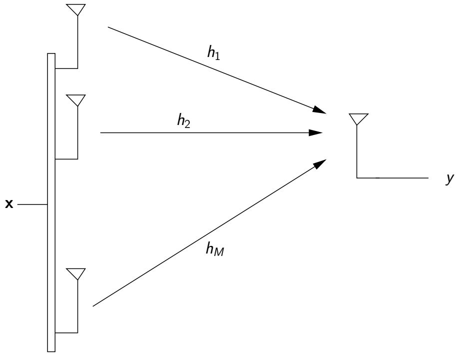
Cases considered:

- Short term power constraint
- Long term power constraint i.e temporal power control

Techniques studied:

- Uniform spatial power allocaton
- Beamforming to the delayed channel feedback
- Optimal spatial power allocation

System Model



$$y = \mathbf{h}^\mathsf{T}\mathbf{x} + z$$

$$\mathbf{h}_{M\times 1} = [h_1 \quad h_2 \quad \cdots \quad h_M]^T \sim \mathcal{CN}(\mathbf{0}, \mathbf{I_{M\times M}})$$

Channel State Information (CSI) assumptions

- Perfect CSI at the receiver (CSIR): $\mathbf{h}$
- CSI at the transmitter (CSIT): $\mathbf{h}_{old}$, a delayed version of $\mathbf{h}$
- Channel correlation is a zeroth-order Bessel function of the First kind ($J_0(x)$)

$$E\left[h_i^{(t)}\left(h_i^{(t+\Delta t)}\right)^*\right] = J_0(\omega_d \Delta t) = \rho,$$

  where ω_d is the Doppler frequency and Δt is the delay.
- For a Gaussian channel, $\mathbf{h}$ and $\mathbf{h}_{old}$ are related as

$$\mathbf{h} = \rho \mathbf{h}_{old} + \sqrt{1-\rho^2}\mathbf{w}$$

Short term power constraint
Known Results

No Temporal Power Control i.e., Transmit Power(P) is fixed.

$$\text{I}(X; Y/\mathbf{h},\mathbf{h}_{old}) = \log(1 + P\mathbf{h}^H\mathbf{Q}\mathbf{h})$$

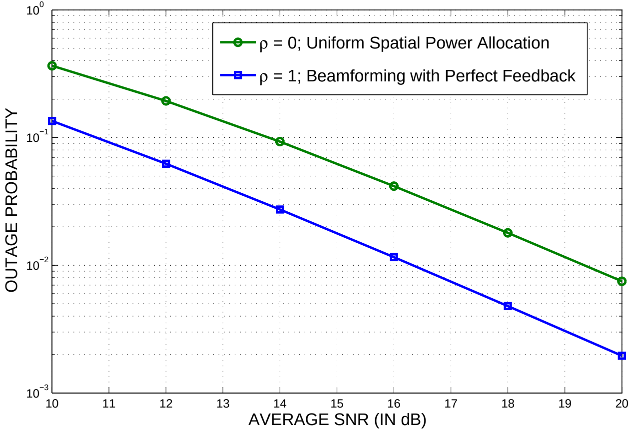
- $\rho = 0 \Rightarrow$ Uniform Spatial Power Allocation (**USPA**):

$$\mathbf{Q} = \frac{\mathbf{I}_{M\times M}}{M} \Rightarrow \text{Pout}_{\mathbf{I}}(M, R, P, \rho = 0) = \Gamma_M\left(\frac{e^R - 1}{P/M}\right)$$

- $\rho = 1 \Rightarrow$ Beamforming (**BF**):

$$\mathbf{Q} = \frac{\mathbf{h}\mathbf{h}^H}{\mathbf{h}^H\mathbf{h}} \Rightarrow \text{Pout}_{\mathbf{BF}}(M, R, P, \rho = 1) = \Gamma_M\left(\frac{e^R - 1}{P}\right)$$

Outage probability with Perfect Feedback and No Feedback



Perfect CSIT is $10\log_{10}M$ dB better

$M = 2$ Tx antennas and $R = 2$ nats/s/Hz

Problem Formulation

For $0 < \rho < 1$:

- What is the outage performance of beamforming using the imperfect CSIT?
- Is beamforming optimal?
- If not, what is the optimal spatial power allocation?

