

# Activity Detection and Channel Estimation for Massive Random Access Systems using Learning-based Sparse Recovery

Srikrishna Bhashyam

IIT Madras

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DRDO

A. P. Sabulal, S. Bhashyam, "Joint Sparse Recovery using Deep Unfolding With Application to Massive Random Access," ICASSP 2020 - 2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Barcelona, Spain, 2020, pp. 5050-5054.

U. K. Sreeshma Shiv, S. Bhashyam, C. R. Srivatsa and C. R. Murthy, "Learning-Based Sparse Recovery for Joint Activity Detection and Channel Estimation in Massive Random Access Systems," in IEEE Wireless Communications Letters, vol. 11, no. 11, pp. 2295-2299, Nov. 2022

# Recent Research: Overview

## Research Interests

- Communication and Information Theory
- Statistical Inference

## Recent work

- MIMO
- Sequential hypothesis testing
- Model-based learning for wireless communication
  - Learning-based sparse recovery for massive random access

More details at <https://www.ee.iitm.ac.in/skrishna/>

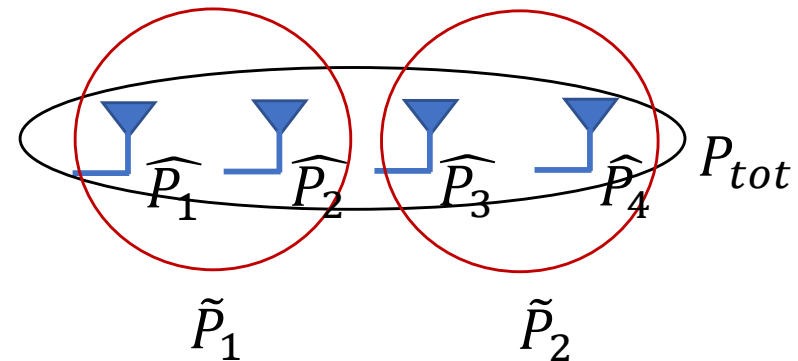
# Multiple-Input Multi-Output (MIMO)

## Capacity under per-group power constraints

- Distributed antennas
  - Cell-free MIMO, CoMP
- Hardware constraints

## Rank-constrained MIMO capacity

- Limited by RF chains
- Iterative algorithm

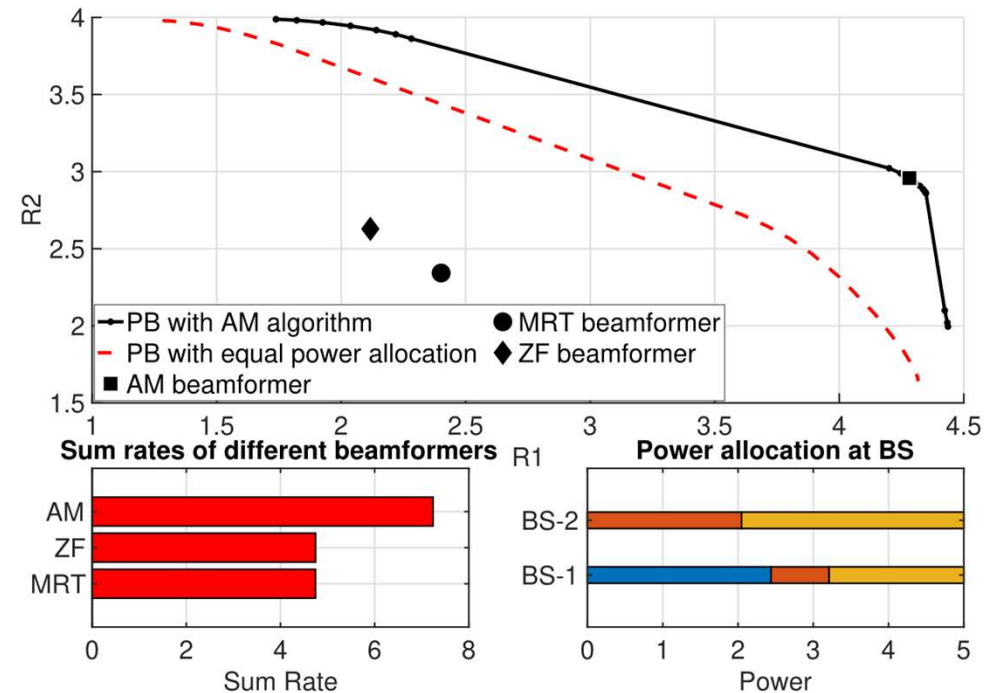


- R. Chaluvadi, S. S. Nair, S. Bhashyam, "Optimal Multi-antenna Transmission with Multiple Power Constraints," IEEE Transactions on Wireless Communications, vol. 18, no. 7, pp. 3382-3394, July 2019.

# Distributed MIMO

## Distributed beamforming

- Limited coordination
- Alternating optimization

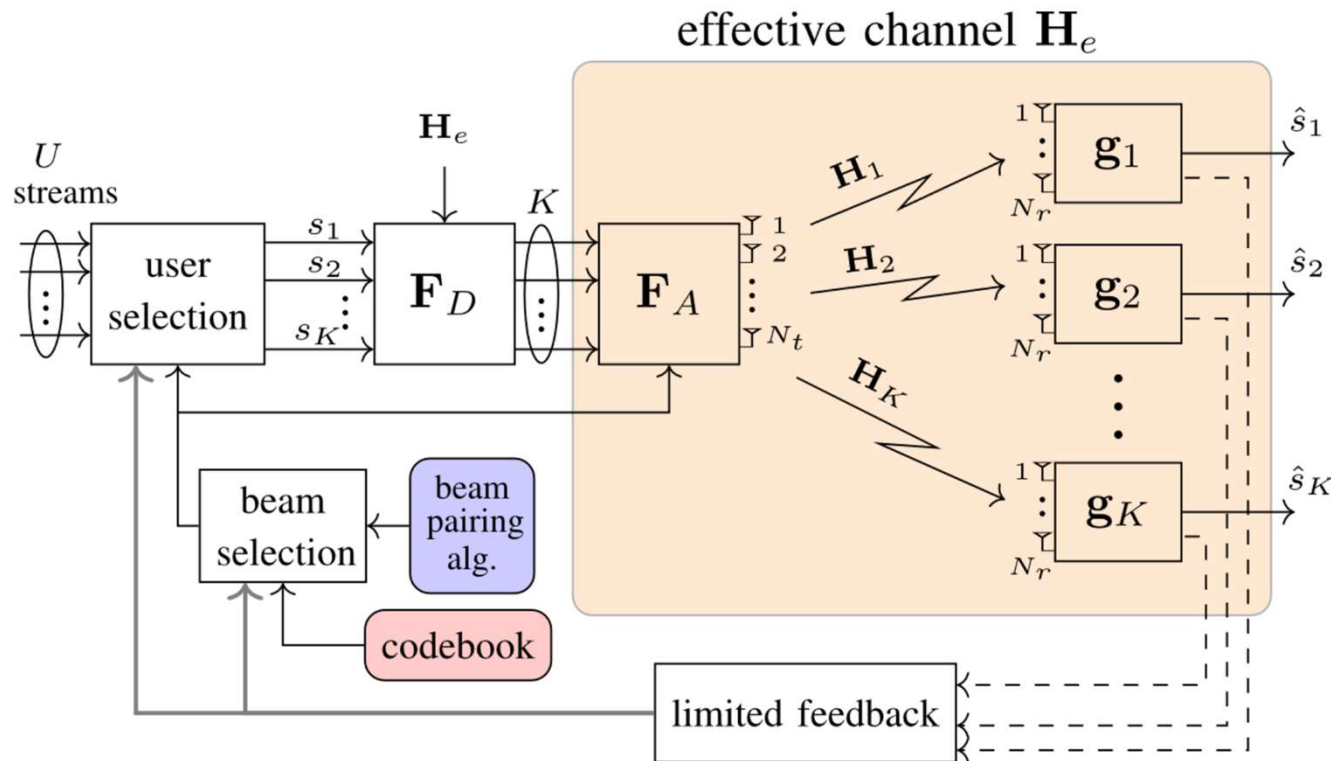


2 cells, 3 bands, 2 antennas per cell

- V. N. Moothedath and S. Bhashyam, "Distributed Pareto Optimal Beamforming for the MISO Multi-band Multi-cell Downlink," in IEEE Transactions on Wireless Communications, vol. 19, no. 11, pp. 7196-7209, Nov. 2020.

# Hybrid beamforming with partial channel knowledge

Top- $p$  beam information for each user

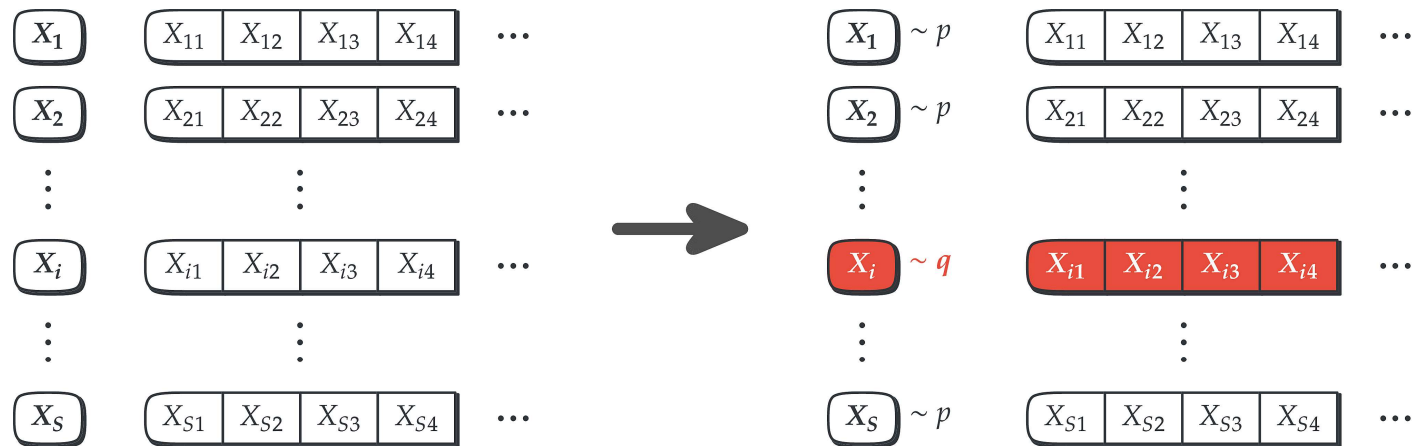


- Digital: Linear and Nonlinear precoding
- Analog: DFT vs Taylor codebook
- Robust precoding

- S. S. Nair and S. Bhashyam, "Hybrid beamforming in MU-MIMO using partial interfering beam feedback," in IEEE Communications Letters, vol. 24, no. 7, pp. 1548-1552, July 2020.
- S. S. Nair and S. Bhashyam, "Robust Nonlinear Precoding in MU-MIMO using Partial Interfering Beam Feedback," 2023 IEEE Wireless Communications and Networking Conference (WCNC), Glasgow, United Kingdom, 2023, pp. 1-6.

