MIMO and Mode Division Multiplexing in Multimode Fibers

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Prerequisites

- Basic EE, physics
- Maxwell's equations
- Linear system theory, communication theory
- Optics, reflection, refraction etc.

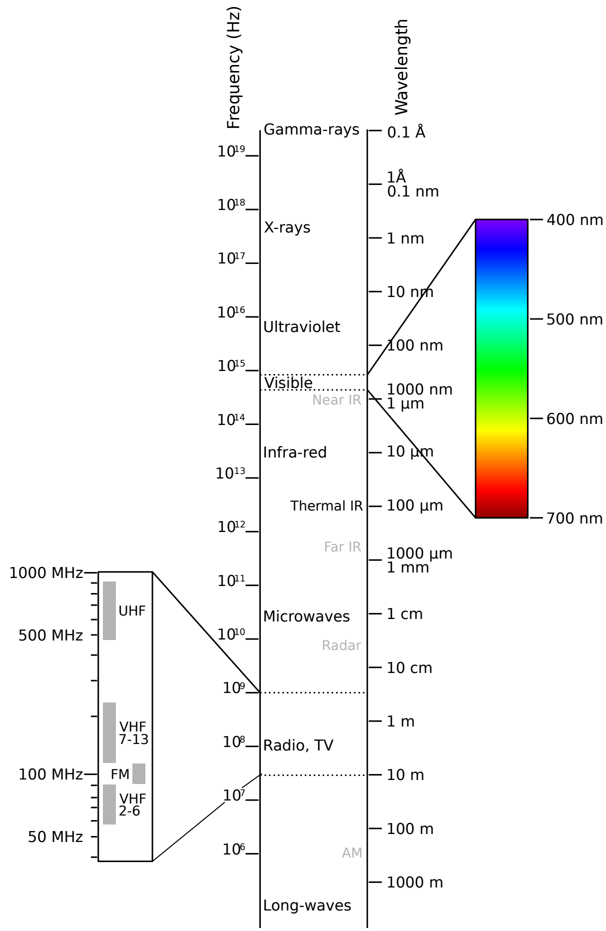
Outline

- Introduction
- Basic optical fiber concepts
- Propagation characteristics
- MIMO, MDM in multimode fibers
- Recent trends, adoption, future
- Summary

Introduction

- Optical fibers: waveguides that carry EM waves
- High bandwidths, long distance carrying abilities
- Challenges: alignment, dispersion, nonlinearities
- Insert picture here

Electromagnetic spectrum



Optical fibre

- Guided medium
- High bandwidth
- Large bandwidth-length product
- Examples:
  - Long undersea links
  - Medium haul links
  - Short links: data centres, vehicles

Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \tag{2}$$

$$\nabla \cdot \mathbf{D} = 0 \tag{3}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{4}$$

where $\mathbf{D} = \epsilon \mathbf{E}$, $\mathbf{B} = \mu \mathbf{H}$.

Maxwell's equations

Take curl of equation (1)

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{E}) = -\mu \frac{\partial}{\partial t}(\boldsymbol{\nabla} \times \mathbf{H}) = -\mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Identity:

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{E}) = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{E}) - \boldsymbol{\nabla}^2 \mathbf{E}$$

Using $\boldsymbol{\nabla} \cdot \mathbf{E} = 0$, we get the wave equations (repeated for $\mathbf{H}$):

$$\boldsymbol{\nabla}^2 \mathbf{E} = \mu\epsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\boldsymbol{\nabla}^2 \mathbf{H} = \mu\epsilon \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

Waveguide equations

Using cylindrical coordinate system:

$$\mathbf{E}(r,\phi,z) = \mathbf{E}_0(r,\phi)e^{j(\omega t-\beta z)}$$
$$\mathbf{H}(r,\phi,z) = \mathbf{H}_0(r,\phi)e^{j(\omega t-\beta z)}$$

β: z-component of propagation vector, determined by the boundary conditions. Substitute above in the curl equations. Use ∇ in cylindrical coordinates:

$$\nabla \times \mathbf{A} = \det\begin{pmatrix} \frac{1}{r}\hat{e}_r & \hat{e}_\phi & \frac{1}{r}\hat{e}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & A_\phi & A_z \end{pmatrix}$$

Waveguide equations

$$\frac{1}{r}\left(\frac{\partial E_z}{\partial \phi} + jr\beta E_\phi\right) = -j\omega\mu H_r$$

$$j\beta E_r + \frac{\partial E_z}{\partial r} = j\omega\mu H_\phi$$

$$\frac{1}{r}\left(\frac{\partial}{\partial r}(rE_\phi) - \frac{\partial E_r}{\partial \phi}\right) = -j\mu\omega H_z$$

$$\frac{1}{r}\left(\frac{\partial H_z}{\partial \phi} + jr\beta H_\phi\right) = j\omega\mu E_r$$

$$j\beta H_r + \frac{\partial H_z}{\partial r} = -j\omega\mu E_\phi$$

$$\frac{1}{r}\left(\frac{\partial}{\partial r}(rH_\phi) - \frac{\partial H_r}{\partial \phi}\right) = -j\mu\omega E_z$$

Six fields are coupled, eliminate to keep only H_z and E_z.