Beamforming to the imperfect CSIT

Transmitter performs beamforming along the direction of the imperfect CSIT.

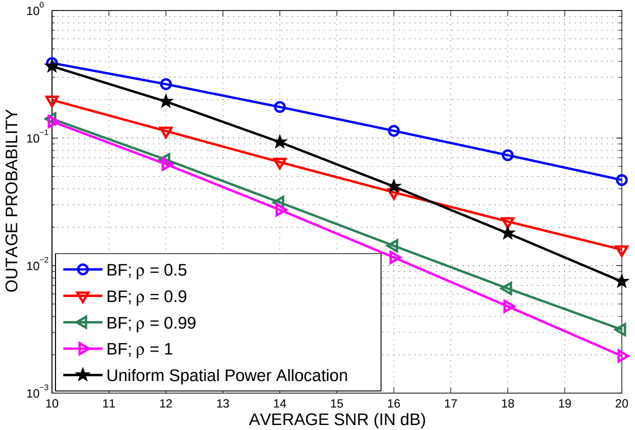
- Fix $\mathbf{h}_{old}$ and compute the outage probability
- Expectation over $\mathbf{h}_{old}$ gives the average outage probability

$$P_{\text{outBF}}(M, R, P, \rho) = \sum_{i=1}^{M} \alpha_i \Gamma_i \left(\frac{e^R - 1}{P} \right),$$

$$\text{where } \alpha_i = \frac{1}{(1+\mu)^{M-1}} \binom{M-1}{i-1} \mu^{i-1} \text{ and } \mu = \frac{\rho^2}{1-\rho^2}.$$

$\Gamma_i\left(\frac{e^R-1}{P}\right)$ is the Outage Probability of MISO channel with i transmit antennas with perfect feedback.

Outage probability with Beamforming to imperfect CSIT



Beamforming - better at low SNR and worse at high SNR

M = 2 Tx antennas and R = 2 nats/s/Hz

Diversity Gain

The asymptotic diversity gain at infinite SNR is defined as

$$\mathrm{d} = -\lim_{\mathrm{SNR}\to\infty} \frac{\log \mathrm{P_{out}}}{\log \mathrm{SNR}}$$

- $0 \leq \rho < 1$

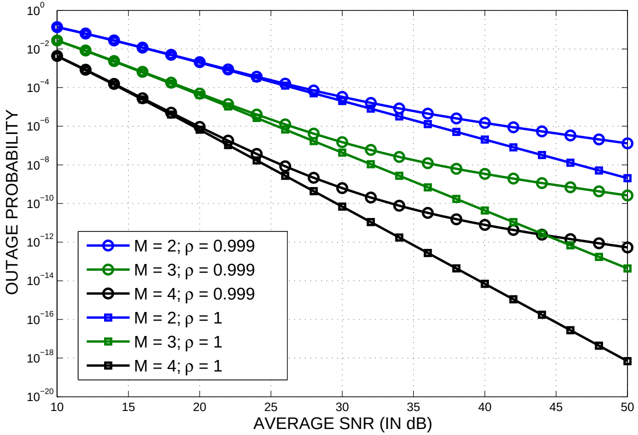
$$\mathrm{P_{outBF}}(M, R, P \to \infty, \rho) \simeq \frac{1}{(1+\mu)^{M-1}} \left(\frac{e^R - 1}{P} \right)$$

- $\rho = 1$

$$\mathrm{P_{outBF}}(M, R, P \to \infty, \rho) \simeq \frac{1}{M!} \left(\frac{e^R - 1}{P} \right)^M$$