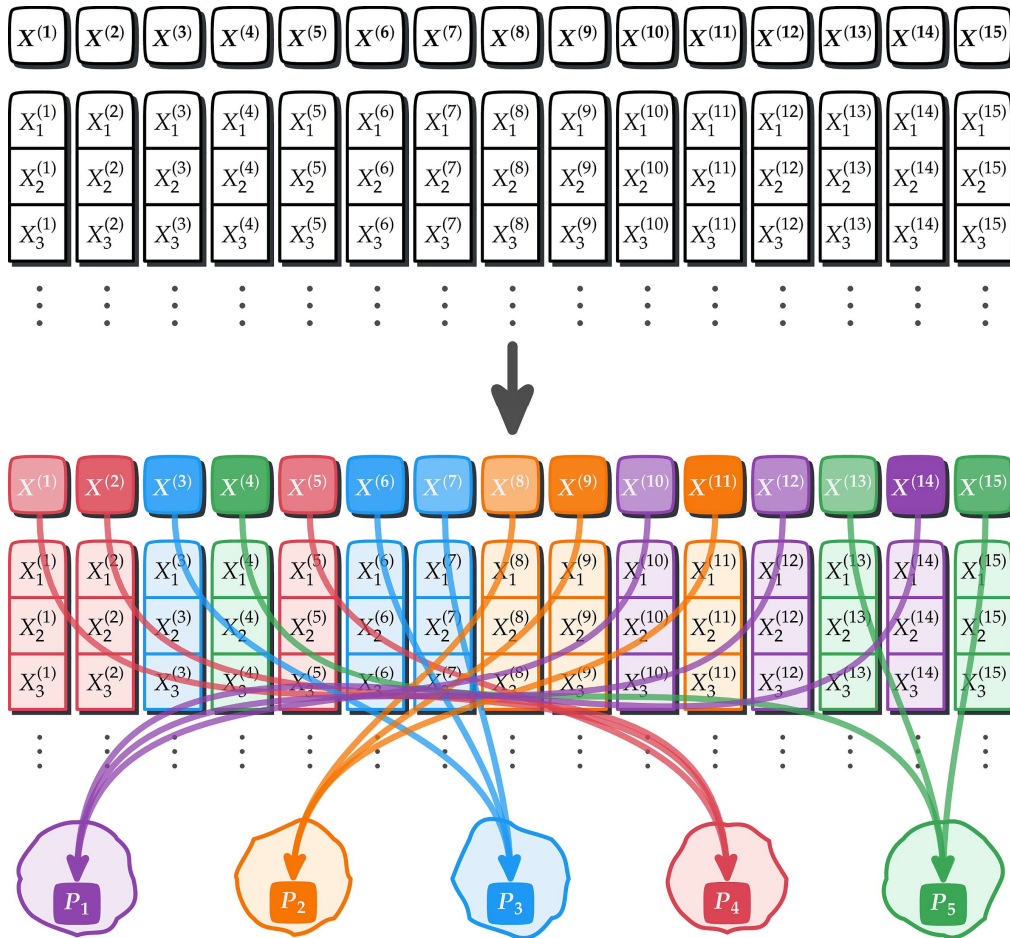
# Sequential hypothesis testing in Multi-Armed Bandits

- Anomaly detection → Generalized hypothesis testing
- Parametric setting: Vector exponential family
- Active sampling under constraints



- G. R. Prabhu, S. Bhashyam, A. Gopalan and R. Sundaresan, "Sequential Multi-Hypothesis Testing in Multi-Armed Bandit Problems: An Approach for Asymptotic Optimality," in IEEE Transactions on Information Theory, vol. 68, no. 7, pp. 4790-4817, July 2022.
- Aditya Deshmukh, Venugopal V. Veeravalli & Srikrishna Bhashyam (2021) Sequential controlled sensing for composite multihypothesis testing, Sequential Analysis, 40:2, 259-289.

# Sequential hypothesis testing



- Nonparametric setting
- Anomaly detection & Clustering

- S. C. Sreenivasan and S. Bhashyam, "Sequential Nonparametric Detection of Anomalous Data Streams," in IEEE Signal Processing Letters, vol. 28, pp. 932-936, 2021.
- S. C. Sreenivasan, S. Bhashyam, Nonparametric Sequential Clustering of Data Streams with Composite Distributions, Signal Processing (2022).

**Model-based learning for wireless  
communication:  
Learning-based sparse recovery for  
massive random access**



# Model-based learning for wireless communication

## Model-based signal processing

Domain knowledge

Analysis and interpretation

## Deep learning

Data-driven, uses large data sets

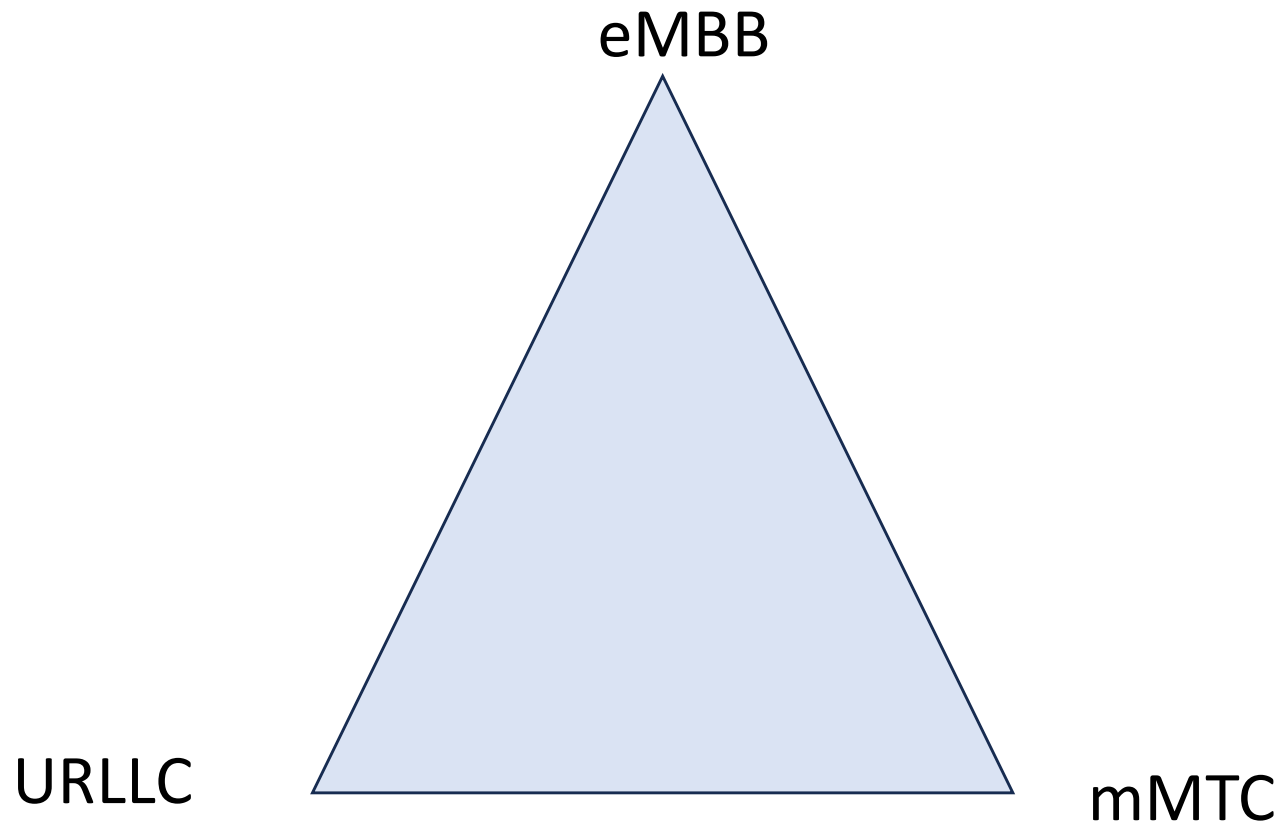
Not easy to interpret or analyse

## Model-based learning: Hybrid approach

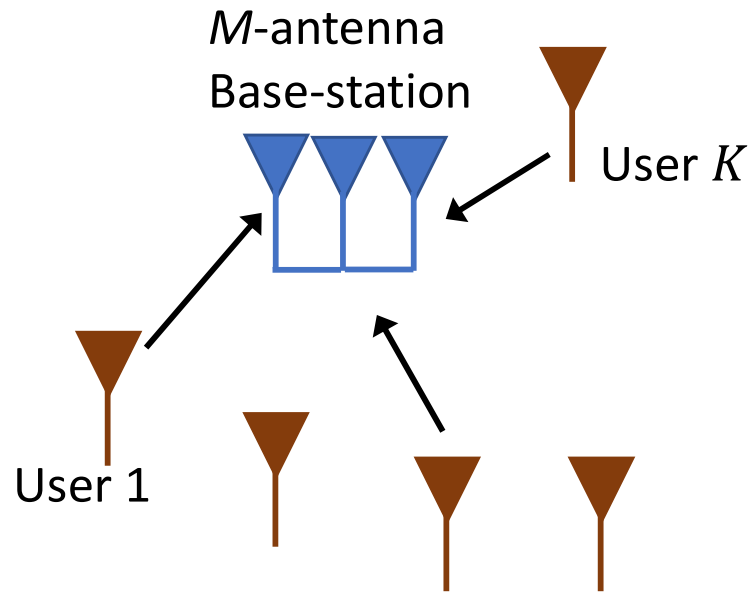
- Deep unfolding
- Model-aided networks

# 5G Cellular Systems

- eMBB: Enhanced Mobile Broadband
- mMTC: Massive Machine Type Communication
- URLLC: Ultra Reliable Low Latency Communication



# Massive Random Access for mMTC

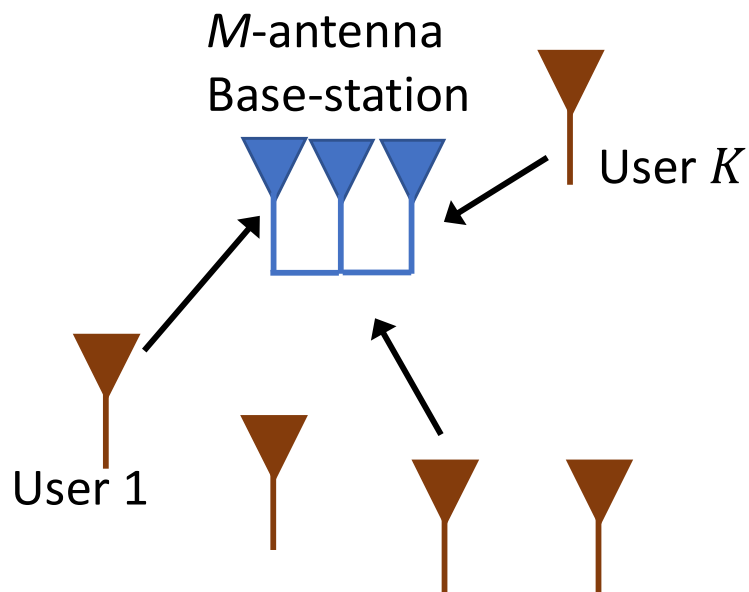


Grant-free  
random access

Multiple  
antennas at BS

- Small fraction of users are active at any given time
- Active users have limited data to send
- Overhead to grant resources large