Waveguide equations

$$E_r = -\frac{j}{q^2}\left(\beta\frac{\partial E_z}{\partial r} + \frac{\mu\omega}{r}\frac{\partial H_z}{\partial \phi}\right)$$

$$E_\phi = -\frac{j}{q^2}\left(\frac{\beta}{r}\frac{\partial E_z}{\partial \phi} - \mu\omega\frac{\partial H_z}{\partial r}\right)$$

$$H_r = -\frac{j}{q^2}\left(\beta\frac{\partial H_z}{\partial r} - \frac{\epsilon\omega}{r}\frac{\partial E_z}{\partial \phi}\right)$$

$$H_\phi = -\frac{j}{q^2}\left(\frac{\beta}{r}\frac{\partial H_z}{\partial \phi} + \epsilon\omega\frac{\partial E_z}{\partial r}\right)$$

We now substitute the above into $\nabla^2\mathbf{E} = \mu\epsilon\frac{\partial^2\mathbf{E}}{\partial t^2}$, to get

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \phi^2} + q^2 E_z = 0$$

where $q^2 = \omega^2\mu\epsilon - \beta^2$.

Waveguide equations

Separation of variables: let's see if this works:

$$E_z = AF_1(r)F_2(\phi)F_3(z)F_4(t)$$

where $F_3(z)F_4(t) = e^{j(\omega t - \beta z)}$ by prior assumption. By symmetry, we also have $F_2(\phi) = e^{j\nu\phi}$. Finally, for $F_1(z)$, we have:

$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \phi^2} + q^2 E_z = 0$$

$$\Rightarrow \frac{\partial^2 F_1}{\partial r^2} + \frac{1}{r}\frac{\partial F_1}{\partial r} + \left(q^2 - \frac{\nu^2}{r^2}\right) F_1 = 0$$

Solution of above differential equation: Bessel functions (Please refer)

Boundary conditions

Boundary conditions lead to Bessel functions J_ν, K_ν.

1. The field can't be ∞ at the centre.

2. The field has to "die" far away from the centre (cladding).

Solution with boundary conditions:

$$E_z(r < a) = AJ_\nu(ur)e^{j\nu\phi}e^{j(\omega t - \beta z)}$$

$$H_z(r < a) = BJ_\nu(ur)e^{j\nu\phi}e^{j(\omega t - \beta z)}$$

$$E_z(r > a) = CK_\nu(wr)e^{j\nu\phi}e^{j(\omega t - \beta z)}$$

$$H_z(r > a) = DK_\nu(wr)e^{j\nu\phi}e^{j(\omega t - \beta z)}$$

where $u^2 = k_1^2 - \beta^2, w^2 = \beta^2 - k_2^2$. We need F_1 to be real everywhere. So, $u^2 \geqslant 0, v^2 \geqslant 0 \Rightarrow k_2 \leqslant \beta \leqslant k_1$.

Boundary conditions

At the core-cladding interface, E_ϕ, E_z, H_ϕ, H_z must be continuous.

$$\Rightarrow AJ_\nu(ua) = CK_\nu(ua)$$

$$\Rightarrow BJ_\nu(ua) = DK_\nu(wa)$$

Similarly, E_ϕ, H_ϕ can be found from E_z, H_z, and the continuity condition may be obtained as

$$\frac{1}{u^2}\left[A\frac{j\nu\beta}{a}J_\nu(ua) - B\omega\mu u J'_\nu(ua)\right] = \frac{1}{w^2}\left[C\frac{j\nu\beta}{a}K_\nu(wa) - D\omega\mu w K'_\nu(wa)\right]$$

$$\frac{1}{u^2}\left[B\frac{j\nu\beta}{a}J_\nu(ua) + A\omega\epsilon_1 u J'_\nu(ua)\right] = \frac{1}{w^2}\left[D\frac{j\nu\beta}{a}K_\nu(wa) + C\omega\epsilon_2 w K'_\nu(wa)\right]$$

Boundary conditions

Consolidating above equation, we get

$$\det \begin{pmatrix} J_\nu(ua) & 0 & -K_\nu(wa) & 0 \\ \frac{\beta\nu}{au^2} J_\nu(ua) & \frac{j\omega\mu}{u} J'_\nu(ua) & \frac{\beta\nu}{aw^2} K_\nu(wa) & \frac{j\omega\mu}{w} K'_\nu(wa) \\ 0 & J_\nu(ua) & 0 & -K_\nu(wa) \\ -\frac{j\omega\epsilon_1}{u} J'_\nu(ua) & \frac{\beta\nu}{au^2} J_\nu(ua) & -\frac{j\omega\epsilon_2}{w} K'_\nu(wa) & \frac{\beta\nu}{aw^2} K_\nu(wa) \end{pmatrix} = 0$$

$$\Rightarrow \left(\frac{J'_\nu(ua)}{uJ_\nu(ua)} + \frac{K'_\nu(wa)}{wK_\nu(wa)} \right) \left(\frac{k_1^2 J'_\nu(ua)}{uJ_\nu(ua)} + \frac{k_2^2 K'_\nu(wa)}{wK_\nu(wa)} \right) = \left(\frac{\beta\nu}{a} \right)^2 \left(\frac{1}{u^2} + \frac{1}{w^2} \right)^2$$

Solving above for β gives us the values of β that correspond to the propagating modes of the fibre.

