Sum Capacity of the Gaussian many-to-one X channel

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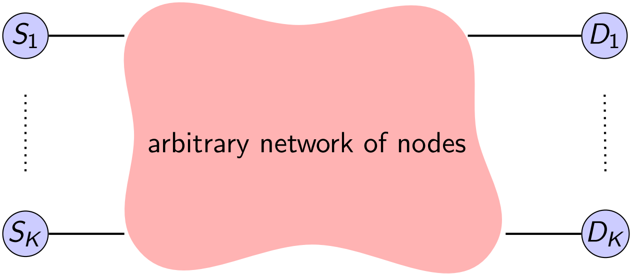
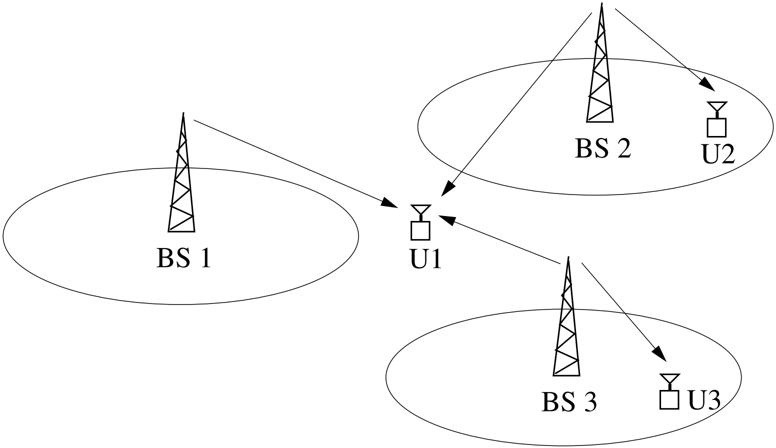
Ultimate goal: Multi-hop multi-flow wireless networks

Fundamental limits: Capacity region



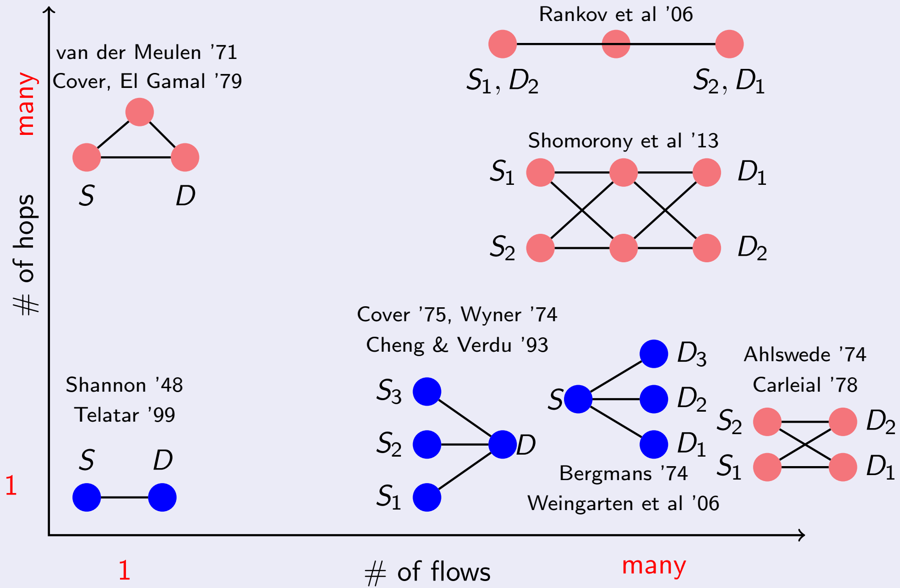
- Network: nodes, bandwidth, power
- R_k: Information flow rate from S_k to D_k
- Is reliable communication at $(R_1, R_2, \cdots, R_K)$ feasible?