# Massive Random Access: Our Setting



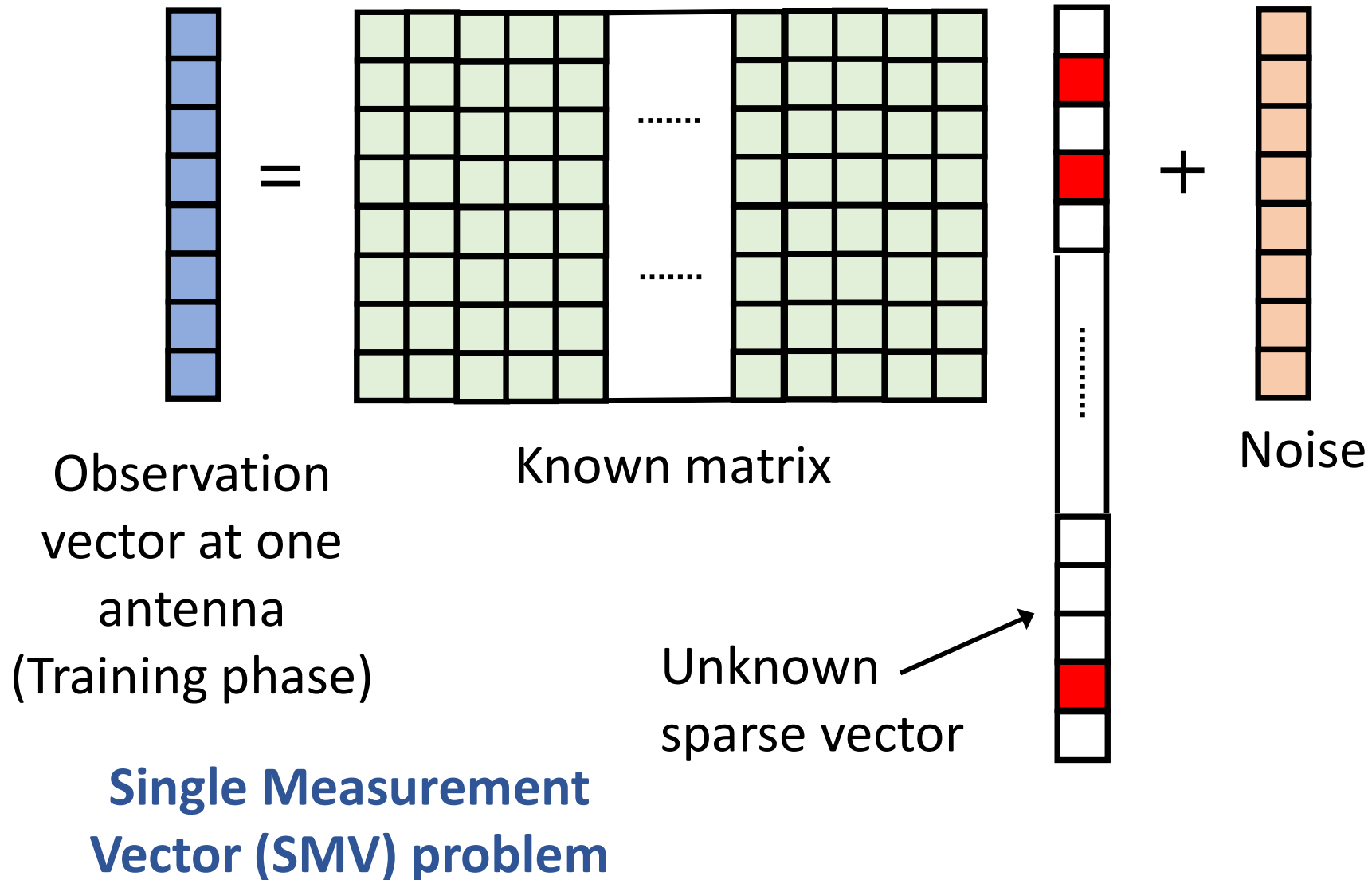
Training phase  
followed by data

- Small fraction of users are active at any given time
- Identify the active users
- Estimate channel corresponding to the active users

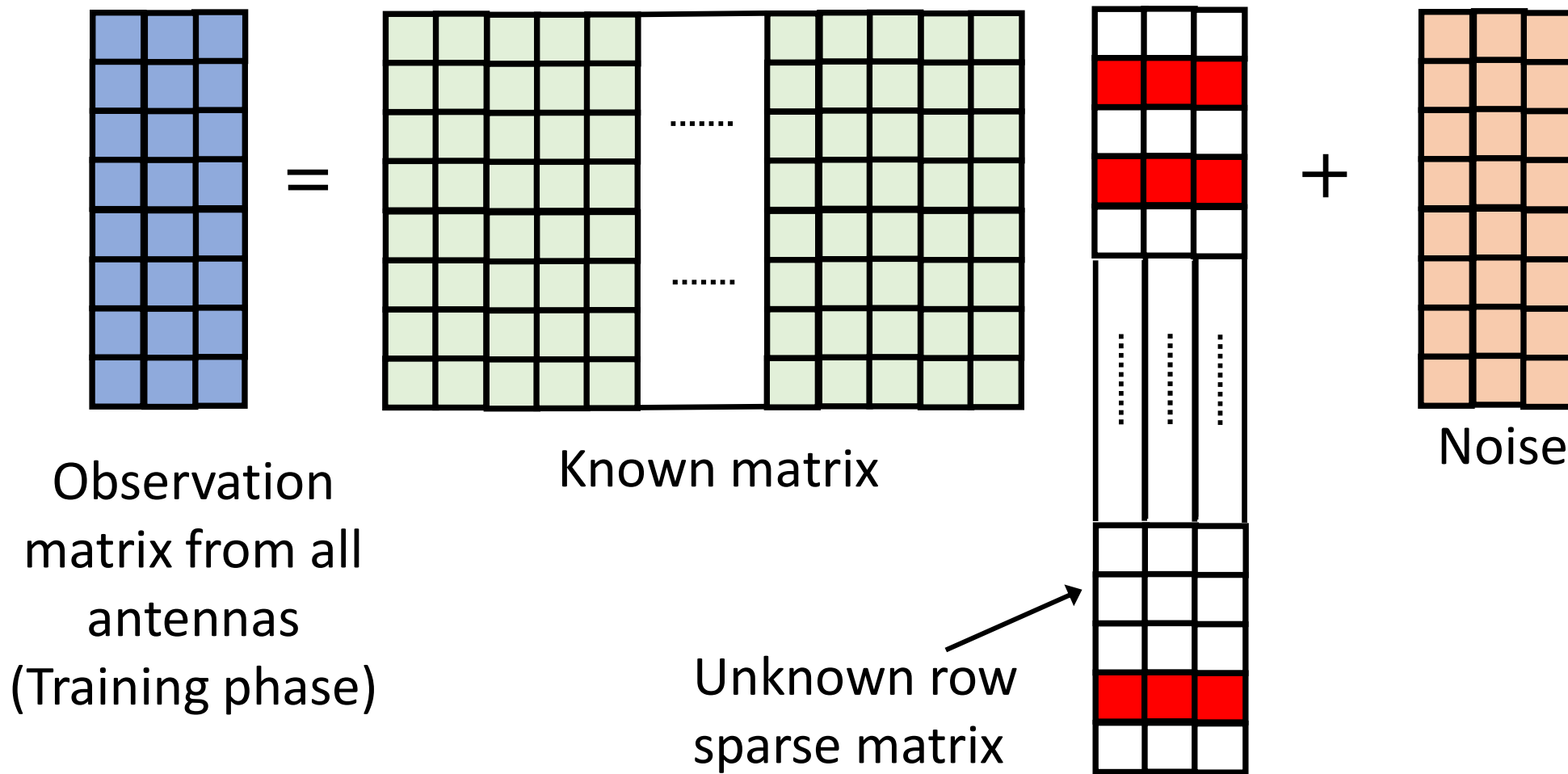


- Active users send training sequences of length  $L$

# Sparse recovery: Activity detection and Channel estimation



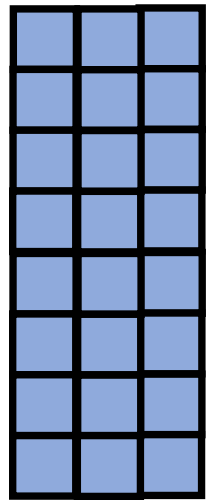
# Joint sparse recovery



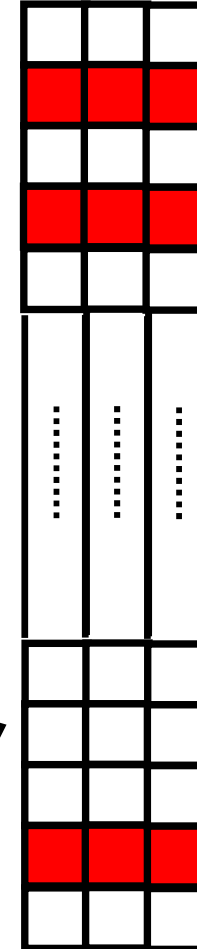
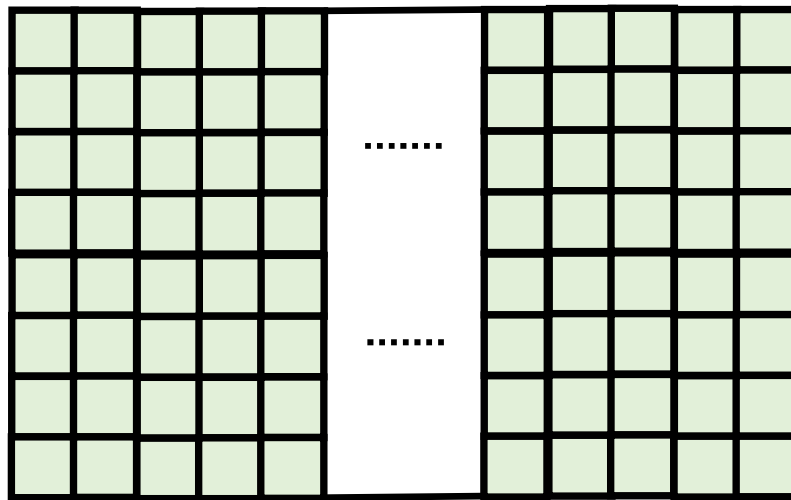
**Multiple Measurement Vector (MMV) problem**

# Joint Sparse Recovery: Activity detection and Channel estimation

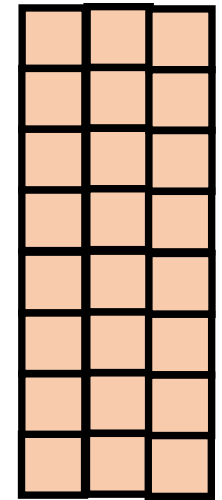
$$Y = A X + N$$



=



+



Observation  
Matrix  $Y$   
 $L \times M$

Training sequence  
matrix  $A$   
 $L \times K$

Row sparse  
channel matrix  $X$   
 $K \times M$

Noise  $N$

**Multiple Measurement  
Vector (MMV) problem**

# Plan

- Some sparse recovery methods
- Learning-based sparse recovery
  - Two proposed methods
- Results and discussion

A. P. Sabulal, S. Bhashyam, "Joint Sparse Recovery using Deep Unfolding With Application to Massive Random Access," ICASSP 2020 - 2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Barcelona, Spain, 2020, pp. 5050-5054.