Fibre modes

Inspecting the equation for $\nu = 0$, we get

$$\left(\frac{J_0'(ua)}{uJ_0(ua)} + \frac{K_0'(wa)}{wK_0(wa)}\right)\left(\frac{k_1^2 J_0'(ua)}{uJ_0(ua)} + \frac{k_2^2 K_0'(wa)}{wK_0(wa)}\right) = 0$$

which implies that

$$\underbrace{\left(\frac{J_0'(ua)}{uJ_0(ua)} + \frac{K_0'(wa)}{wK_0(wa)}\right) = 0}_{\text{TE modes}} \text{ or } \underbrace{\left(\frac{k_1^2 J_0'(ua)}{uJ_0(ua)} + \frac{k_2^2 K_0'(wa)}{wK_0(wa)}\right)}_{\text{TM modes}} = 0$$

Proof that these are TE/TM: homework. ☺

For $\nu \geqslant 1$, solution is more complex.

Fibre modes

Convenience substitutions: Normalized frequency or V-number, defined by:

$$V^2 = (u^2 + w^2)a^2 = \left(\frac{2\pi a}{\lambda}\right)^2 (n_1^2 - n_2^2) = \left(\frac{2\pi a}{\lambda}\right)^2 \mathrm{NA}^2$$

Normalized propagation constant b, defined as:

$$b = \frac{a^2 w^2}{V^2} = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2}$$

Linearly polarized modes

- In the weakly guiding case (i.e. $\Delta << 1$ or $n_1 - n_2 << 1$), several mode pairs are almost degenerate. That is, $HE_{\nu+1,m}$ and $EH_{\nu-1,m}$ are very similar. Similar is the case for TE_{0m}, TM_{0m} and HE_{2m}. So, only four field components need to be considered, instead of six.

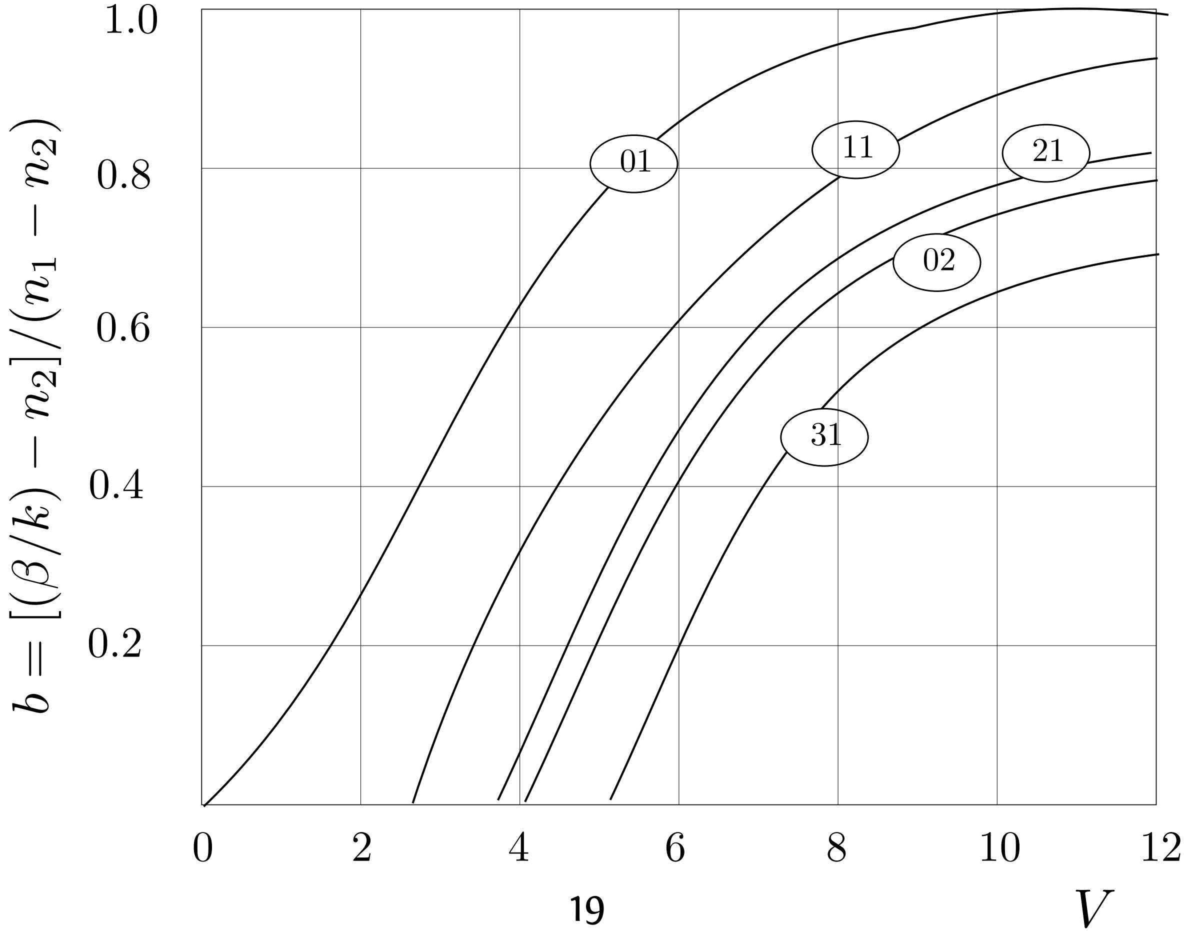
- Linearly polarized modes: Simplification