Example network

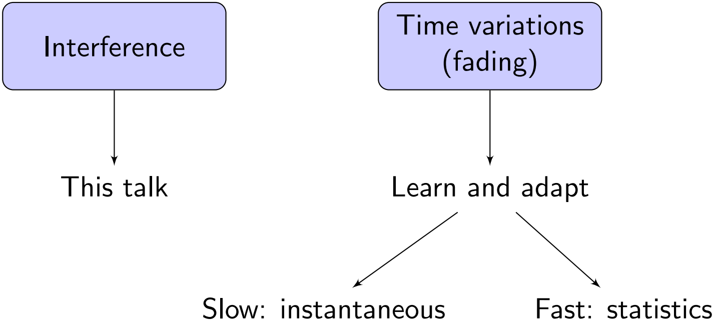


Three source-destination pairs
BS1 $\rightarrow$ U1, BS2 $\rightarrow$ U2, and BS3 $\rightarrow$ U3

A classification & known results and open problems

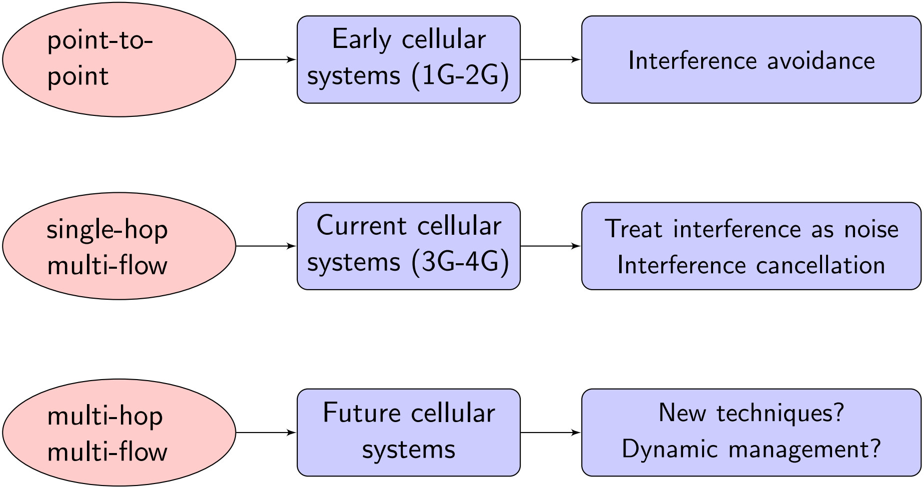


Wireless Channels: Main Issues



Evolution of Cellular Systems: Interference viewpoint

Treat network as a network of well-understood building blocks



Summary of Contributions

Time-variations

New resource allocation algorithms

- Throughput-optimal scheduling
- Partial/imperfect state information, strategic users

Single-hop single-flow	Effect of delayed channel feedback on adaptive MIMO [TCOM09, TWC09] Co-ordinate interleaved spatial multiplexing [TWC09]
Single-hop multi-flow	Joint subcarrier and power allocation for OFDM [COMML05, 183 citations] Cross-layer scheduling for downlink OFDM [TWC07, 76 citations] Scheduling with delayed/infrequent channel feedback [TWC09] Scheduling with partial feedback (best sub-band) [TWC15] Pricing and efficient resource allocation to strategic agents [TASE11]

Summary of Contributions

Interference

- Capacity bounds, Sum capacity:
  MIMO X, Many-to-one X, Multi-stage relaying, Two-way relaying
- Protocols:
  Multi-stage relaying, two-way relaying, multicast on a random network

Multi-hop single-flow	New decode-and-forward protocol for multi-stage relaying [TCOM12] New amplify-and-forward protocol for multi-stage relaying [TSP14]
Single-hop multi-flow	Sum capacity of MIMO X channels under weak interference [COMML14] Sum capacity bounds for the MIMO Z and MIMO X channels [TCOM15]
Multi-hop multi-flow	Coding scheme for compute-and-forward in two-way relaying [TCOM15] Asymptotically optimal algorithm for multicast in random networks [TIT13]

Sum capacity of the Gaussian many-to-one X channel[2]

[2] Joint work with Ranga Prasad (IISc) and A. Chockalingam(IISc). Preprint available at http://arxiv.org/abs/1403.5089
R. Prasad, S. Bhashyam, A. Chockalingam, "On the Gaussian many-to-one X channel," Submitted to IEEE Transactions on Information Theory in March 2014, Revised June 2015.

