

# On the Capacity of Interference Networks

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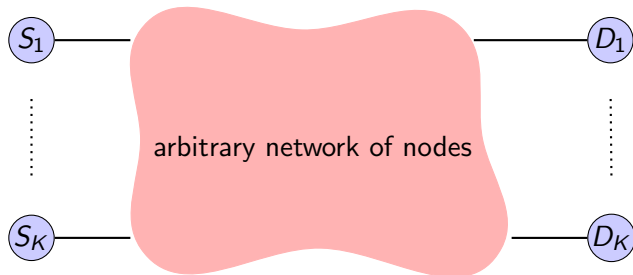
July 3, 2015

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<sup>1</sup>Acknowledgement: Students, Collaborators, Sponsors

# Ultimate goal: Multi-hop multi-flow wireless networks

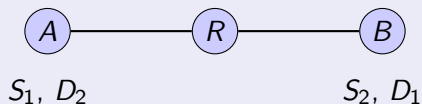
## Fundamental limits: Capacity region



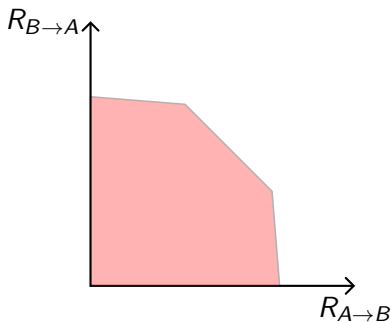
- Network: nodes, bandwidth, power
- $R_k$ : Information flow rate from  $S_k$  to  $D_k$
- Is **reliable communication** at  $(R_1, R_2, \dots, R_K)$  feasible?

## Example: Two-way relaying

Two flows ( $A \rightarrow B$ ,  $B \rightarrow A$ ) and two hops

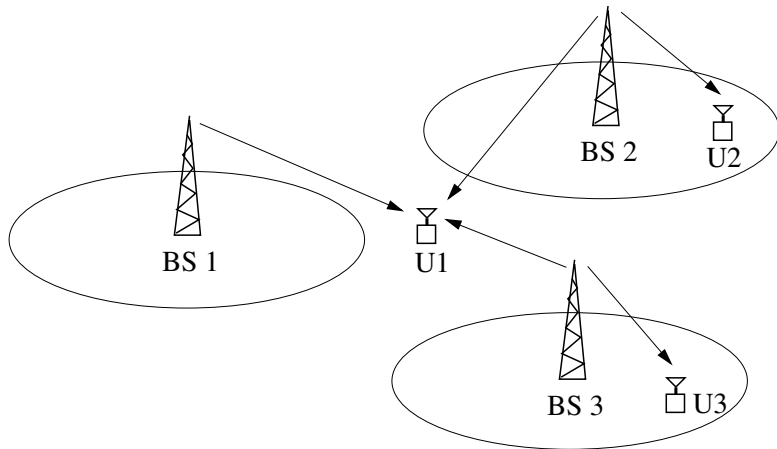


- Capacity region: Set of all achievable  $(R_{A \rightarrow B}, R_{B \rightarrow A})$



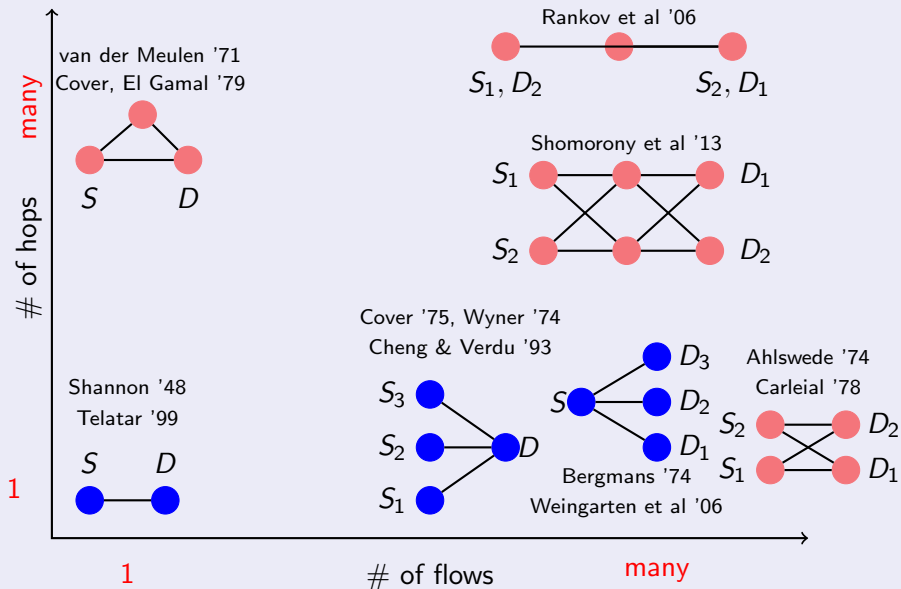
- Exact capacity region unknown

## Example network

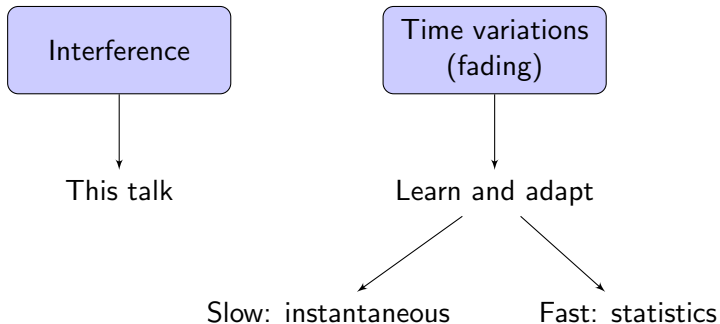


Three source-destination pairs  
 $BS1 \rightarrow U1$ ,  $BS2 \rightarrow U2$ , and  $BS3 \rightarrow U3$