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# Sparse recovery

Iterative soft  
thresholding

ISTA

Approximate  
message  
passing

AMP  
OAMP  
Vector AMP

Matching  
Pursuit

OMP  
CoSaMP

Alternating  
direction  
method of  
multipliers

MMV-ADM

Sparse  
Bayesian  
learning

SBL  
M-SBL

# Learning-based sparse recovery

Iterative soft  
thresholding

ISTA

LISTA

TISTA, MMV-TISTA

L-MMSE-MMV-TISTA

Approximate  
message  
passing

AMP

OAMP

Vector AMP

L-AMP

VAMP-net

Matching  
Pursuit

OMP

CoSaMP

Alternating  
direction  
method of  
multipliers

MMV-ADM

MMV-MADM

L-MMV-MADM

Sparse  
Bayesian  
learning

SBL

M-SBL

L-SBL

# Our Work

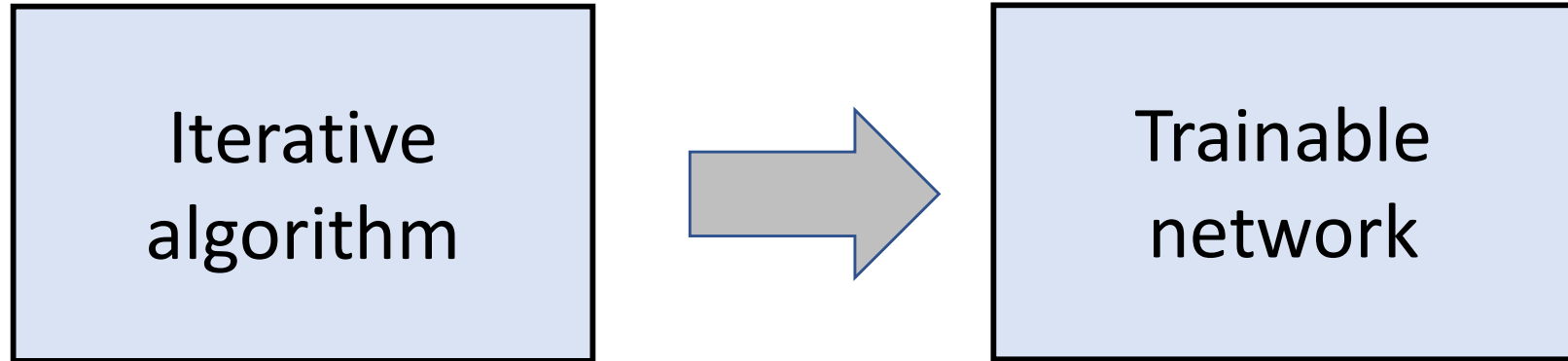
- Proposed methods
  - MMV-MADM and LMMV-MADM
    - Uses deep unfolding, modified cost
  - MMV-TISTA and learnt version
    - Replaces denoiser with a model-based neural network
- New comparisons
  - Performance-complexity trade-offs

A. P. Sabulal, S. Bhashyam, "Joint Sparse Recovery using Deep Unfolding With Application to Massive Random Access," ICASSP 2020 - 2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), Barcelona, Spain, 2020, pp. 5050-5054.

U. K. Sreeshma Shiv, S. Bhashyam, C. R. Srivatsa and C. R. Murthy, "Learning-Based Sparse Recovery for Joint Activity Detection and Channel Estimation in Massive Random Access Systems," in IEEE Wireless Communications Letters, vol. 11, no. 11, pp. 2295-2299, Nov. 2022

# Deep Unfolding

# Technique 1: Deep unfolding



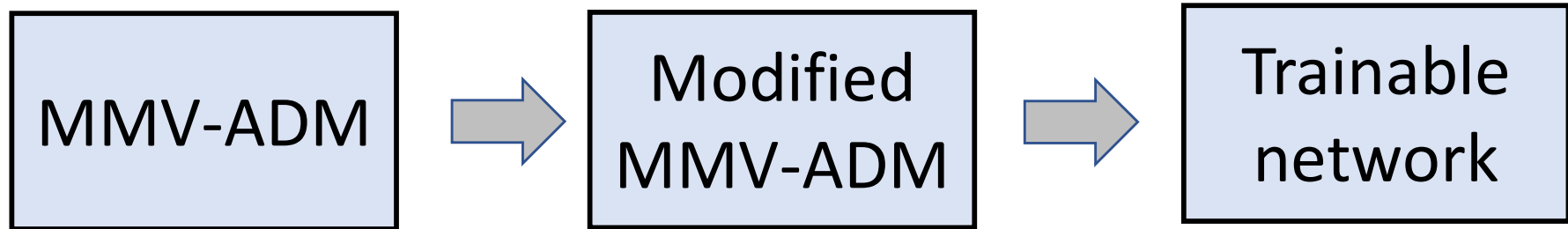
- Each iteration is a layer
- Parameters in each layer untied and trained

John R. Hershey, Jonathan Le Roux, and Felix Weninger, "Deep unfolding: Model-based inspiration of novel deep architectures," CoRR, vol. abs/1409.2574, 2014.

Alexios Balatsoukas-Stimming and Christoph Studer, "Deep unfolding for communications systems: A survey and some new directions," arXiv preprint arXiv:1906.05774, 2019.

V. Monga, Y. Li, and Y. C. Eldar, "Algorithm unrolling: Interpretable, efficient deep learning for signal and image processing," IEEE Signal Processing Magazine, vol. 38, no. 2, pp. 18–44, 2021.

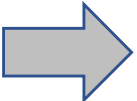
# Proposed method: LMMV-MADM



- MMV-ADM
  - Based on alternating direction method of multipliers
- Modification of existing algorithm to help learning
  - Back-projected error
- Unfolding: **Significant reduction in training overhead**
- Two learning approaches: Supervised, **Unsupervised**

# MMV-ADM

$$\min_{\mathbf{X}} \|\mathbf{X}\|_{2,1} + \frac{1}{2\mu} \|\mathbf{Y} - \mathbf{A}\mathbf{X}\|_2^2$$

- Alternating direction method
- No matrix inversions  fast, scalable
- Convergence analysis feasible

# MMV-ADM

$$\min_{\mathbf{X}} \|\mathbf{X}\|_{2,1} + \frac{1}{2\mu} \|\mathbf{Y} - \mathbf{A}\mathbf{X}\|_2^2$$

$$\min_{\mathbf{X}} \|\mathbf{X}\|_{2,1} + \frac{1}{2\mu} \|\mathbf{E}\|_2^2 \quad s. t. \quad \mathbf{A}\mathbf{X} + \mathbf{E} = \mathbf{Y}$$

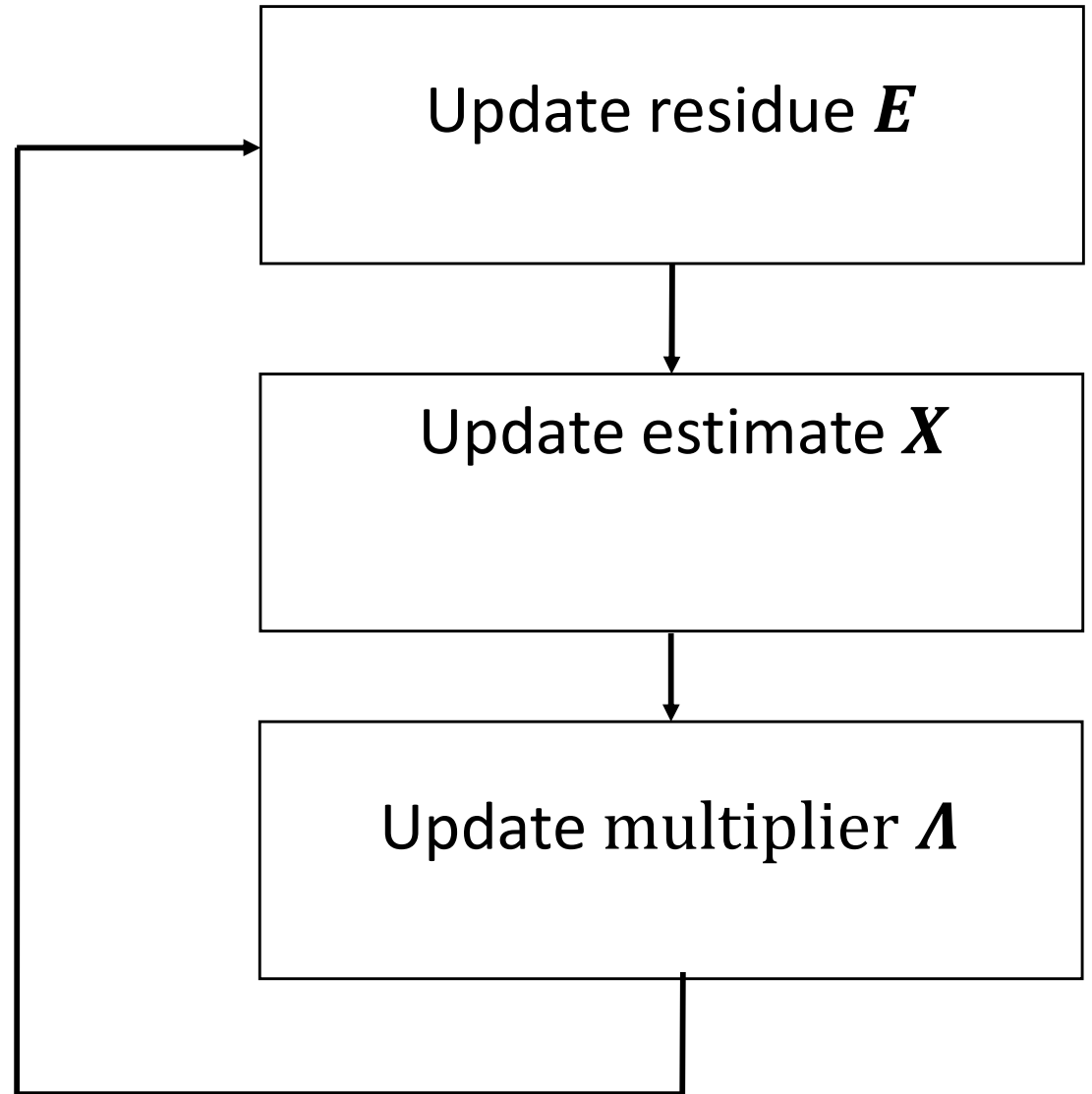
- $\mathbf{E} = \mathbf{Y} - \mathbf{A}\mathbf{X}$
- Augmented Lagrangian

$$\begin{aligned} L(\mathbf{X}, \mathbf{E}, \Lambda) &= \|\mathbf{X}\|_{2,1} + \frac{1}{2\mu} \|\mathbf{E}\|_2^2 - \langle \Lambda, \mathbf{A}\mathbf{X} + \mathbf{E} - \mathbf{Y} \rangle \\ &+ \frac{\beta}{2} \|\mathbf{A}\mathbf{X} + \mathbf{E} - \mathbf{Y}\|_F^2 \end{aligned}$$



# MMV-ADM

- Augmented Lagrangian  
 $L(\mathbf{X}, \mathbf{E}, \boldsymbol{\Lambda})$
- $\mathbf{E} = \mathbf{Y} - \mathbf{A}\mathbf{X}$
- Initialize  $\mathbf{X}, \boldsymbol{\Lambda}$



# MMV-ADM

Initialize  $\Lambda, \mathbf{X}$

Update residue  $\mathbf{E}^{k+1} = \frac{\mu\beta}{1 + \mu\beta} \left[ \frac{1}{\beta} \Lambda^k - (\mathbf{A}\mathbf{X}^k - \mathbf{Y}) \right]$

$$\mathbf{G}^k = \mathbf{A}^T \left[ \mathbf{A}\mathbf{X}^k + \mathbf{E}^{k+1} - \mathbf{Y} - \frac{1}{\beta} \Lambda^k \right]$$

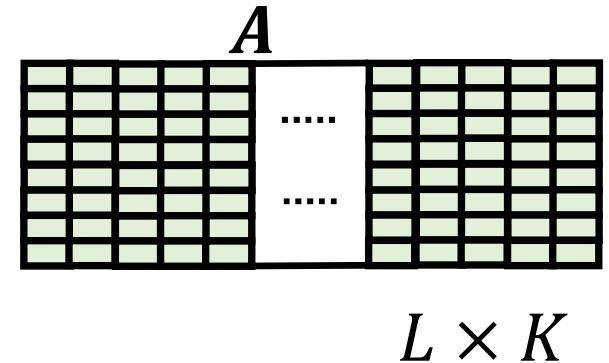
Update estimate  $\mathbf{X}^{k+1} = \text{Row\_shrink} \left[ \mathbf{X}^k - \tau \mathbf{G}^k, \frac{\tau}{\beta} \right]$

Update multiplier  $\Lambda^{k+1} = \Lambda^k - \gamma\beta [\mathbf{A}\mathbf{X}^k + \mathbf{E}^{k+1} - \mathbf{Y}]$