1. LP_{0m} is derived from HE_{1m} mode

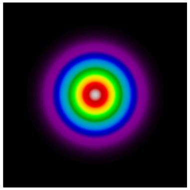
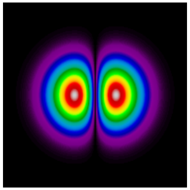
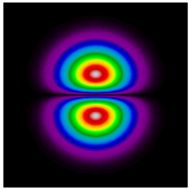
2. LP_{1m} is derived from HE_{2m}, TE_{0m} and TM_{0m} modes

3. $LP_{\nu m}$ for $\nu \geqslant 2$ is derived from an $HE_{\nu+1,m}$ and an $EH_{\nu-1,m}$ mode

Fibre modes



Ideal modes

- Ideal modes have specific field distributions
- Different modes may have different propagation constants
- Orthonormal: overlap integral zero
- Provide “spatial degrees of freedom”

Ideal modes are orthogonal

$$\iint_{-\infty}^{\infty} E^*_{p'q'}(x,y)E_{pq}(x,y)\,dx\,dy = 0 \qquad \text{if} \quad p \neq p' \quad \text{or} \quad q \neq q'$$

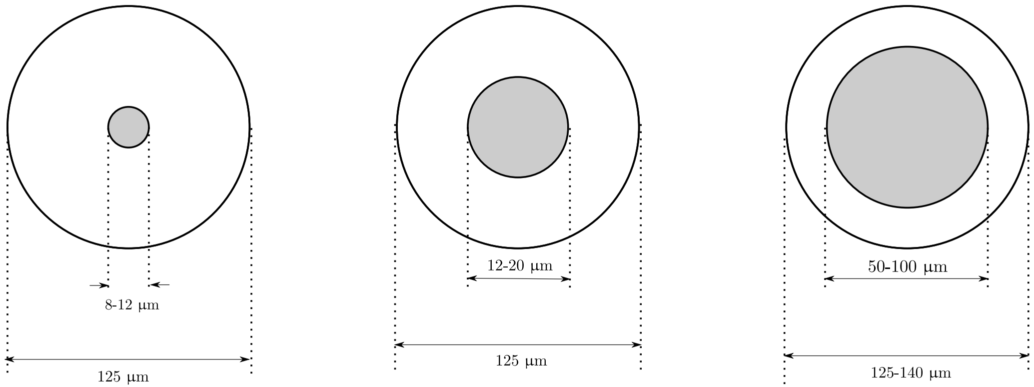
$$= 1 \qquad \text{if} \quad p = p' \quad \text{and} \quad q = q'$$

- Any arbitrary pattern propagating in the fiber can be represented as the linear combination of ideal modes.
- Since each mode can be in x polarized or in y polarized, pattern $E(x,y)$ propagating inside fiber (supporting M spatial modes) can be written as

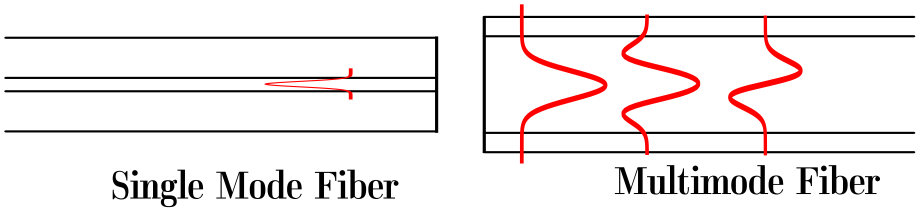
$$\overrightarrow{E}_{2M\times 1} = [a_1^x \; a_2^x \; ... \; a_M^x \; a_1^y \; a_2^y \; ... \; a_M^y]^T$$

Types of fiber

Criterion	Single-mode	Few-mode	Multimode
Core diameter	< 10 µm	< 14 µm	$\geqslant 50$ µm
Alignment tolerance	~0.5 µm	~0.5 µm	>3 µm
Data rate limits	100 Gb/s-km	$>$ 200 Gb/s-km	2 Gb/s-km
Length scales	10-100 km	10-100 km	10 m - a few km
Materials	Silica	Silica	Silica, plastic