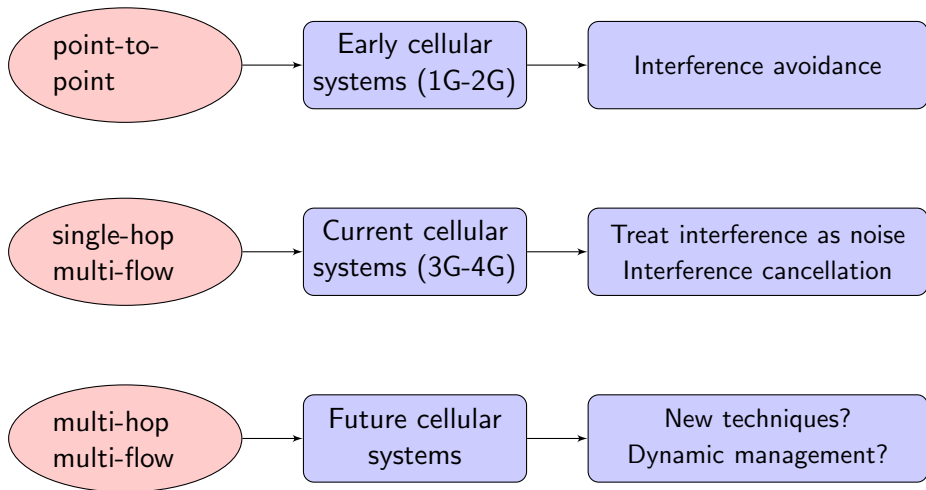
# A classification & known results and open problems



# Wireless Channels: Main Issues

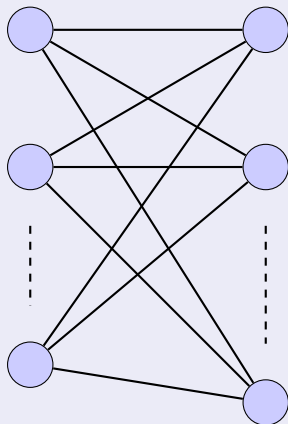


# Evolution of Cellular Systems: Interference viewpoint



Treat network as a network of **well-understood building blocks**

## Can we understand Interference Networks?



- $K$  transmitters,  $N$  receivers, single-hop
- Transmission from each transmitter to each subset of receivers
- $K > 1$  and  $N > 1$  is hard



# Importance of interference networks

## Scenario

- Full frequency reuse
- Dense deployment
- No strong association with a single basestation
- Possibility of coordination over backhaul
- Relay deployment

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- Full frequency reuse
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## Observations/Questions

- Interference avoidance: inefficient use of spectrum/bandwidth
- Treating interference as noise: not good for dense deployment
- No single strategy good for all scenarios
- Need dynamic interference management strategy
- Under what channel conditions is a given strategy good?

# Brief summary of work (1)

## Time variations (point-to-point)

- Adaptive point-to-point MIMO [TCOM 09, TWC 09, TWC 09]

V. S. Annapureddy, D. V. Marathe, T. R. Ramya, S. Bhashyam, "Outage Probability of Multiple-Input Single-Output (MISO) Systems with Delayed Feedback," IEEE Transactions on Communications, Feb 2009.

T. R. Ramya, S. Bhashyam, "Using delayed feedback for antenna selection in MIMO systems," IEEE Transactions on Wireless Communications, Dec. 2009.

K. V. Srinivas, R. D. Koilpillai, S. Bhashyam, K. Giridhar, "Co-ordinate Interleaved Spatial Multiplexing with Channel State Information," IEEE Transactions on Wireless Communications, Jun. 2009.

## Brief summary of work (2)

### Time variations (multi-flow)

- Joint subcarrier and power allocation, scheduling [COMML 05, TWC 07]

C. Mohanram, S. Bhashyam, "A Sub-optimal Joint Subcarrier and Power Allocation Algorithm for Multiuser OFDM," IEEE Communications Letters, Aug. 2005.

C. Mohanram, S. Bhashyam, "Joint Subcarrier and Power Allocation in Channel-Aware Queue-Aware Scheduling for Multiuser OFDM," IEEE Transactions on Wireless Communications, Sep. 2007.

## Brief summary of work (3)

### Time variations (multi-flow)

- Scheduling with delayed channel information [TWC 09]
- Scheduling with partial channel information (order statistics) [TWC15]

C. Manikandan, S. Bhashyam, R. Sundaresan, "Cross-layer scheduling with infrequent channel and queue measurements," IEEE Transactions on Wireless Communications, Dec. 2009.

H. Ahmed, K. Jagannathan, S. Bhashyam, "Queue-Aware Optimal Resource Allocation for the LTE Downlink with Best M Sub-band Feedback," To appear in the IEEE Transactions on Wireless Communications.

## Brief summary of work (4)

### Time variations (multi-flow)

- Pricing mechanism for resource allocation to strategic agents [TASE 11]

A. K. Chorppath, S. Bhashyam, R. Sundaresan, "A convex optimization framework for almost budget balanced allocation of a divisible good," IEEE Transactions on Automation Science and Engineering, Jul. 2011.

D. Thirumulanathan, H. Vinay, S. Bhashyam, R. Sundaresan, "Almost Budget Balanced Mechanisms with Scalar Bids For Allocation of a Divisible Good," Submitted to Operations Research, Apr. 2015.

## Brief summary of work (5)

### Interference

- Multi-hop single-flow: layered relay networks [TCOM 12, TSP 14]

Bama Muthuramalingam, S. Bhashyam, A. Thangaraj, "A Decode and Forward Protocol for Two-stage Gaussian Relay Networks," IEEE Transactions on Communications, Jan. 2012.

P. S. Elamvazhuthi, B. K. Dey, S. Bhashyam, An MMSE strategy at relays with partial CSI for a multi-layer relay network, IEEE Transactions on Signal Processing, Jan. 15, 2014.

## Brief summary of work (6)

### Interference

- Single-hop multi-flow: X channel [COMML 14, TCOM 15]

Praneeth Kumar V., S. Bhashyam, "MIMO Gaussian X Channel: Noisy Interference Regime," IEEE Communications Letters, Aug. 2014.

R. Prasad, S. Bhashyam, A. Chockalingam, "On the Sum-Rate of the Gaussian MIMO Z Channel and the Gaussian MIMO X Channel," IEEE Transactions on Communications, Feb. 2015.