Parameters to choose:  $\mu, \beta, \gamma, \tau$

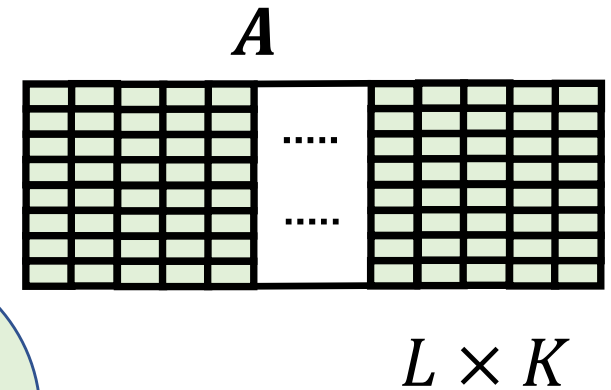
# Modified MMV-ADM

$$\min_X \|X\|_{2,1} + \frac{1}{2\mu} \|A^\dagger Y - A^\dagger AX\|_2$$



- Backprojected LS error instead of LS error
  - $A^\dagger Y - A^\dagger AX$  instead of  $Y - AX$
  - $A^\dagger = A^T [AA^T]^{-1}$
- Modified algorithm also fast, scalable
- Unfolding results in a easily trainable network

# Back-projected error



$$\|A\mathbf{x}_0 - A\mathbf{x}\|_2^2 = \sum_{i=1}^L \lambda_i^2 |\mathbf{v}_i^T (\mathbf{x}_0 - \mathbf{x})|^2$$

$$\|A^\dagger(A\mathbf{x}_0 - A\mathbf{x})\|_2^2 = \sum_{i=1}^L |\mathbf{v}_i^T (\mathbf{x}_0 - \mathbf{x})|^2$$

$$\|\mathbf{x}_0 - \mathbf{x}\|_2^2 = \sum_{i=1}^K |\mathbf{v}_i^T (\mathbf{x}_0 - \mathbf{x})|^2$$

- $\lambda_i$ :  $i$  th singular value of  $A$
- $\mathbf{v}_i$ :  $i$  th right singular vector of  $A$

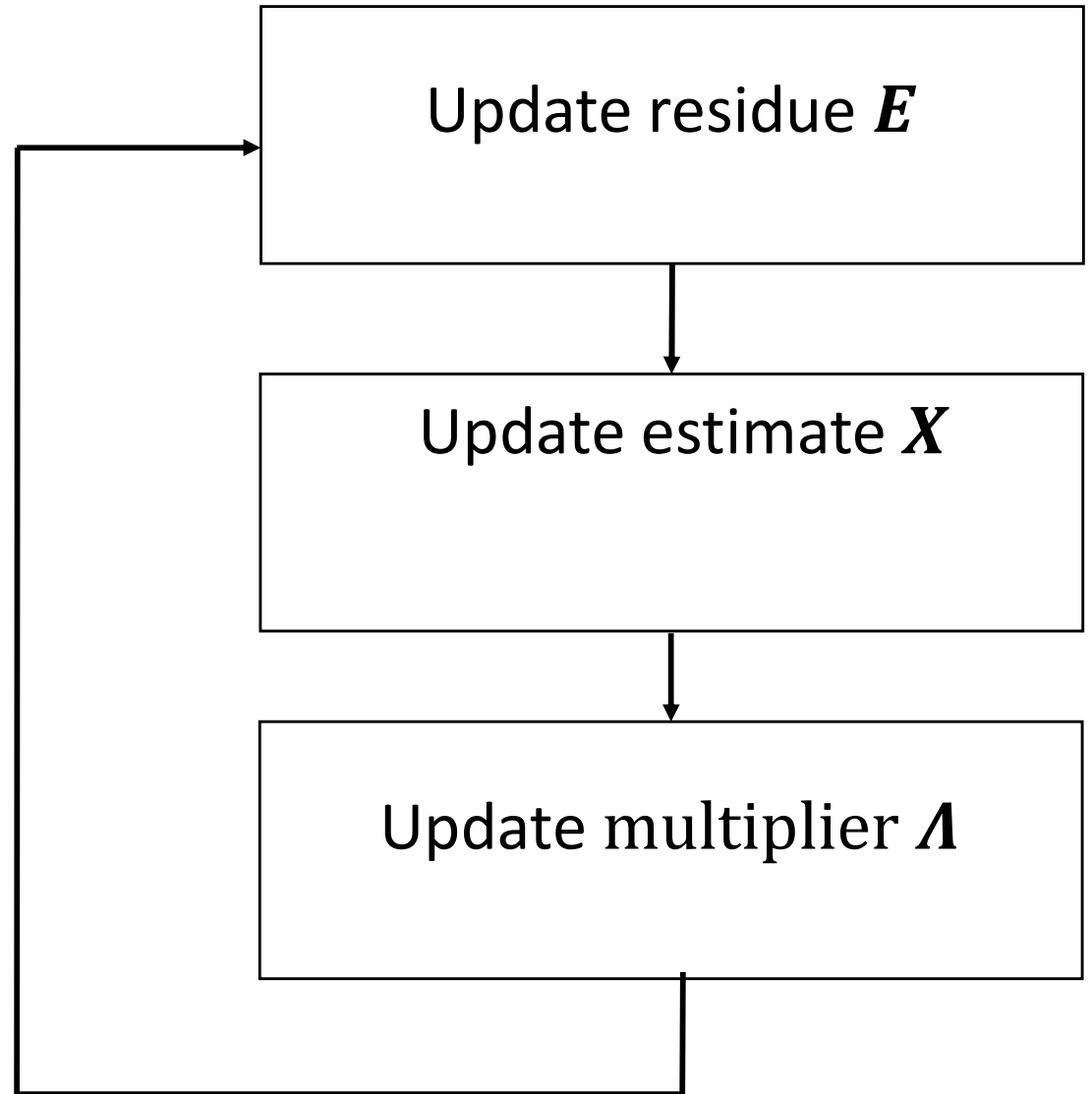
# Modified MMV-ADM

- Augmented Lagrangian  
 $L(\mathbf{X}, \mathbf{E}, \boldsymbol{\Lambda})$

- $\mathbf{E} = \mathbf{A}^\dagger \mathbf{Y} - \mathbf{A}^\dagger \mathbf{A} \mathbf{X}$

- Initialize  $\mathbf{X}, \boldsymbol{\Lambda}$

4 scalar parameters to  
choose:  $\mu, \beta, \gamma, \tau$



# MMV-MADM

Initialize  $\Lambda, \mathbf{X}$

Update residue 
$$\tilde{\mathbf{E}}^{k+1} = \frac{\mu\beta}{1 + \mu\beta} \left[ \frac{1}{\beta} \tilde{\Lambda}^k - (\mathbf{A}\mathbf{X}^k - \mathbf{Y}) \right]$$

$$\mathbf{G}^k = \mathbf{A}^\dagger \left[ \mathbf{A}\mathbf{X}^k + \tilde{\mathbf{E}}^{k+1} - \mathbf{Y} - \frac{1}{\beta} \tilde{\Lambda}^k \right]$$

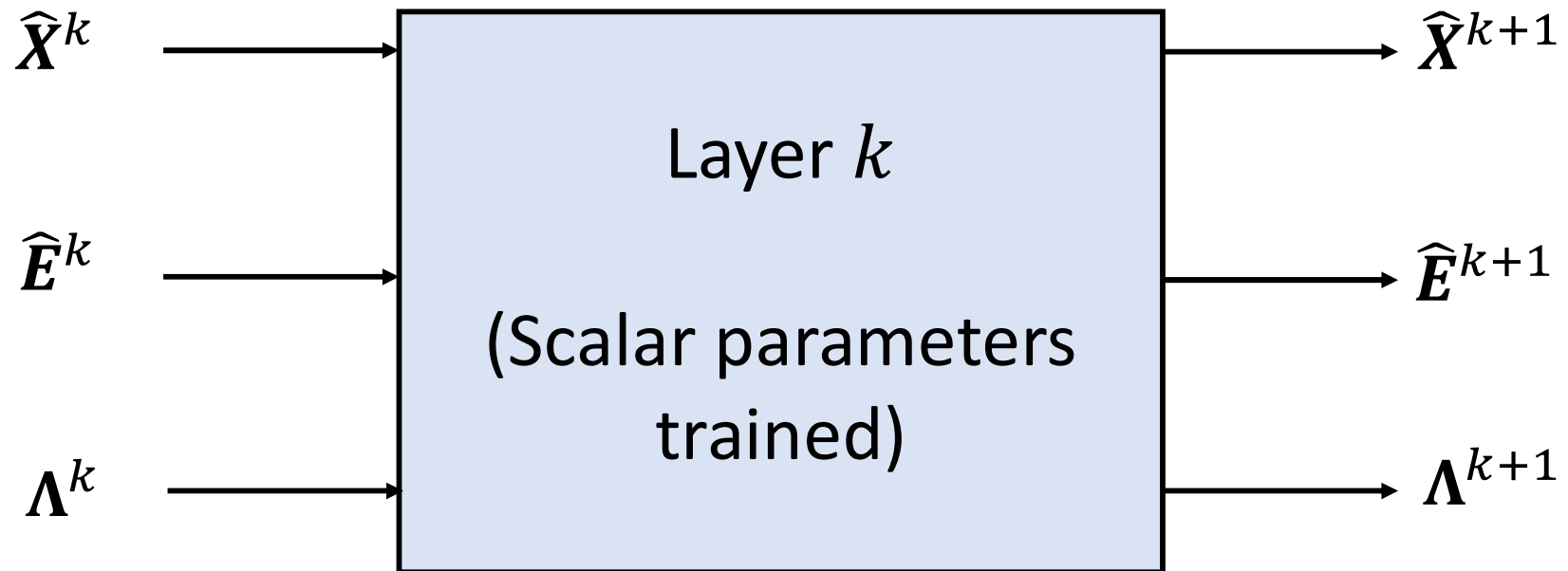
Update estimate 
$$\mathbf{X}^{k+1} = \text{Row\_shrink} \left[ \mathbf{X}^k - \tau \mathbf{G}^k, \frac{\tau}{\beta} \right]$$

Update multiplier 
$$\tilde{\Lambda}^{k+1} = \tilde{\Lambda}^k - \gamma\beta [\mathbf{A}\mathbf{X}^k + \tilde{\mathbf{E}}^{k+1} - \mathbf{Y}]$$

$$\tilde{\Lambda} = \mathbf{A}\Lambda, \tilde{\mathbf{E}} = \mathbf{A}\mathbf{E}$$

Parameters to choose:  $\mu, \beta, \gamma, \tau$

# Unfolded network



- One iteration of ADM algorithm is one layer

# Training the network: Supervised

- True  $X, Y$  pairs available
  - Generated using a channel model for training
- 
- Layers trained sequentially
  - MSE between layer output  $\hat{X}^{k+1}$  and true  $X$  used as loss function for training