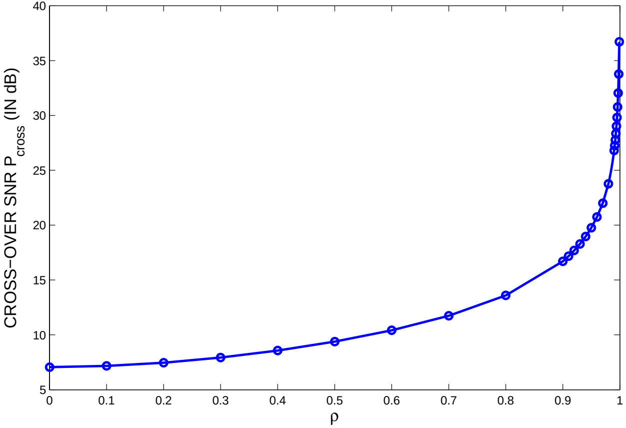
$$\text{Diversity Gain } d = \begin{cases} 1 & \text{for } 0 \leq \rho < 1 \\ M & \text{for } \rho = 1 \end{cases}$$

Outage Probability for different values of M



Diversity Gain $= 1$, for any M and $\rho < 1$

Cross-over SNR



M = 2 Tx antennas and R = 2 nats/s/Hz

Optimal Spatial Power Allocation

Beamforming

- Total Power is spent in the direction of $\mathbf{h}_{old}$
- Not Optimal

Optimal Strategy

- Spend only a fraction (λ) of power in the direction of $\mathbf{h}_{old}$.
- Rest of the (M-1) orthogonal directions are identical, which share the rest of power equally.
- Find the optimal value of (λ) for each $\mathbf{h}_{old}$ to minimize P(outage/$\mathbf{h}_{old}$)

Extreme Cases

- $\rho = 0$; All directions are identical, so optimal $\lambda = \frac{1}{M}$ (**USPA**)
- $\rho = 1$; Perfect CSIT, so optimal $\lambda = 1$ (**BF**)

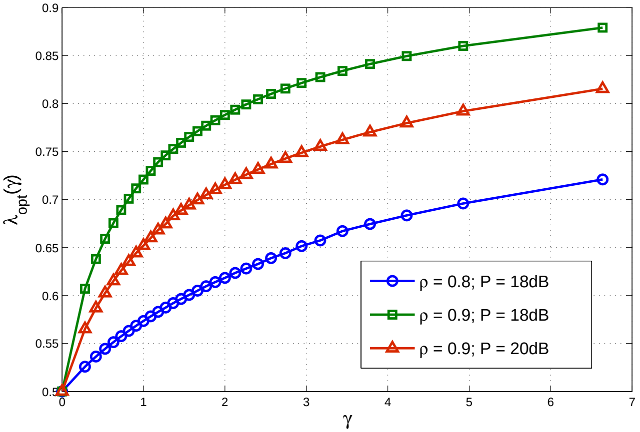
Optimal λ: $0 \leq \rho \leq 1$

Optimal λ as a function of γ; $(\lambda_{opt}(\gamma))$; $\gamma = ||\mathbf{h}_{old}||^2$

- $\gamma = 0$, equivalent to no feedback, so $\lambda_{opt}(\gamma = 0) = \frac{1}{M}$
- $\gamma \to \infty$, equivalent to perfect feedback, so $\lambda_{opt}(\gamma \to \infty) = 1$

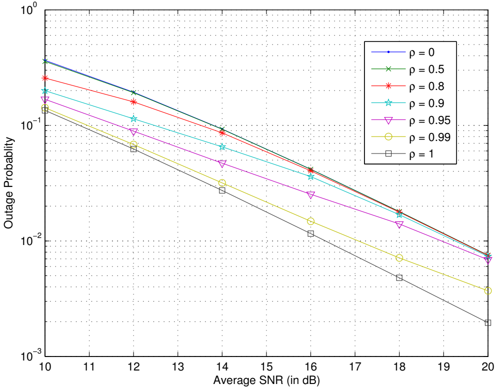
Thus, $\lambda_{opt}(\gamma)$ should start from $\frac{1}{M}$ at $\gamma = 0$ and approach 1 as γ increases.