## Brief summary of work (7)

### Interference

- Multi-hop multi-flow: two-way relaying, multiple allcast [TCOM 15, TIT 13]

V. N. Swamy, S. Bhashyam, R. Sundaresan, P. Viswanath, "An asymptotically optimal push-pull method for multicasting over a random network," IEEE Transactions on Information Theory, Aug. 2013.

K. Ravindran, A. Thangaraj, S. Bhashyam, "LDPC Codes for Network-coded Bidirectional Relaying with Higher Order Modulation," IEEE Transactions on Communications, Jun 2015.

# Sum capacity of the Gaussian many-to-one X channel<sup>2</sup>

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<sup>2</sup>Joint work with Ranga Prasad (IISc) and A. Chockalingam(IISc). Preprint available at <http://arxiv.org/abs/1403.5089>  
R. Prasad, S. Bhashyam, A. Chockalingam, "On the Gaussian many-to-one X channel," Submitted to IEEE Transactions on Information Theory in March 2014, Revised June 2015.

# Single-hop interference networks: History

$K \times N$  Interference network (Carleial '78)

Interference channel (IC)

## 2-user IC

- Strong int.: Car75, Sato78
- Best inner bound: HK81
- Noisy int.: SKC09, AV09, MK09
- Mixed int.: MK09
- Capacity within half bit: ETW08

## K-user IC

- Noisy int.: SKC09, AV09
- Approx. noisy int.: GNAJ13

## Many-to-one IC

- Approx. capacity: BPT10, JWV10
- Noisy int.: AV09, CJ09

X channel (XC)

## $2 \times 2$ XC

- GDoF: JS09, MMK08, HCJ12
- Noisy int.: HCJ12
- Approx. sum capacity: NM13

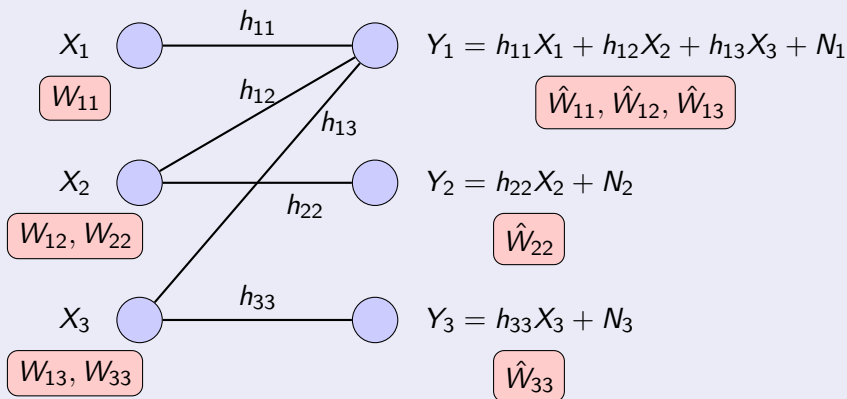
## $K \times K$ XC

- DoF: CJ09
- Approx. noisy int.: GSJ14

## Many-to-one XC

- This talk

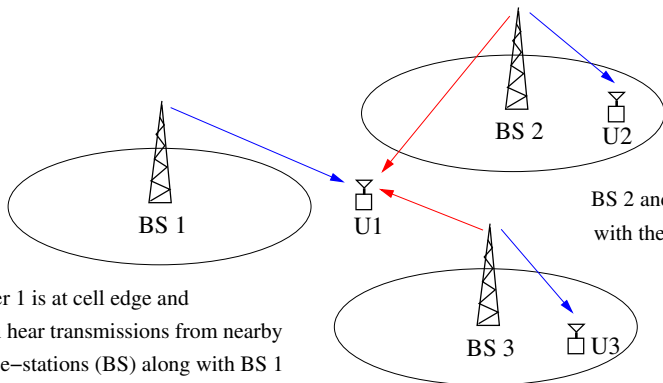
## $3 \times 3$ Gaussian many-to-one X channel



- One flow on each link ( $R_{ij}$ : Rate from Tx  $j$  to Rx  $i$ )

# Motivation

- Possible scenario

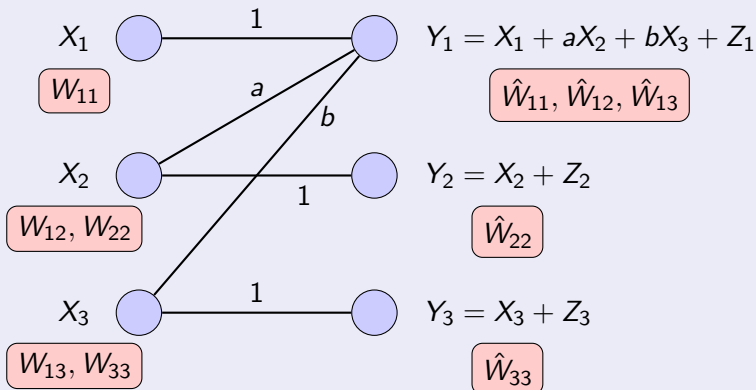


User 1 is at cell edge and can hear transmissions from nearby base-stations (BS) along with BS 1

- Captures essential features, easier for analysis
- Results can be used to find bounds for more general topologies

## Channel in standard form

Reduce the number of parameters required



- $\mathcal{C}(\mathbf{P}', \mathbf{h}, \mathbf{N}) = \mathcal{C}_{standard}(\mathbf{P}, a, b)$
- $Z_i$  IID  $\sim N(0, 1)$ ,  $\mathbf{P}, \mathbf{P}'$ : power constraints,  $\mathbf{N}$ : noise variance vector

## Sum capacity

Capacity region (5-dimensional) not easy to characterize

- $\mathcal{C}$  = Set of all achievable  $\mathbf{R} = (R_{11}, R_{22}, R_{12}, R_{33}, R_{13})$

# Sum capacity

Capacity region (5-dimensional) not easy to characterize

- $\mathcal{C}$  = Set of all achievable  $\mathbf{R} = (R_{11}, R_{22}, R_{12}, R_{33}, R_{13})$

## Alternatives

- Partial characterization: Sum capacity  $C_{sum}$ , Weighted sum capacity

$$C_{sum} = \max_{\mathbf{R} \in \mathcal{C}} [R_{11} + R_{22} + R_{12} + R_{33} + R_{13}]$$