Single-hop interference networks: History

$K \times N$ Interference network (Carleial '78)

Interference channel (IC) | X channel (XC)

2-user IC

- Strong int.: Car75, Sato78
- Best inner bound: HK81
- Noisy int.: SKC09, AV09, MK09
- Mixed int.: MK09
- Capacity within half bit: ETW08

K-user IC

- Noisy int.: SKC09, AV09
- Approx. noisy int.: GNAJ13

Many-to-one IC

- Approx. capacity: BPT10, JWV10
- Noisy int.: AV09, CJ09

2 × 2 XC

- GDoF: JS09, MMK08, HCJ12
- Noisy int.: HCJ12
- Approx. sum capacity: NM13

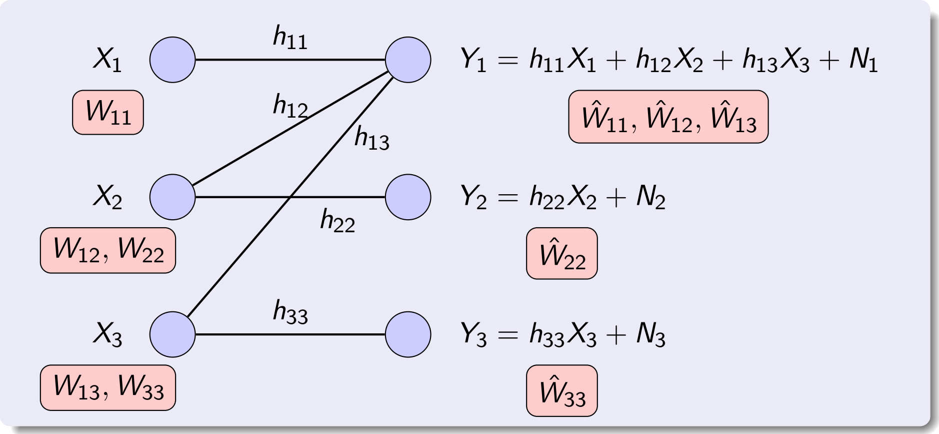
K × K XC

- DoF: CJ09
- Approx. noisy int.: GSJ14

Many-to-one XC

- This talk

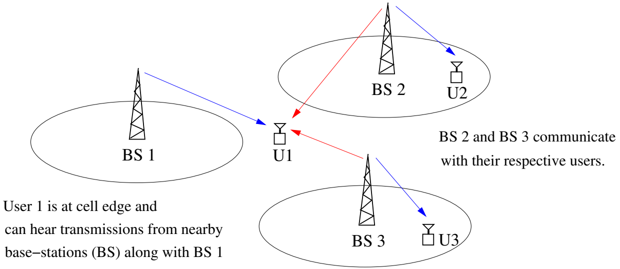
3×3 Gaussian many-to-one X channel



- One flow on each link (R_{ij}: Rate from Tx j to Rx i)
- Special case: Many-to-one interference channel (IC)
  - Messages W_{11}, W_{22}, and W_{33} only

Motivation

- Possible scenario



- Captures essential features, easier for analysis
- Results can be used to find bounds for more general topologies
- Prior work: Many-to-one IC
  - Sum capacity in a low-interference regime[3]
  - Capacity within a constant gap[4]

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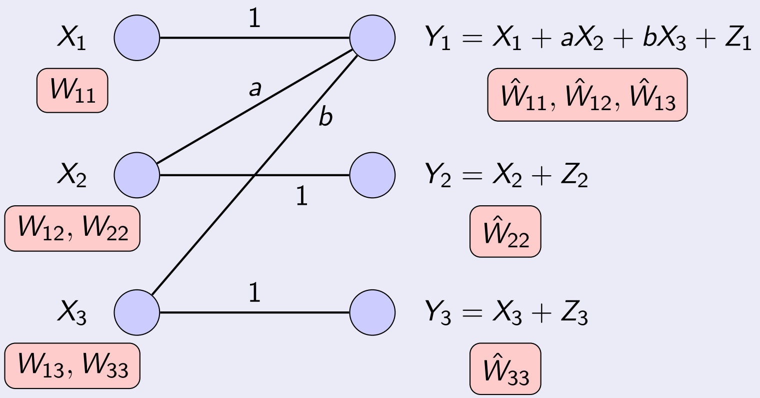
[3] Annapureddy & Veeravalli 2009, Cadambe & Jafar 2009

[4] Bresler, Parekh & Tse 2010, Jovicic, Wang, & Viswanath 2010

Channel in standard form

Reduce the number of parameters required



- $\mathcal{C}(\mathbf{P}', \mathbf{h}, \mathbf{N}) = \mathcal{C}_{standard}(\mathbf{P}, a, b)$, Z_i IID $\sim N(0,1)$[5]

$$\text{Sum capacity } C_{sum} = \max_{\mathbf{R} \in \mathcal{C}} [R_{11} + R_{22} + R_{12} + R_{33} + R_{13}]$$