Optimal λ: $\lambda_{opt}(\gamma)$



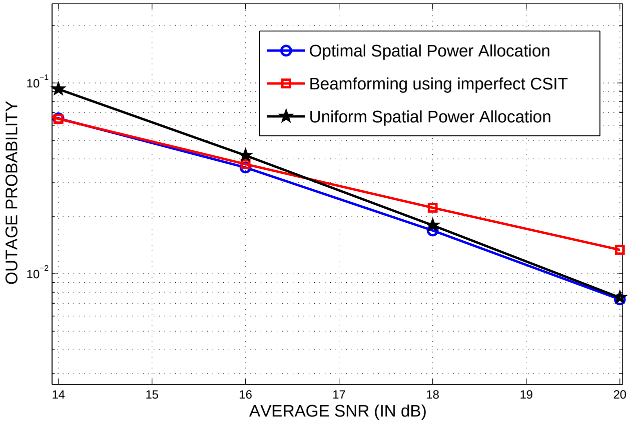
$\rho = 0.9, P = 18\text{dB}, \ M = 2$ and $R = 2$ nats/s/Hz

Outage probability with optimal spatial power control



$M = 2$ and $R = 2$ nats/s/Hz

Comparison



No significant perfromance improvement with OSPA

$\rho = 0.9,\ M = 2$ and $R = 2$ nats/s/Hz

Switching between Beamforming and Uniform Spatial Power Allocation

In practice, it would be sufficient to switch between BF and USPA.

- Optimal spatial power allocation does not improve the outage probability significantly
- Any mismatch between the estimated ρ and the actual ρ will hurt the performance of optimal spatial power control
- Optimal spatial power allocation requires computing $\lambda_{opt}(\gamma)$ for each channel realization

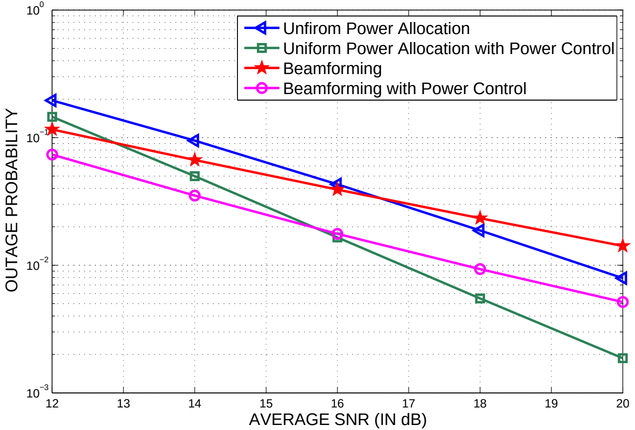
Optimal Spatio-Temporal Power Control

- Temporal power control ($p(\gamma)$) with an average power constraint: $\int_0^\infty f_\Gamma(\gamma)p(\gamma)d\gamma = 1$

$$\min_{p(\gamma)} \int_0^\infty f_\Gamma(\gamma) \min_\lambda \mathrm{P_{out}}(\gamma, p(\gamma), \lambda) d\gamma$$

- No closed form expression exists for optimal λ
- Finding the optimal $p(\gamma)$ and the corresponding λ is computationally intensive.
- Temporal power control for Uniform Spatial Power Allocation and Beamforming are considered.

Outage probability



Even with temporal power control, BF is better only at low SNR

$\rho = 0.9$, $M = 2$ and $R = 2$ nats/s/Hz.

Summary

Beamforming in the presence of delayed feedback

- Closed form expression for outage probability
- Close to optimal at lower SNR

USPA

- For $\rho < 1$, optimal at high SNR
- Switching between Beamforming and USPA
- Cross-over SNR can be determined

Optimal Spatial Power Allocation

- Allocate only a fraction of power in the direction of the imperfect feedback
- Not much gain over switching between BF and USPA

Temporal Power Control for Beamforming with delayed feedback

- Similar results for BF and USPA
- Optimal strategy hard to obtain

Thank You