- Asymptotics: Generalized degrees of freedom region (set of achievable  $\mathbf{d} = (d_{11}, d_{22}, d_{12}, d_{33}, d_{13})$ )

$$d_{ij} = \lim_{P \rightarrow \infty} \frac{R_{ij}(P)}{\log P}$$

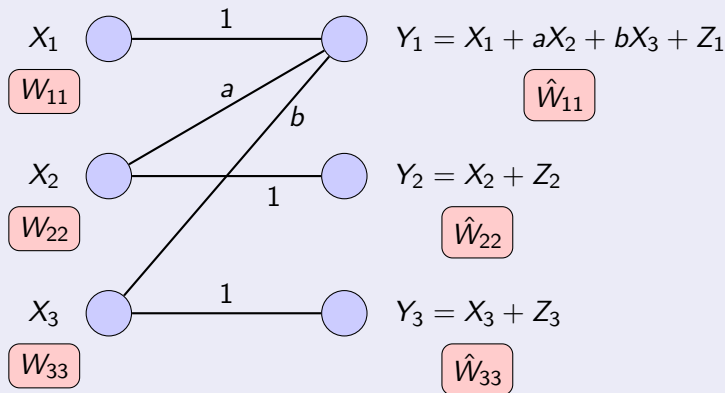
- Approximations and Bounds: Within a constant gap

Sum capacity in this talk



# Many-to-one Interference Channel (IC)

A special case of the many-to-one XC



- Sum capacity in a low-interference regime<sup>3</sup>
- Capacity within a constant gap<sup>4</sup>

<sup>3</sup> Annapureddy & Veeravalli 2009, Cadambe & Jafar 2009

<sup>4</sup> Bresler, Parekh & Tse 2010, Jovicic, Wang, & Viswanath 2010

# Rest of this talk

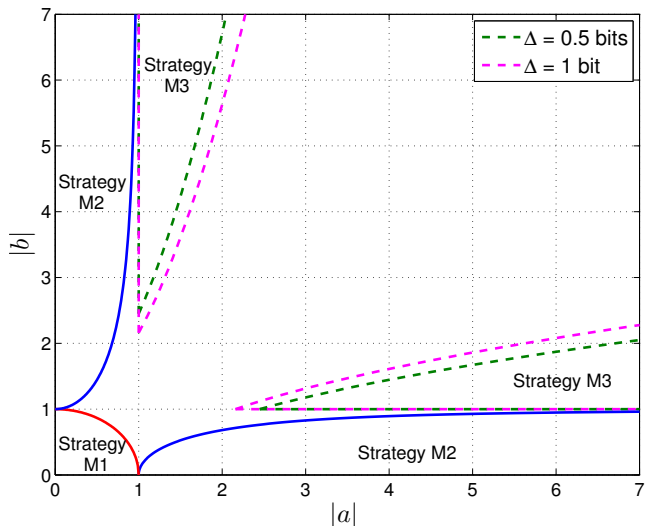
## $3 \times 3$ Many-to-one XC

- Transmission strategies for the many-to-one XC
  - ▶ Treat interference from a **subset** of transmitters as noise
  - ▶ Use of Gaussian codebooks
- Conditions for sum rate optimality

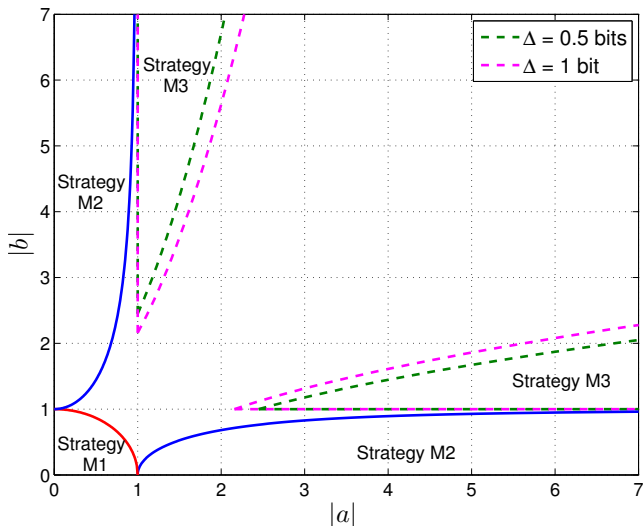
## Extensions to $K \times K$ Many-to-one XC

## Results for $K \times K$ Many-to-one IC

# Preview of result

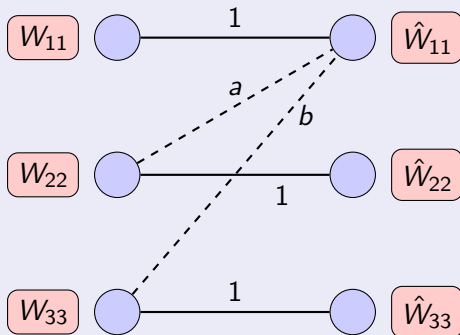


# Preview of result



- Strategy M1: optimal for many-to-one IC under the same conditions

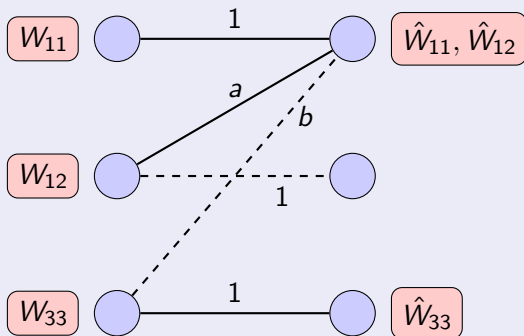
## Strategy M1: Treating Interference as Noise (TIN)



Achieved sum-rate

$$R_{sum} = \frac{1}{2} \log_2 \left( 1 + \frac{P_1}{a^2 P_2 + b^2 P_3 + 1} \right) + \frac{1}{2} \log_2 (1 + P_2) + \frac{1}{2} \log_2 (1 + P_3)$$