[5] $\mathbf{P}$, $\mathbf{P}'$: power constraints, $\mathbf{N}$: noise variance vector

Results

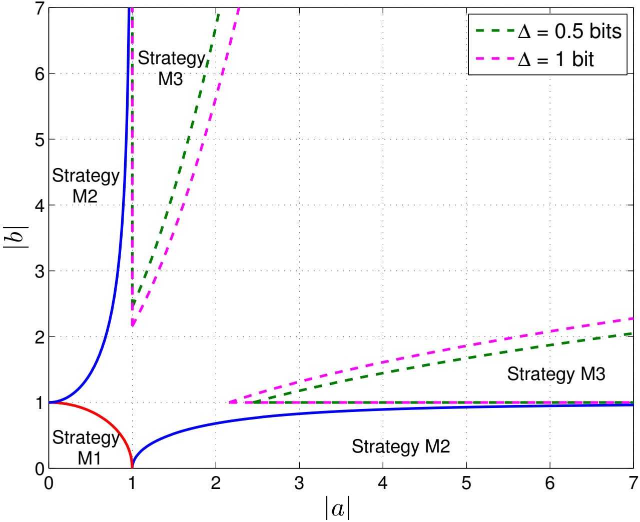
3×3 Many-to-one XC

- Transmission strategies for the many-to-one XC
  - Treat interference from a subset of transmitters as noise
  - Use of Gaussian codebooks
- Conditions for sum rate optimality

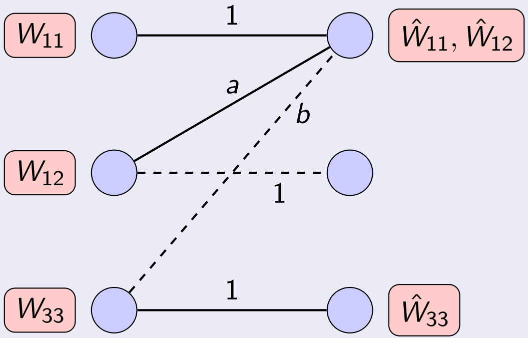
Extensions to $K \times K$ Many-to-one XC

Results for $K \times K$ Many-to-one IC

Result: 3 × 3 many-to-one X channel



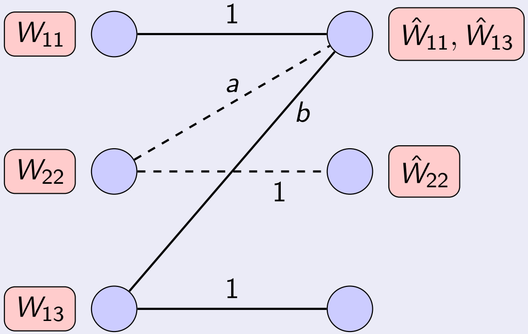
Strategy M2



Achieved sum-rate

$$R_{sum} = \frac{1}{2}\log_2\left(1 + \frac{P_1 + a^2 P_2}{b^2 P_3 + 1}\right) + \frac{1}{2}\log_2(1 + P_3)$$

Strategy M2



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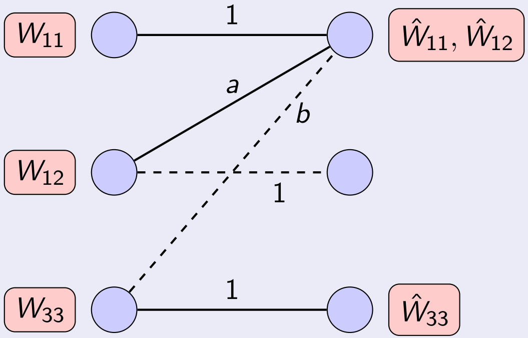
Sum-rate optimality of Strategy M2



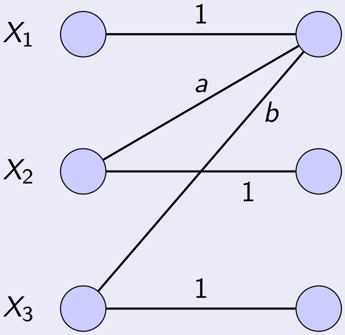
Strategy M2 achieves sum capacity if $b^2 < 1$ and $a^2 \geq \frac{(1+b^2P_3)^2}{1-b^2}$

Proof of sum-rate optimality of Strategy M2 (1)

Need upper bound that matches achievable sum-rate

Want to show $S \leq I(x_{1G}, x_{2G}; y_{1G}) + I(x_{3G}; y_{3G})$.