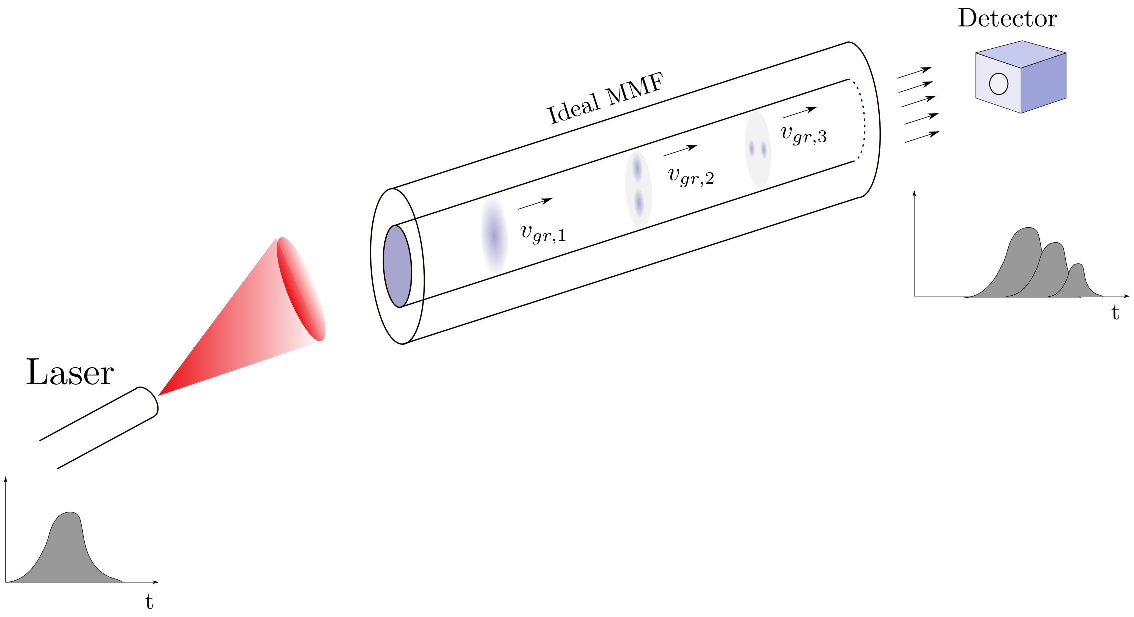


Comparison of SMF and MMF



- SMF: no modal dispersion
- MMF: easier to couple
- Tradeoff: bandwidth-length product