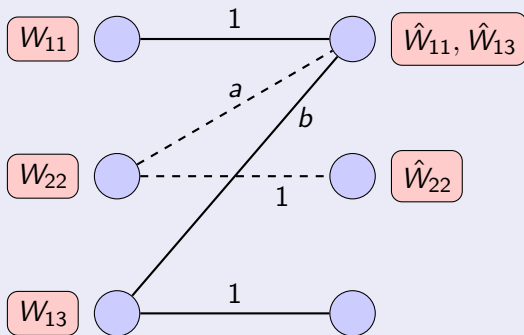
## Strategy M2



Achieved sum-rate

$$R_{sum} = \frac{1}{2} \log_2 \left( 1 + \frac{P_1 + a^2 P_2}{b^2 P_3 + 1} \right) + \frac{1}{2} \log_2 (1 + P_3)$$

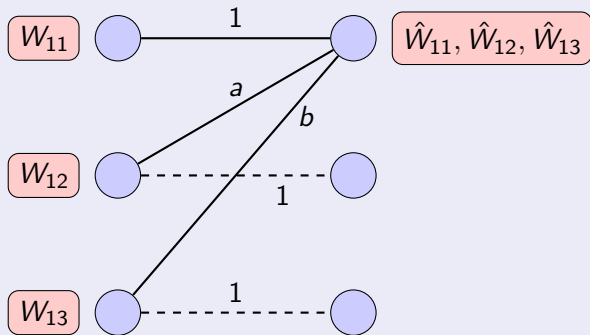
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## Strategy M3

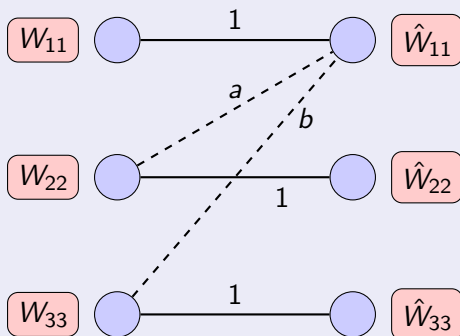


Achieved sum-rate

$$R_{sum} = \frac{1}{2} \log_2 (1 + P_1 + a^2 P_2 + b^2 P_3)$$

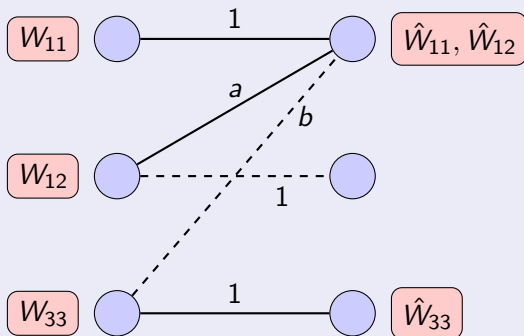


## Sum-rate optimality of Strategy M1 (TIN)



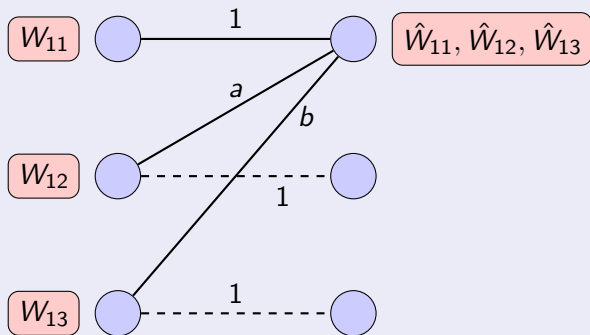
Strategy M1 achieves sum capacity if  $a^2 + b^2 \leq 1$

## Sum-rate optimality of Strategy M2



Strategy M2 achieves sum capacity if  $b^2 < 1$  and  $a^2 \geq \frac{(1+b^2P_3)^2}{1-b^2}$

## Approximate sum-rate optimality of Strategy M3



Strategy M3 achieves rates within

$$\frac{1}{2} \log_2 \left( \frac{1 - (1 + b^2 P_3)^{-1} \rho^2}{1 - \rho^2} \right) \text{ bits}$$

of sum capacity if  $b^2 \geq 1$  and  $a^2 \geq \frac{(1 + b^2 P_3)^2}{\rho^2}$

# Sum-rate optimality proofs: Outline

Need an upper bound that matches achievable sum-rate

Upper bound using

- Fano's inequality
- Worst-case additive noise result (or) Extremal inequality (or) Entropy-Power inequality (EPI)
- Genie-aided channel/Channel with side information (M2 & M3)

## Preliminaries

$X, Y \sim p(x, y)$ : Random variables/vectors

- Measure of information: Entropy  $H(X)$  or Differential entropy  $h(X)$
  - Conditional entropy:  $H(X|Y = y)$ ,  $H(X|Y)$
  - Conditioning reduces entropy:  $H(X|Y) \leq H(X)$
  - Mutual information between  $X$  and  $Y$ :  
$$I(X; Y) = H(Y) - H(Y|X) = H(X) - H(X|Y) \geq 0$$
  - $h(X)$  is maximized by Gaussian  $X$  under a covariance constraint
-

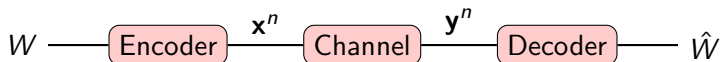
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Coding over  $n$  channel uses



$W \in \mathcal{W} = \{1, 2, \dots, 2^{nR}\} \implies R$  bits/channel use

## Fano's inequality

Relates probability of error to conditional entropy

Let  $(W, V) \sim p(w, v)$  and  $P_e = P[W \neq V]$ . Then

$$H(W|V) \leq 1 + P_e \log |\mathcal{W}|.$$

## Fano's inequality

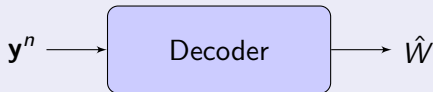
Relates probability of error to conditional entropy

Let  $(W, V) \sim p(w, v)$  and  $P_e = P[W \neq V]$ . Then

$$H(W|V) \leq 1 + P_e \log |\mathcal{W}|.$$

How are we going to use this?

$$W \in \mathcal{W} = \{1, 2, \dots, 2^{nR}\} \implies 1 + P_e^{(n)} \log(2^{nR}) = n(RP_e^{(n)} + \frac{1}{n})$$