$S_1 = X_1 + aX_2 + \eta N_1$



$Y_1 = X_1 + aX_2 + bX_3 + Z_1$

$Y_2 = X_2 + Z_2$

$Y_3 = X_3 + Z_3$

- Show $S \leq I(x_{1G}, x_{2G}; y_{1G}, s_{1G}) + I(x_{3G}; y_{3G})$
- $E[N_1 Z_1] = \rho$, $\eta > 0$ chosen later

Proof of sum-rate optimality of Strategy M2 (2)

$$
\begin{aligned}
nS &\leq H(W_{11}, W_{12}, W_{22}) + H(W_{13}, W_{33}) \\
&\stackrel{(a)}{\leq} I(\mathbf{x}_1^n, \mathbf{x}_2^n\,;\, \mathbf{y}_1^n, \mathbf{s}_1^n) + I(\mathbf{x}_3^n\,;\, \mathbf{y}_3^n) + 5n\epsilon_n \\
&\stackrel{(b)}{\leq} n\,I(x_{1G}, x_{2G}\,;\, y_{1G}, s_{1G}) + n\,I(x_{3G}\,;\, y_{3G}) + 5n\epsilon_n \\
&\stackrel{(c)}{=} n\,I(x_{1G}, x_{2G}\,;\, y_{1G}) + n\,I(x_{3G}\,;\, y_{3G}) + 5n\epsilon_n
\end{aligned}
$$

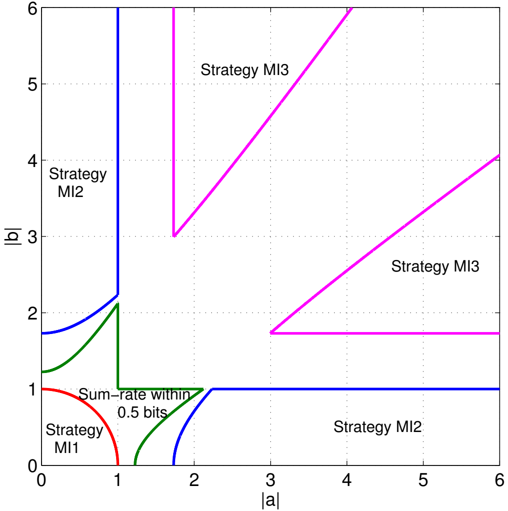
(a): $\eta^2 \leq a^2$ (W_{22} decodable at Rx 1), $b^2 \leq 1$ (W_{13} decodable at Rx 3)
(b): $b^2 \leq 1 - \rho^2$ (Gaussian inputs optimal for genie-aided channel)
(c): $\eta\rho = 1 + b^2 P_3$ (Genie does not increase sum capacity)

Then, choose $\rho^2 = 1 - b^2$ to get the final result

$$b^2 < 1 \quad \text{and} \quad a^2 \geq \frac{(1 + b^2 P_3)^2}{1 - b^2}$$

Result for the 3 × 3 many-to-one IC

Only sum rate optimality of Strategy MI1 known prior to this work



$P_1 = P_2 = P_3 = 3\text{dB}$

Summary: This work

Many-to-one XC

- Strategies where a subset of interfering signals are treated as noise
- Conditions for sum-rate optimality
- 3×3 case
- Extension to $K \times K$ case

Many-to-one IC

- Strategies MIk and conditions for sum-rate optimality

Current work

- Sum capacity for other channel conditions
- More general topologies: Approximate sum-rate optimality
- Recent results for strategy M1 (TIN) by Geng, Sun & Jafar 2014

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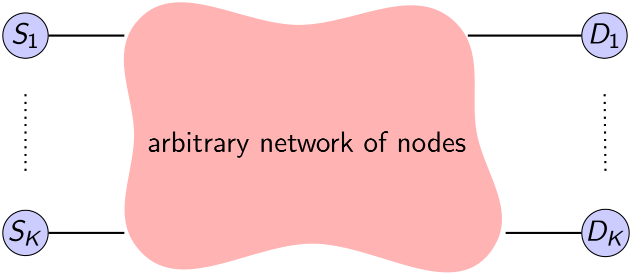

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Thank you!