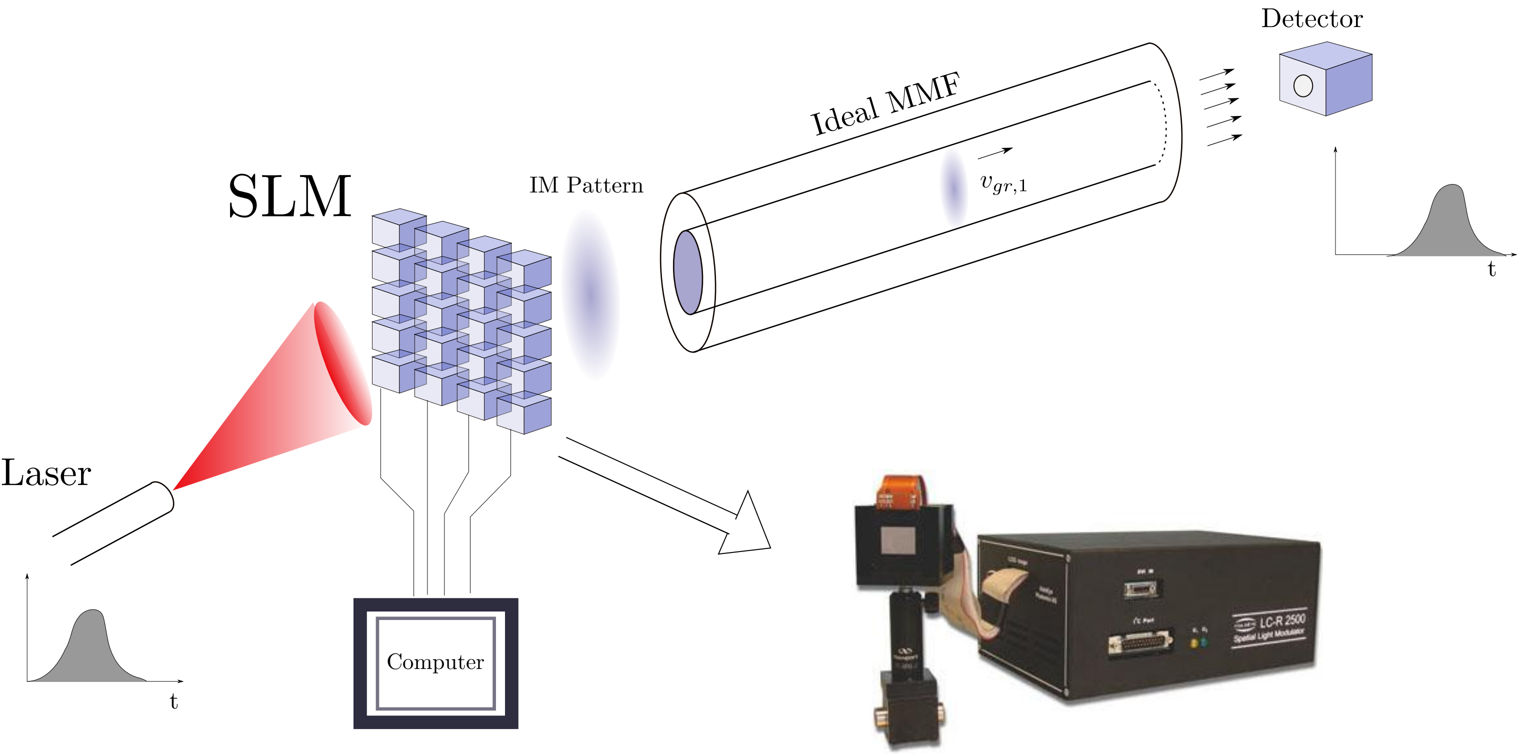
Modal Dispersion



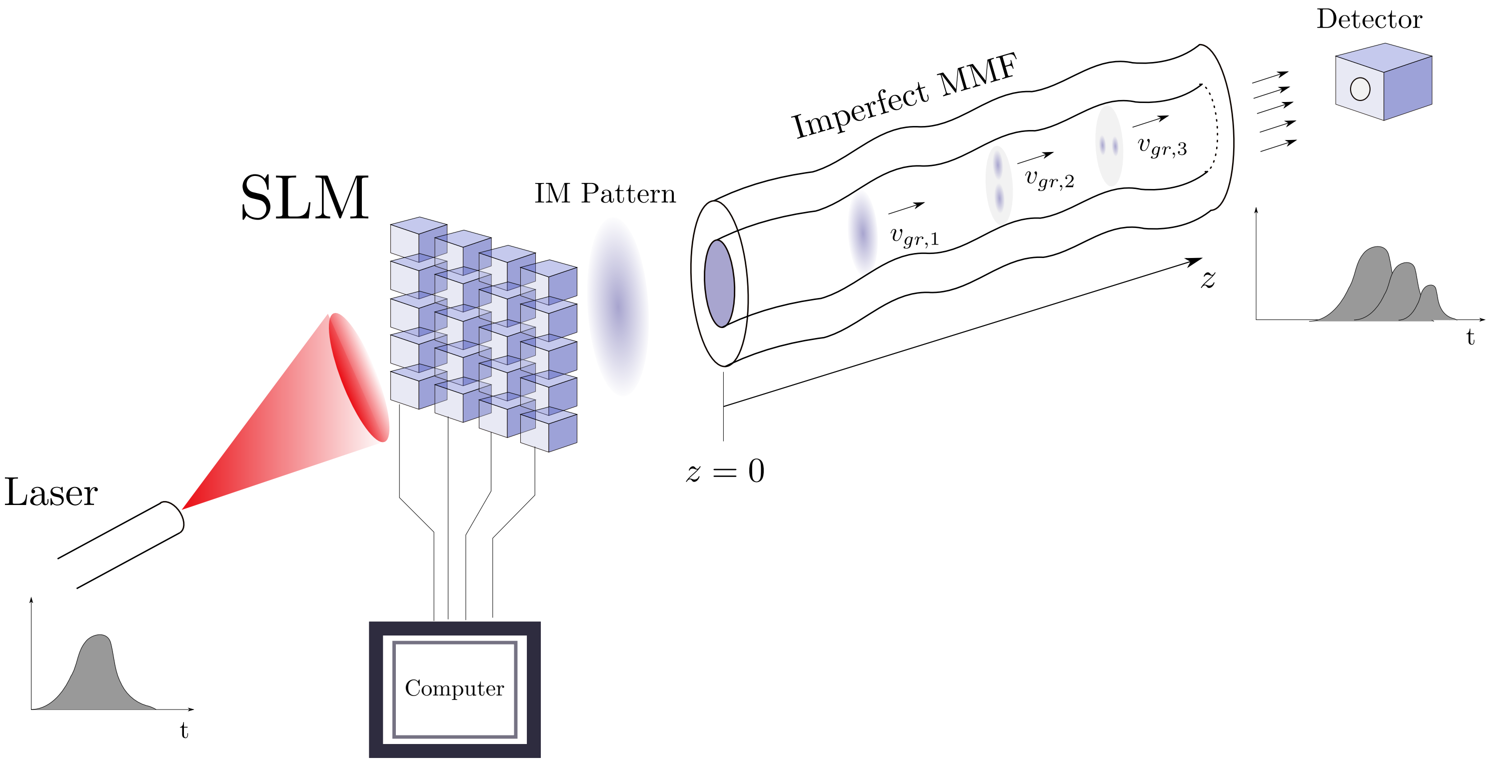
Modal Dispersion

- Different modes travel at different speeds
- Different have different spatial distributions
- Modes "detected" at the photodetector retain individual propagation delays
- Pulse spreading: dispersion, limits data rates
- Conventional MMFs: BW-length product below 1 Gb/s km

Selective excitation of modes



Intermodal coupling



Multiplexing using fiber modes

- Idea: Shape inputs to match each mode shape
- Shape outputs to match each mode shape
- Ideal fiber: spatial multiplexing!
- Issue: Intermodal coupling mixes the signal

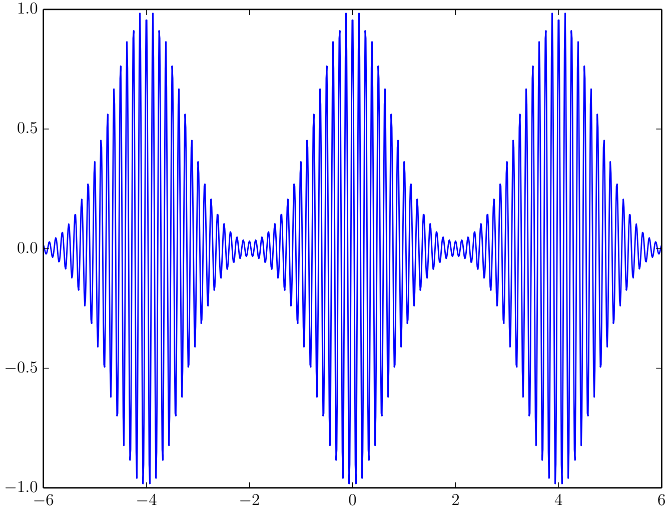
Handling mode mixing

- Mode mixing: linear or nonlinear?
- Coherent or incoherent?
- Depends on what? Fiber length? What else?
- Complexity of implementation?