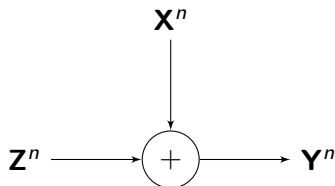
Suppose  $P_e^{(n)} \rightarrow 0$  as  $n \rightarrow \infty$ . Then

$$H(W|\mathbf{y}^n) \leq H(W|\hat{W}) \leq n\epsilon_n$$

where  $\epsilon_n \rightarrow 0$  as  $n \rightarrow \infty$



## Worst-case additive noise



- $\mathbf{Z}^n \sim \mathcal{N}(\mathbf{0}, \Sigma_Z)$  IID
- $\mathbf{X}^n$ : average covariance constraint  $\Sigma_X$

Worst case noise result (Diggavi & Cover 01, Annapureddy & Veeravalli 09)

$$h(\mathbf{X}^n) - h(\mathbf{X}^n + \mathbf{Z}^n) \leq nh(\mathbf{X}_G) - nh(\mathbf{X}_G + \mathbf{Z}),$$

where  $\mathbf{X}_G \sim \mathcal{N}(\mathbf{0}, \Sigma_X)$ .

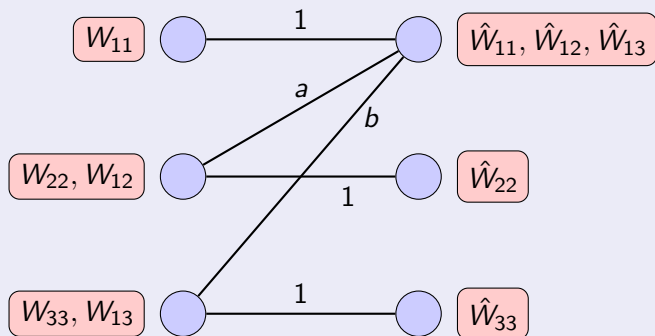
## A more general result<sup>5</sup>

$$\begin{aligned} \sum_{i=1}^K h(X_i^n + Z_i^n) &= h\left(\sum_{i=1}^K c_i X_i^n + Z_1^n\right) \\ &\leq n \sum_{i=1}^K h(X_{iG} + Z_i) - nh\left(\sum_{i=1}^K c_i X_{iG} + Z_1\right), \\ &\quad \text{if } \sum_{i=1}^K c_i^2 \leq \sigma^2 \end{aligned}$$

- $X_i^n$  with power constraint  $\sum_{j=1}^n \mathbb{E}[(X_{ij}^2)] \leq nP_i$
- $Z_1^n$  vector with IID  $\mathcal{N}(0, \sigma^2)$  components
- $Z_i^n, i \neq 1$  vector with IID  $\mathcal{N}(0, 1)$  components
- $X_i^n$  are independent of  $Z_i^n$
- $X_{iG} \sim \mathcal{N}(0, P_i)$

<sup>5</sup>Lemma 5 from Annapureddy & Veeravalli 2009 in different form

## Degraded receivers



- If  $a^2 \leq 1$ , Rx 1 is a degraded version of Rx 2 w.r.t.  $W_{12}$
- If  $b^2 \leq 1$ , Rx 1 is a degraded version of Rx 3 w.r.t.  $W_{13}$

# Proof of sum-rate optimality of Strategy M1 (1)

Let  $S$  denote any achievable sum-rate. Want to show

$$S \leq I(x_{1G}; y_{1G}) + I(x_{2G}; y_{2G}) + I(x_{3G}; y_{3G}).$$

$$\begin{aligned} nS &\leq H(W_{11}) + H(W_{12}, W_{22}) + H(W_{13}, W_{33}) \\ &= I(W_{11}; \mathbf{y}_1^n) + \sum_{i=2}^3 I(W_{1i}, W_{ii}; \mathbf{y}_i^n) \\ &\quad + H(W_{11} | \mathbf{y}_1^n) + \sum_{i=2}^3 H(W_{1i}, W_{ii} | \mathbf{y}_i^n) \\ &\leq I(\mathbf{x}_1^n; \mathbf{y}_1^n) + \sum_{i=2}^3 I(\mathbf{x}_i^n; \mathbf{y}_i^n) \\ &\quad + H(W_{11} | \mathbf{y}_1^n) + \sum_{i=2}^3 H(W_{1i}, W_{ii} | \mathbf{y}_i^n) \end{aligned}$$

## Proof of sum-rate optimality of Strategy M1 (2)

$$nS \leq I(\mathbf{x}_1^n; \mathbf{y}_1^n) + \sum_{i=2}^3 I(\mathbf{x}_i^n; \mathbf{y}_i^n) + H(W_{11} | \mathbf{y}_1^n) + \sum_{i=2}^3 H(W_{1i}, W_{ii} | \mathbf{y}_i^n)$$

$$\stackrel{(a)}{\leq} h(\mathbf{y}_1^n) - h(a\mathbf{x}_2^n + b\mathbf{x}_3^n + \mathbf{n}_1^n) + h(\mathbf{x}_2^n + \mathbf{n}_2^n) - h(\mathbf{n}_2^n) + h(\mathbf{x}_3^n + \mathbf{n}_3^n) - h(\mathbf{n}_3^n) + 5\epsilon_n$$

$$\stackrel{(b)}{\leq} nh(y_{1G}) - nh(ax_{2G} + bx_{3G} + n_1) + nh(x_{2G} + n_2) + nh(x_{3G} + n_3) - nh(n_2) - nh(n_3) + 5\epsilon_n$$

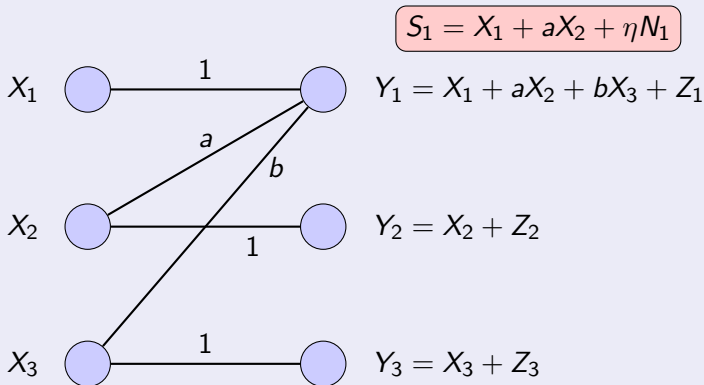
$$= nI(x_{1G}; y_{1G}) + nI(x_{2G}; y_{2G}) + nI(x_{3G}; y_{3G}) + 5\epsilon_n,$$

(a): Fano's inequality,  $a^2 \leq 1$  and  $b^2 \leq 1$

(b): Generalized form of worst-case noise result,  $a^2 + b^2 \leq 1$

# Proof of sum-rate optimality of Strategy M2 (1)

Want to show  $S \leq I(x_{1G}, x_{2G}; y_{1G}) + I(x_{3G}; y_{3G})$ .