# Training the network: Unsupervised

- True  $X, Y$  pairs not needed

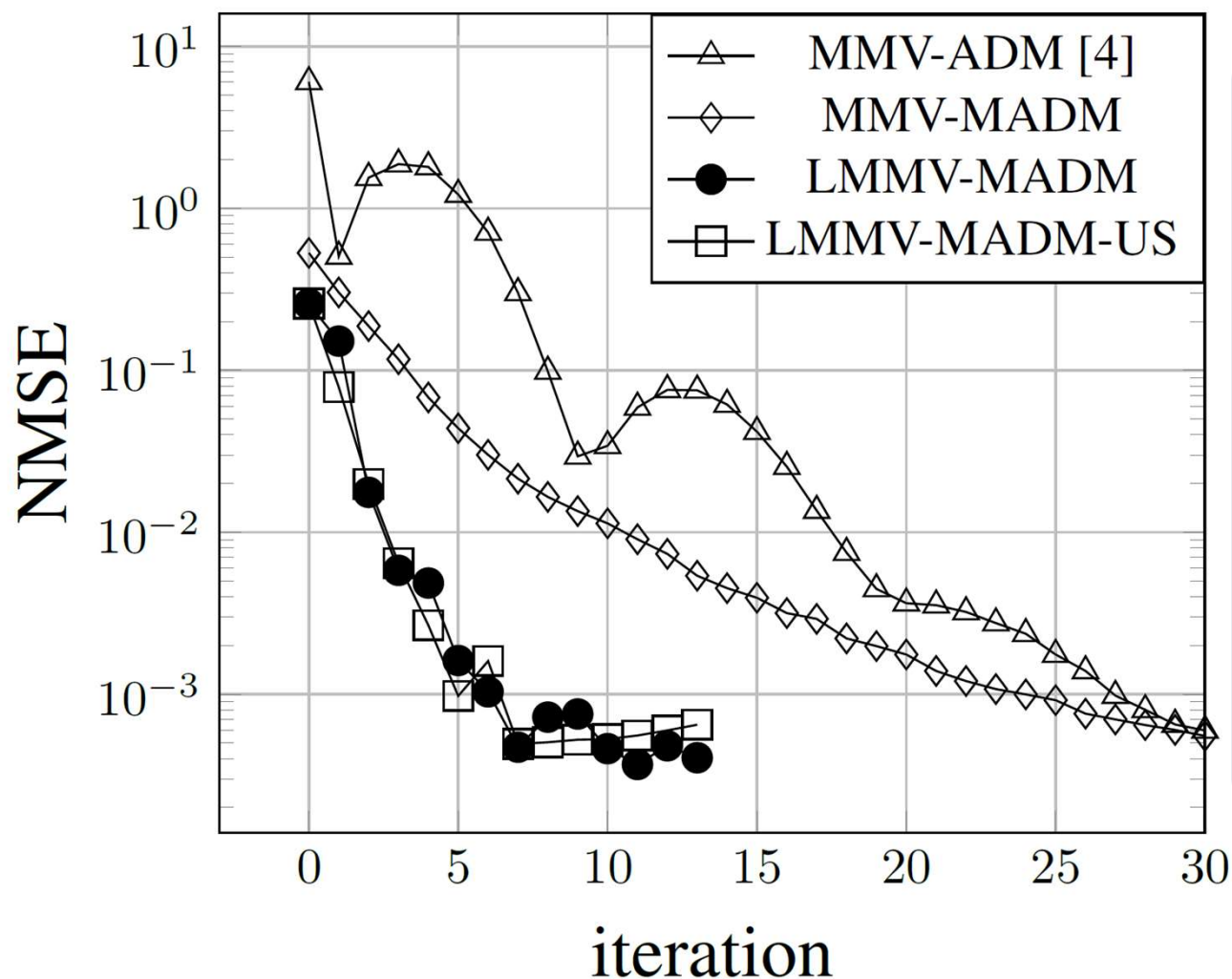
- Layers trained sequentially

- Loss function for training

- $\lambda \|\hat{X}^{k+1}\|_{2,1/p}^{1/p} + \|Y - A\hat{X}^{k+1}\|_F^2$

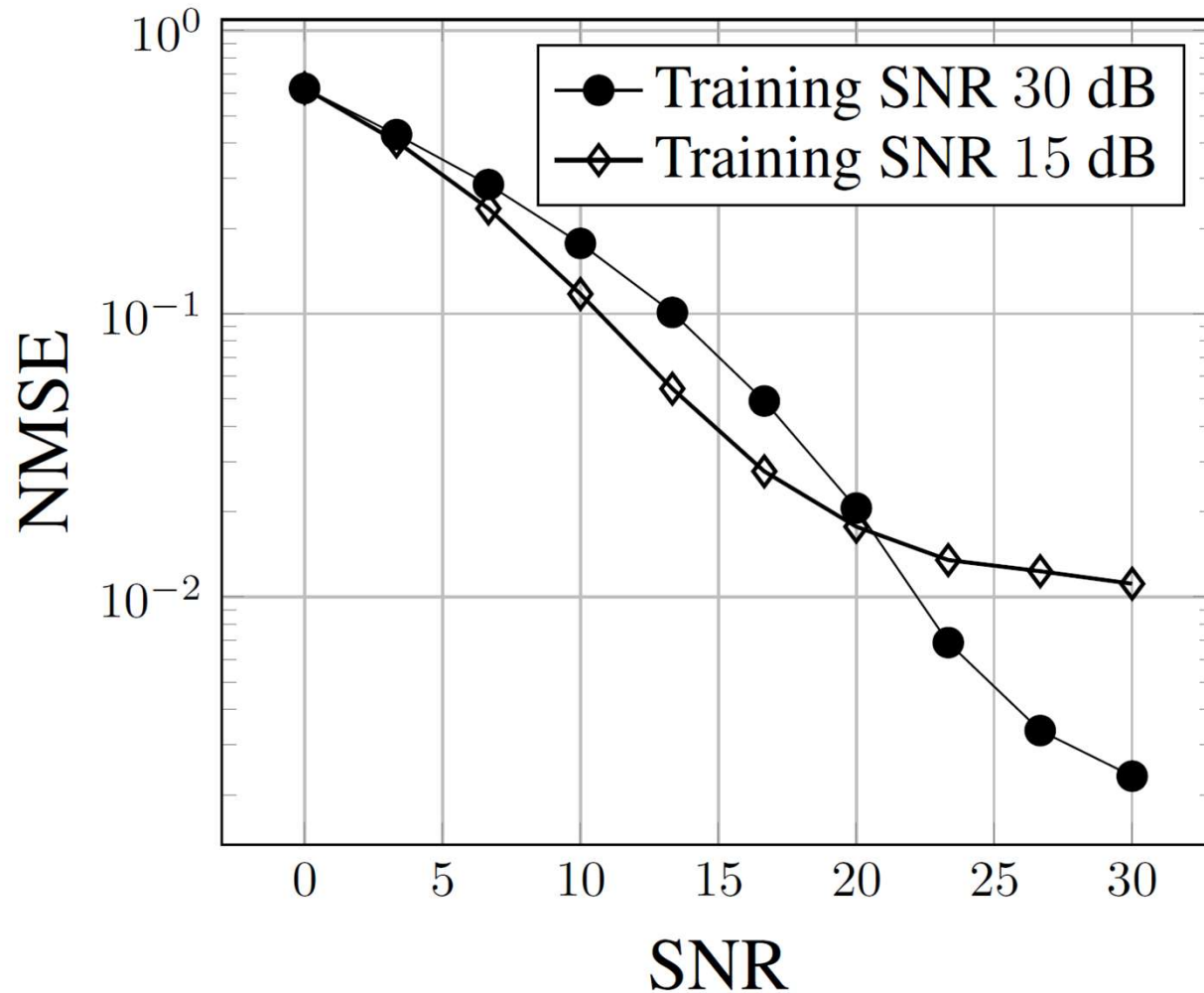
- Unsupervised method can be used after initialization with the supervised method

# Performance



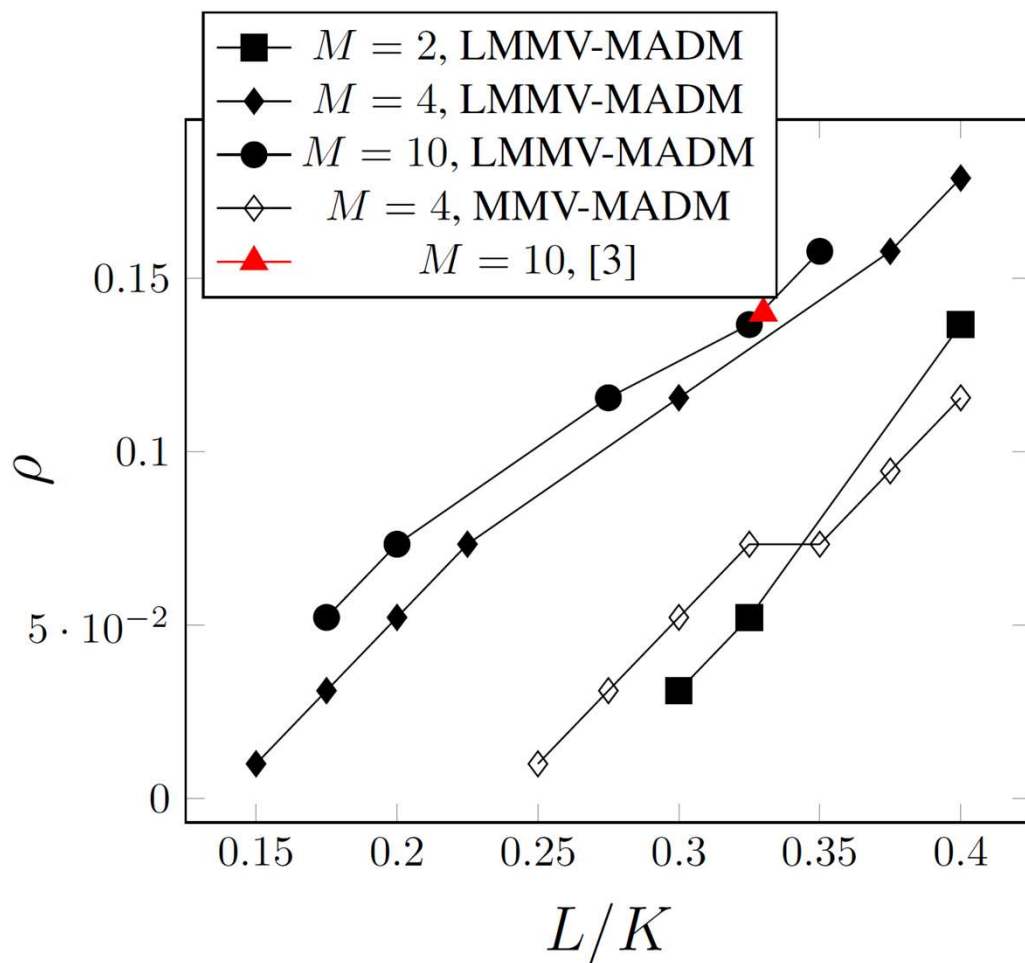
- $K = 2000, L = 500, \rho = 0.05, M = 10$
- Use of back-projected error results in smooth convergence
- Unfolded network converges faster
- Unsupervised training also works well

# Training SNR



- $K = 2000, L = 300, \rho = 0.05, M = 10$
- Training at  $\rho = 0.07$
- Network trained at higher SNR preferable

# Performance: Phase transition



- $K = 500$ , 20 layer network, MMV-MADM with 40 iterations
- Minimum  $L/K$  for a given activity probability  $\rho$
- Training and test SNR at 30 dB
- Training at  $\rho = 0.2$
- Success if NMSE < -20 dB

[3] T. Jiang, Y. Shi, J. Zhang, and K. B. Letaief, "Joint activity detection and channel estimation for IoT networks: Phase transition and computation-estimation tradeoff," IEEE Internet of Things Journal, vol. 6, no. 4, pp. 6212–6225, Aug 2019.