Next topics

- Incoherent multiplexing techniques
- Coherent multiplexing techniques
- Mode generation, shaping
- DSP implementation issues

Incoherent systems

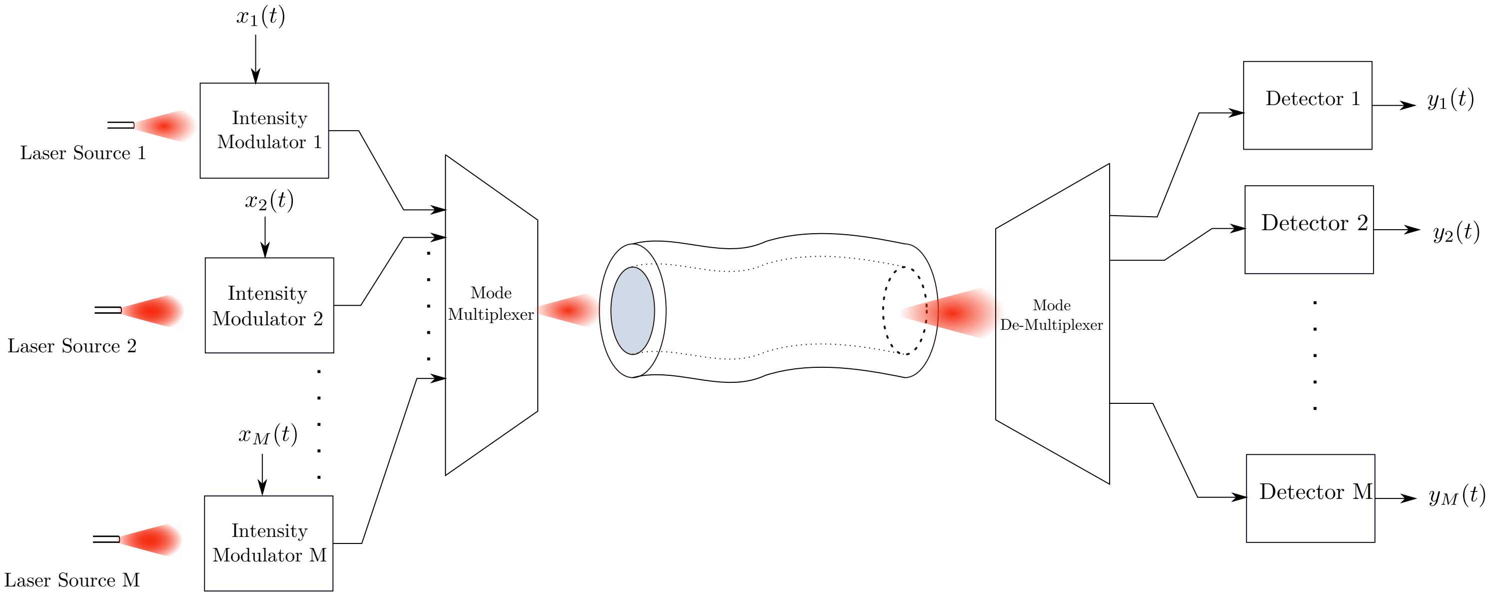


- Incoherent in laser
- Like amplitude modulation

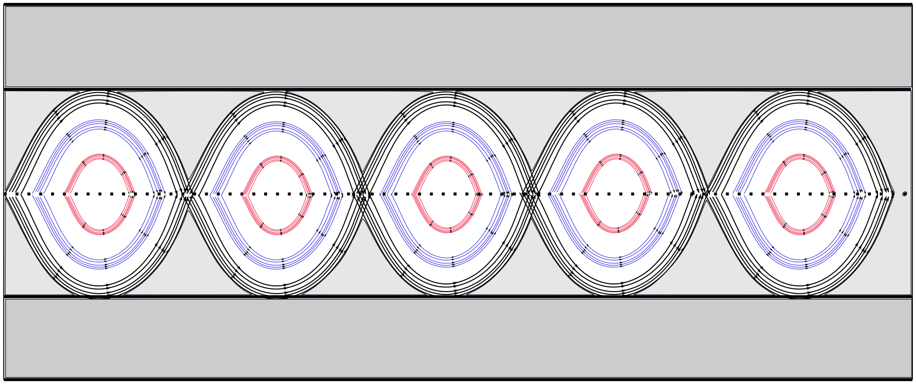
Incoherent multiplexing

- Stuart: dispersive multiplexing
- Offset coupling based solutions
- Mode group diversity multiplexing
- Issues and limitations