- Show  $S \leq I(x_{1G}, x_{2G}; y_{1G}, s_{1G}) + I(x_{3G}; y_{3G})$
- $E[N_1 Z_1] = \rho$ ,  $\eta > 0$  chosen later

## Proof of sum-rate optimality of Strategy M2 (2)

$$\begin{aligned} nS &\leq H(W_{11}, W_{12}, W_{22}) + H(W_{13}, W_{33}) \\ &= I(W_{11}, W_{12}, W_{22}; \mathbf{y}_1^n, \mathbf{s}_1^n) + H(W_{11} | \mathbf{y}_1^n, \mathbf{s}_1^n) + H(W_{12} | \mathbf{y}_1^n, \mathbf{s}_1^n, \mathbf{x}_1^n) \\ &\quad + H(W_{22} | \mathbf{y}_1^n, \mathbf{s}_1^n, \mathbf{x}_1^n, W_{12}) + I(W_{13}, W_{33}; \mathbf{y}_3^n) + H(W_{13} | \mathbf{y}_3^n) \\ &\quad + H(W_{33} | \mathbf{y}_3^n, W_{13}) \\ &\leq I(\mathbf{x}_1^n, \mathbf{x}_2^n; \mathbf{y}_1^n, \mathbf{s}_1^n) + H(W_{11} | \mathbf{y}_1^n) + H(W_{12} | \mathbf{y}_1^n) \\ &\quad + H(W_{22} | \mathbf{s}_1^n, \mathbf{x}_1^n) + I(\mathbf{x}_3^n; \mathbf{y}_3^n) + H(W_{13} | \mathbf{y}_3^n) + H(W_{33} | \mathbf{y}_3^n), \\ &\stackrel{(a)}{\leq} I(\mathbf{x}_1^n, \mathbf{x}_2^n; \mathbf{y}_1^n, \mathbf{s}_1^n) + I(\mathbf{x}_3^n; \mathbf{y}_3^n) + 5n\epsilon_n \end{aligned} \tag{1}$$

(a):  $\eta^2 \leq a^2$  and  $b^2 \leq 1$

## Proof of sum-rate optimality of Strategy M2 (3)

$$\begin{aligned} nS &\leq I(\mathbf{x}_1^n, \mathbf{x}_2^n; \mathbf{y}_1^n, \mathbf{s}_1^n) + I(\mathbf{x}_3^n; \mathbf{y}_3^n) + 5n\epsilon_n \\ &= I(\mathbf{x}_1^n, \mathbf{x}_2^n; \mathbf{s}_1^n) + I(\mathbf{x}_1^n, \mathbf{x}_2^n; \mathbf{y}_1^n | \mathbf{s}_1^n) + I(\mathbf{x}_3^n; \mathbf{y}_3^n) + 5n\epsilon_n \\ &= h(\mathbf{s}_1^n) - h(\mathbf{s}_1^n | \mathbf{x}_1^n, \mathbf{x}_2^n) + h(\mathbf{y}_1^n | \mathbf{s}_1^n) \\ &\quad - h(\mathbf{y}_1^n | \mathbf{s}_1^n, \mathbf{x}_1^n, \mathbf{x}_2^n) + h(\mathbf{y}_3^n) - h(\mathbf{y}_3^n | \mathbf{x}_3^n) + 5n\epsilon_n \\ &\leq nh(s_{1G}) - nh(\eta z_1) + nh(y_{1G} | s_{1G}) \\ &\quad - h(b\mathbf{x}_3^n + \tilde{\mathbf{n}}_1^n) + h(\mathbf{x}_3^n + \mathbf{n}_3^n) - nh(n_3) + 5n\epsilon_n \\ &\stackrel{(b)}{\leq} nh(s_{1G}) - nh(\eta z_1) + nh(y_{1G} | s_{1G}) \\ &\quad - nh(bx_{3G} + \tilde{n}_1) + nh(x_{3G} + n_3) - nh(n_3) + 5n\epsilon_n \\ &= nI(x_{1G}, x_{2G}; y_{1G}, s_{1G}) + nI(x_{3G}; y_{3G}) + 5n\epsilon_n, \end{aligned}$$

$$(b): b^2 \leq 1 - \rho^2$$



## Proof of sum-rate optimality of Strategy M2 (4)

Choose

$$\eta\rho = 1 + b^2P_3$$

to get

$$I(x_{1G}, x_{2G}; y_{1G}, s_{1G}) = I(x_{1G}, x_{2G}; y_{1G})$$

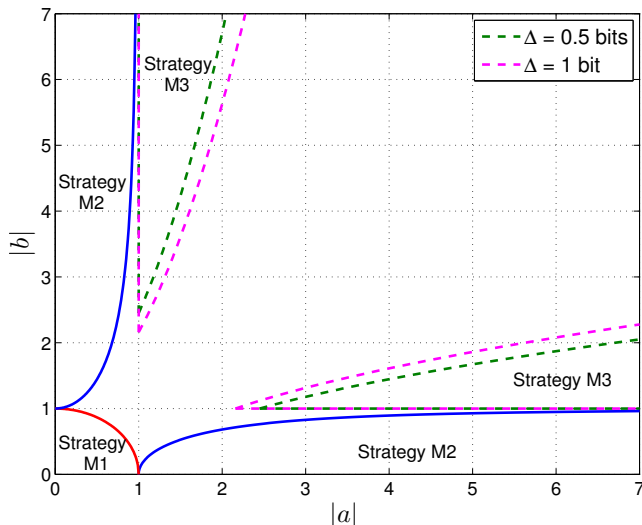
Then, choose

$$\rho^2 = 1 - b^2$$

to get the final result

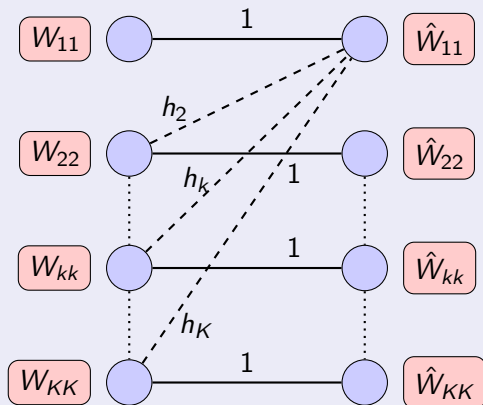
$$b^2 < 1 \quad \text{and} \quad a^2 \geq \frac{(1 + b^2P_3)^2}{1 - b^2}$$