# **Model-based Neural Network for Denoising**

# Trainable ISTA (TISTA)

$\mathbf{S}_t$ : Estimate of  $\mathbf{X}$  at iteration  $t$

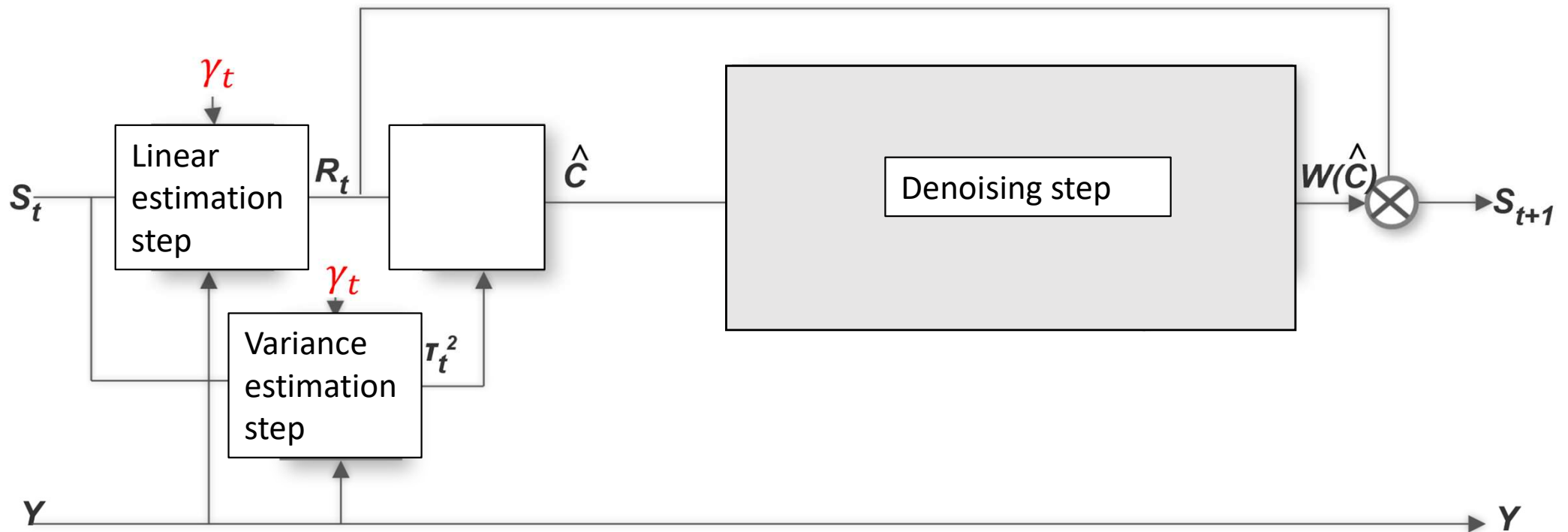
$$\mathbf{R}_t = \mathbf{S}_t + \gamma_t \mathbf{A}^\dagger (\mathbf{Y} - \mathbf{A} \mathbf{S}_t)$$

$$\mathbf{S}_{t+1} = \eta_{MMSE}(\mathbf{R}_t, \tau_t^2)$$

$\tau_t^2$ : Estimated using  $\mathbf{Y}$ ,  $\mathbf{S}_t$ ,  $\mathbf{A}$  and noise variance  $\sigma^2$

- $\gamma_t$ : Learnt from data
- Denoising step based on an **approximate model**

# Trainable ISTA (TISTA)



- Already uses deep unfolding
- Denoising step based on an **approximate model**

# Technique 2: Model-based neural network (for row-wise denoising)

$$\mathbf{y}_t = \mathbf{h}_t + \mathbf{z}_t, \quad \mathbf{Y} = [\mathbf{y}_1 \ \mathbf{y}_2 \ \dots \ \mathbf{y}_T]$$

- $\mathbf{h}_t$  : Conditionally Gaussian ( $\mathbf{0}, \mathbf{C}_\delta$ ) given parameters  $\delta$
- $\mathbf{z}_t$  : Gaussian ( $\mathbf{0}, \sigma^2 \mathbf{I}$ ) noise

- $\delta \sim \mathbf{p}(\delta)$

- MMSE estimate of  $\mathbf{h}_t = \widehat{\mathbf{W}}(\widehat{\mathbf{C}})\mathbf{y}_t$

$$\widehat{\mathbf{C}} = \frac{1}{\sigma^2} \sum_{t=1}^T \mathbf{y}_t \mathbf{y}_t^H$$

$$\widehat{\mathbf{W}}(\widehat{\mathbf{C}}) = \frac{\int \exp(\text{tr}(\mathbf{W}_\delta \widehat{\mathbf{C}}) + T \log|\mathbf{I} - \mathbf{W}_\delta|) \mathbf{W}_\delta \mathbf{p}(\delta) d\delta}{\int \exp(\text{tr}(\mathbf{W}_\delta \widehat{\mathbf{C}}) + T \log|\mathbf{I} - \mathbf{W}_\delta|) \mathbf{p}(\delta) d\delta}$$

$$\mathbf{W}_\delta = \mathbf{C}_\delta (\mathbf{C}_\delta + \sigma^2 \mathbf{I})^{-1}$$



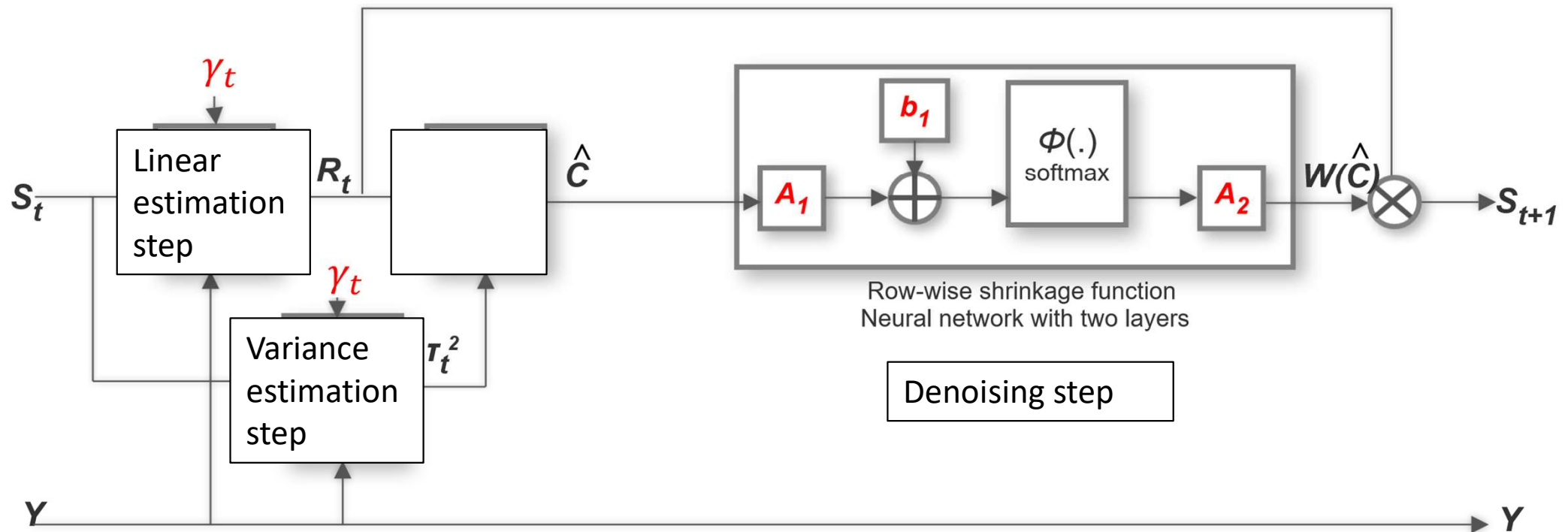
# Model-based neural network

$$\widehat{\mathbf{W}}(\widehat{\mathbf{C}}) = \frac{\sum_{i=1}^N \exp(\text{tr}(\mathbf{W}_{\delta_i} \widehat{\mathbf{C}}) + b_i) \mathbf{W}_{\delta_i} p_i}{\sum_{i=1}^N \exp(\text{tr}(\mathbf{W}_{\delta_i} \widehat{\mathbf{C}}) + b_i) p_i}$$

$$\text{vec}(\widehat{\mathbf{W}}(\widehat{\mathbf{C}})) = \mathbf{A} \frac{\exp(\text{tr}(\mathbf{A}^T \text{vec}(\widehat{\mathbf{C}})) + \mathbf{b})}{\mathbf{1}^T \exp(\text{tr}(\mathbf{A}^T \text{vec}(\widehat{\mathbf{C}})) + \mathbf{b})}$$

- MMSE estimator of  $\mathbf{h}_t$ : a two-stage neural network with linear layers and soft-max activation function
- Use a trained network for the denoising step
  - Parameters learnt from training data
  - Reduces modelling approximation error

# Trainable ISTA (TISTA) and modification

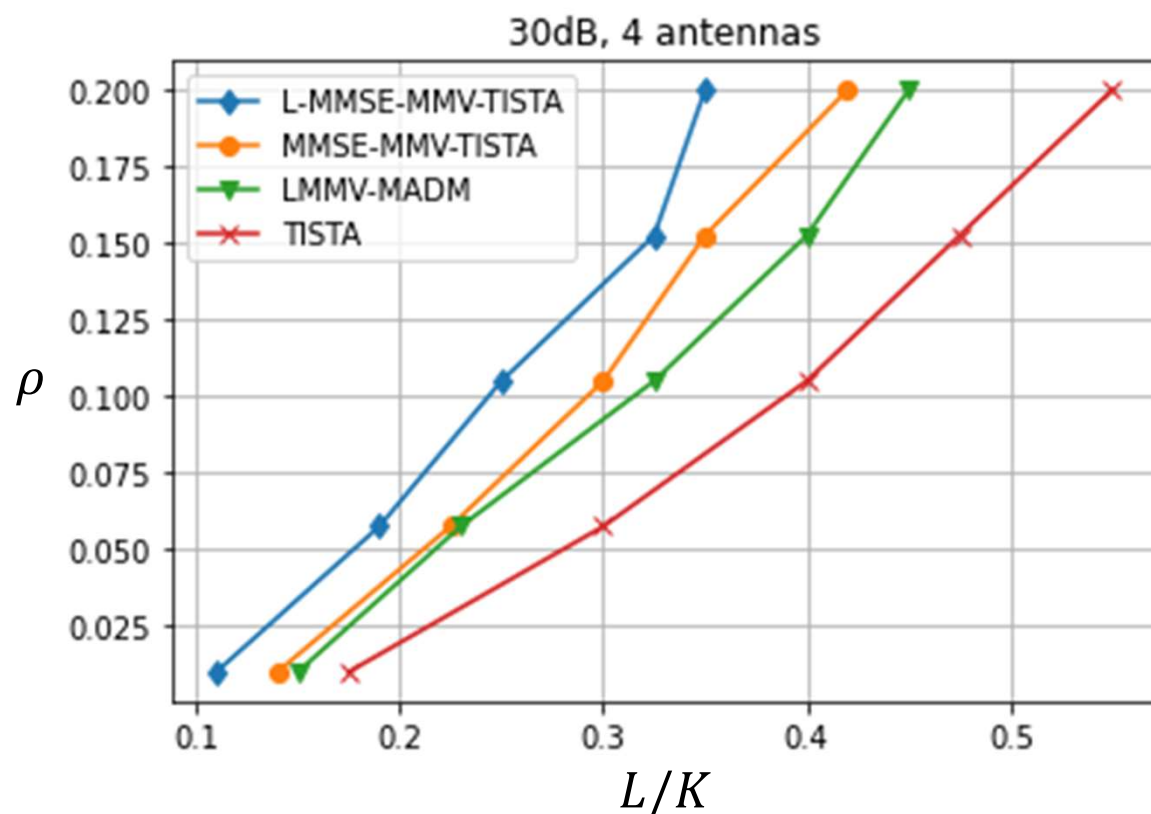


D. Ito, S. Takabe, and T. Wadayama, "Trainable ISTA for sparse signal recovery," IEEE Transactions on Signal Processing, vol. 67, no. 12, pp. 3113–3125, June 2019.

D. Neumann, T. Wiese and W. Utschick, "Learning the MMSE Channel Estimator," in IEEE Transactions on Signal Processing, vol. 66, no. 11, pp. 2905-2917, 1 June 1, 2018.