## Back to the numerical result



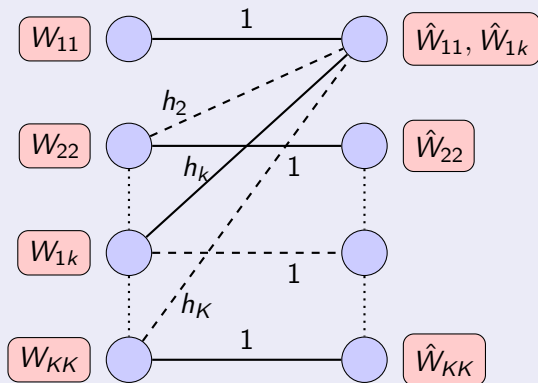
- $P_1 = P_2 = P_3 = 0$  dB

## Strategy M1 for the $K \times K$ many-to-one XC



Strategy M1 achieves sum capacity if  $\sum_{j=2}^K h_j^2 < 1$

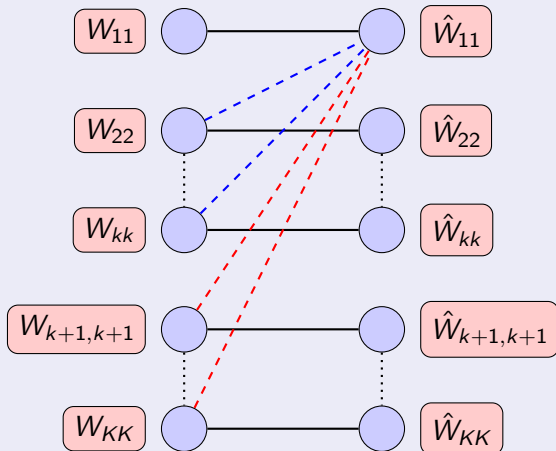
## Strategy M2 for the $K \times K$ many-to-one XC



Strategy M2 achieves sum capacity if

$$\sum_{j=2, j \neq k}^K h_j^2 < 1 \text{ and } h_k^2 \geq \frac{(1 + \sum_{j=2}^K h_j^2 P_j)^2}{1 - \sum_{j=2, j \neq k}^K h_j^2}$$

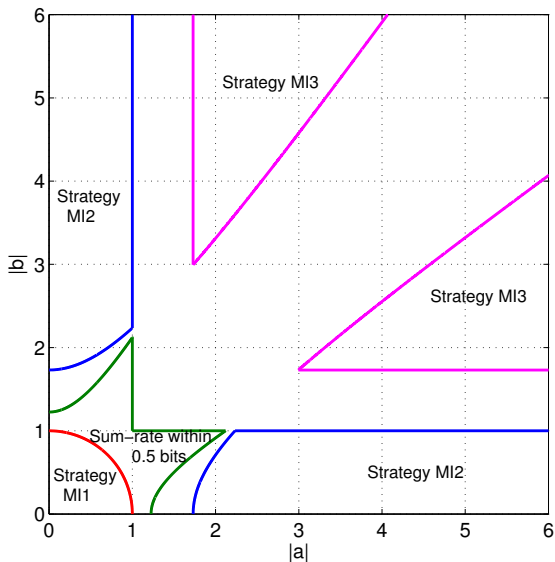
## $K \times K$ many-to-one IC



Strategies  $MI_k$  for  $k = 1, 2, \dots, K$

- Decode interference from transmitters 2 to  $k$  (for  $k \geq 2$ )
- Treat interference from transmitters  $k + 1$  to  $K$  as noise

# Result for the $3 \times 3$ many-to-one IC



$$P_1 = P_2 = P_3 = 3\text{dB}$$

# Summary

## Many-to-one XC

- Strategies where a **subset** of interfering signals are treated as noise
- Conditions for sum-rate optimality
- $3 \times 3$  case
- $K \times K$  case

## Many-to-one IC

- Strategies MIM and conditions for sum-rate optimality

# Summary

## Many-to-one XC

- Strategies where a **subset** of interfering signals are treated as noise
- Conditions for sum-rate optimality
- $3 \times 3$  case
- $K \times K$  case

## Many-to-one IC

- Strategies M1k and conditions for sum-rate optimality

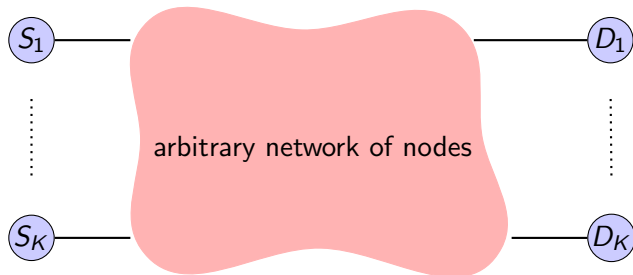
## Current work

- Sum capacity for other channel conditions
- More general topologies: Approximate sum-rate optimality
- Recent results for strategy M1 (TIN) by Geng, Sun & Jafar 2014



# Ultimate goal: Multi-hop multi-flow wireless networks

## Fundamental limits: Capacity region



- Network: nodes, bandwidth, power
- $R_k$ : Information flow rate from  $S_k$  to  $D_k$
- Is reliable communication at  $(R_1, R_2, \dots, R_K)$  feasible?

Thank you

<http://www.ee.iitm.ac.in/~skrishna/>