# Simulation Results

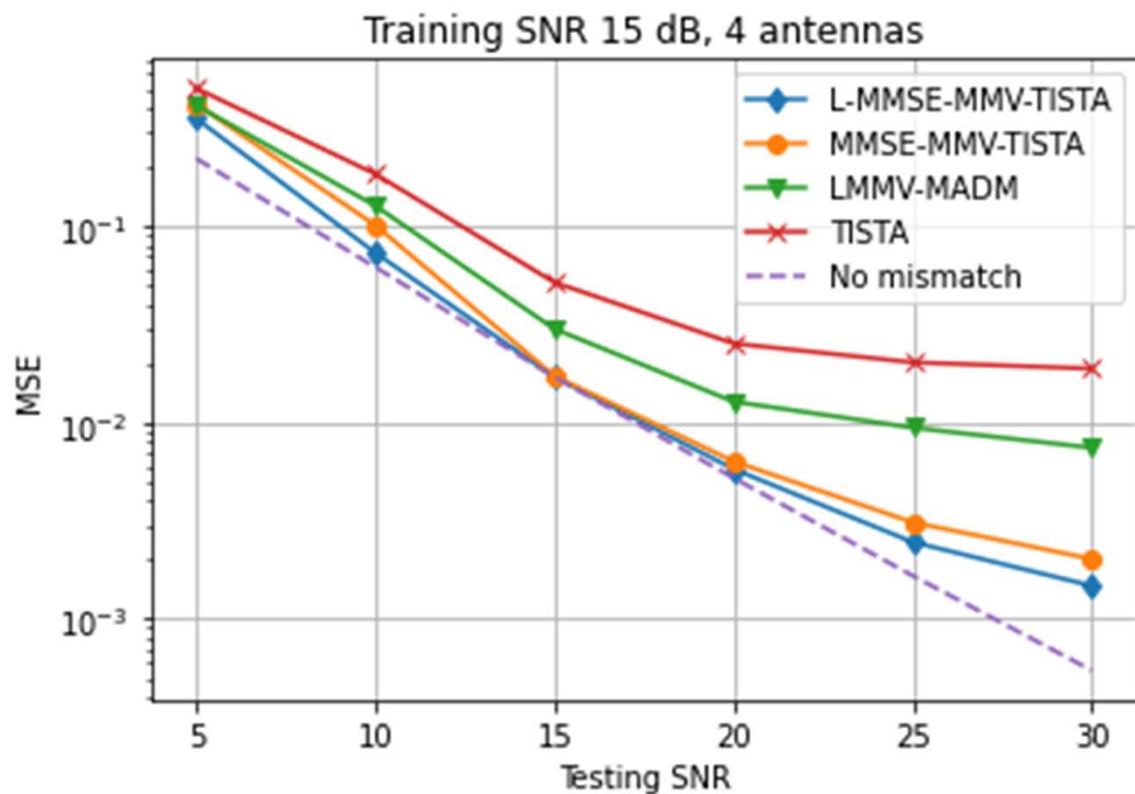
# Performance



Unfolded network needs fewer iterations  
Learnt denoiser gives better performance

- $K = 500$  users
- 12 layer network
- Minimum  $L/K$  for a given activity probability  $\rho$
- Training and test SNR at 30 dB
- Success if NMSE  $< -20$  dB
- Correlated channel

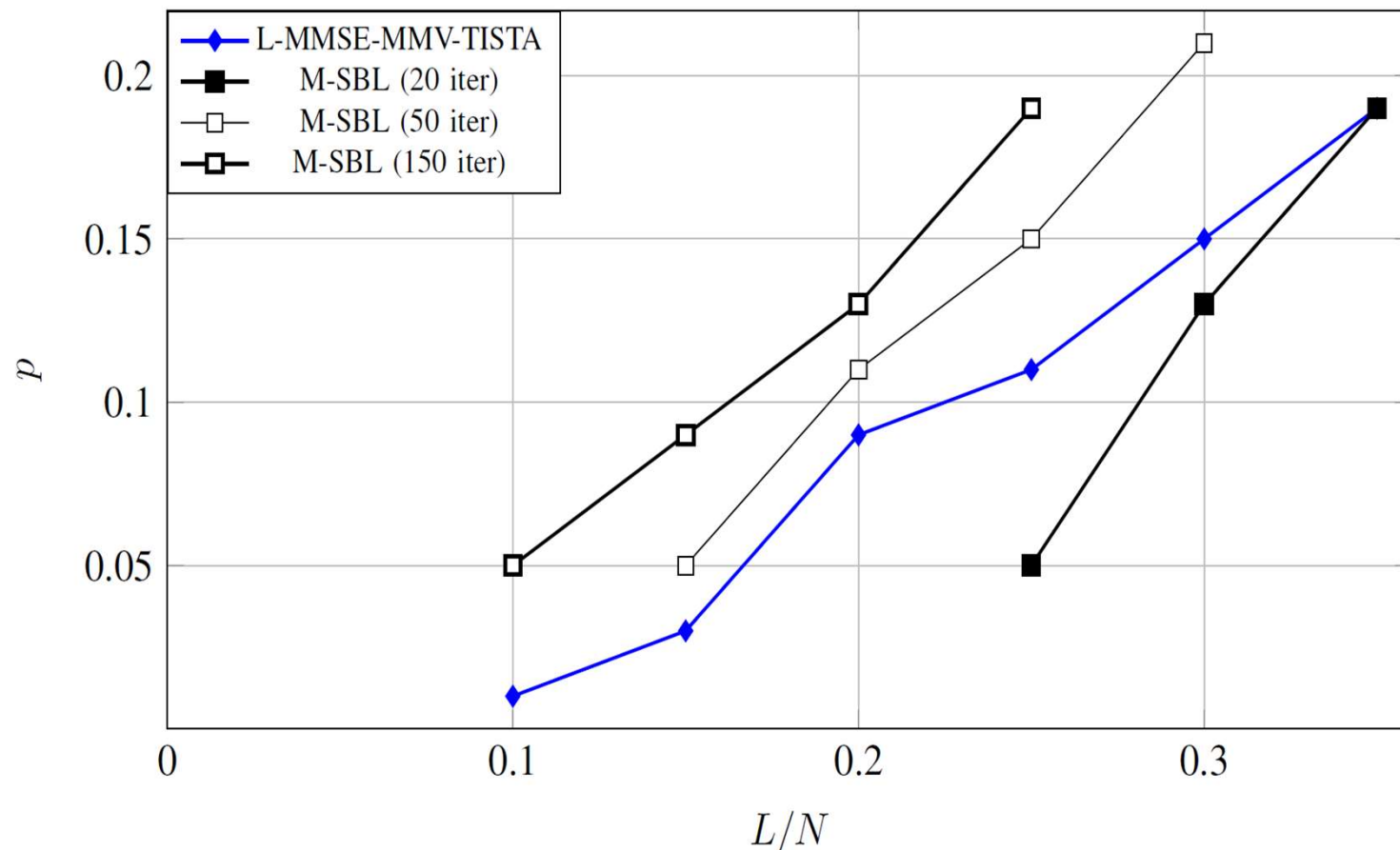
# Robustness



Robust to differences in training and testing SNR

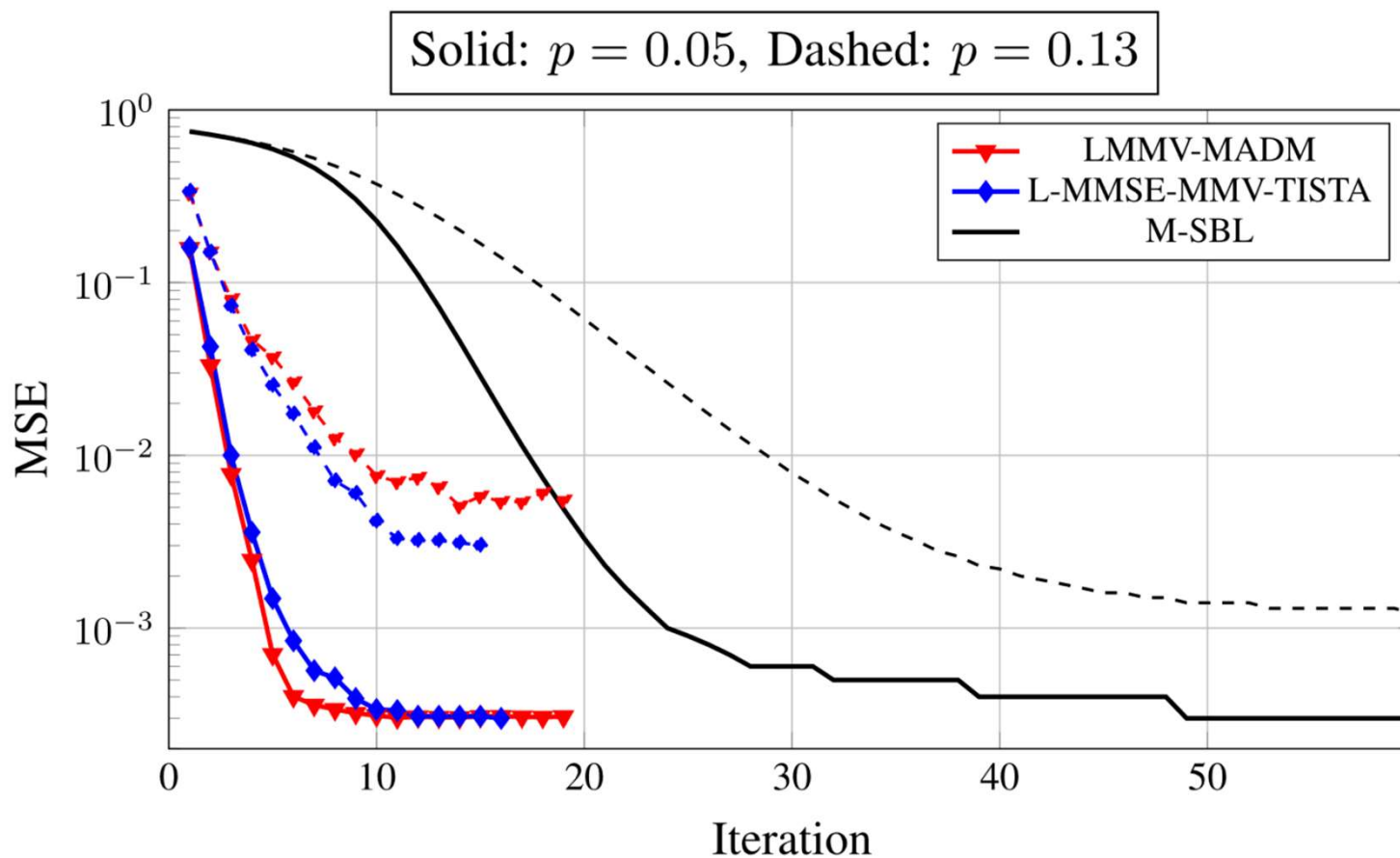
- Robustness study
- $K = 500$ , 12 layer network
- $\rho = 0.1$
- Training SNR at 15 dB
- $L = 200$ ,  $M = 4$
- Correlated channel

# Comparison with M-SBL



- 500 users, 30 dB, 4 antennas, correlated channel
- M-SBL can perform better with higher complexity

# Comparison with M-SBL



- 500 users, 30 dB, 10 antennas,  $L/N = 0.3$
- Complexity advantage for smaller  $p$

# Summary

- New learning-based sparse recovery methods
  - Back-projected error
  - Deep unfolding
  - Model-based neural network
  - Both supervised and unsupervised training
- Massive random access
  - Reduction in pilot overhead
- Ongoing work
  - Probability of error threshold
  - Large scale fading effects and estimation for MMV-TISTA
  - Unsourced random access
  - More sparsity structures

**Thank you**