Incoherent multiplexing



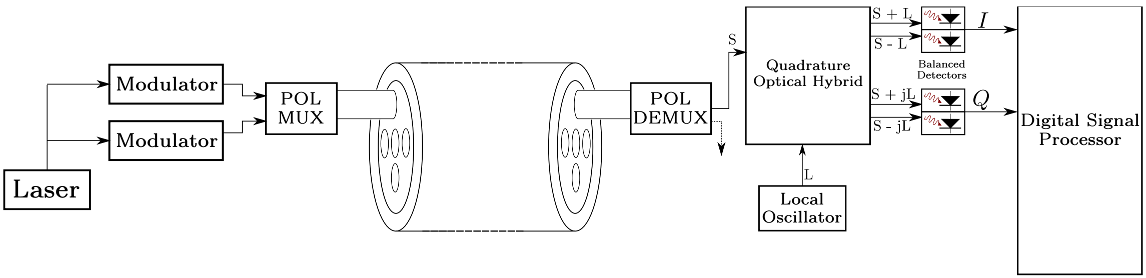
Mode group diversity multiplexing



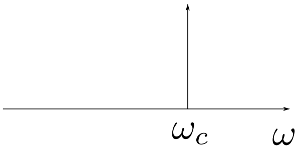
- Idea: Modes with close β values travel "together", so use them to multiplex
- Intermodal mixing: linear methods help reduce cross-talk

Coherent multiplexing

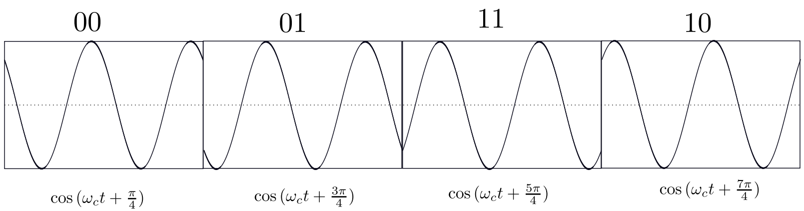
- Coherent: laser + polarization recovered at receiver
- Requires PLL (optical/electrical) or homodyne
- Improves receiver sensitivity
- Price: requires balanced detection



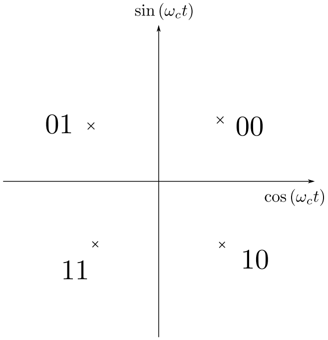
PSK and DPSK



ω_c
ω

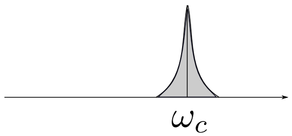


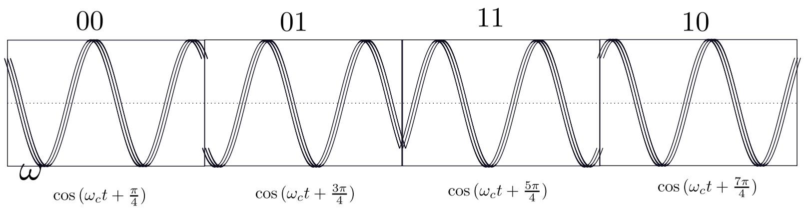
00
01
11
10
$\cos(\omega_c t + \frac{\pi}{4})$
$\cos(\omega_c t + \frac{3\pi}{4})$
$\cos(\omega_c t + \frac{5\pi}{4})$
$\cos(\omega_c t + \frac{7\pi}{4})$

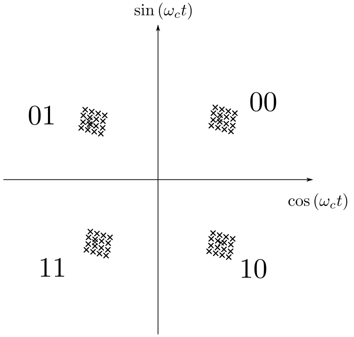


$\sin(\omega_c t)$
01
00
$\cos(\omega_c t)$
11
10

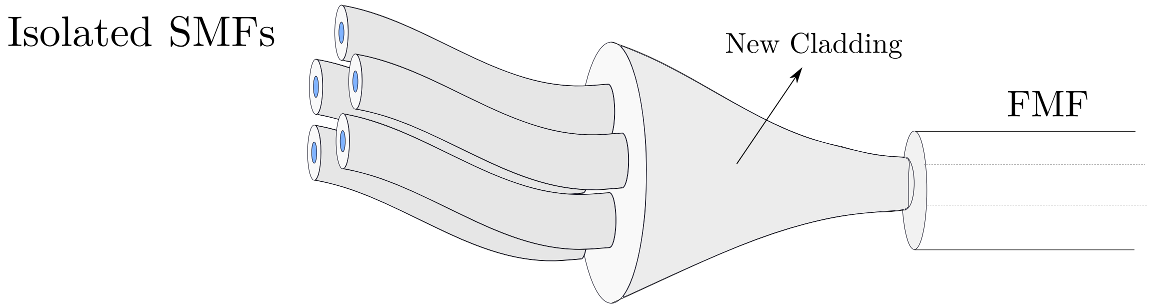
PSK: Phase noise





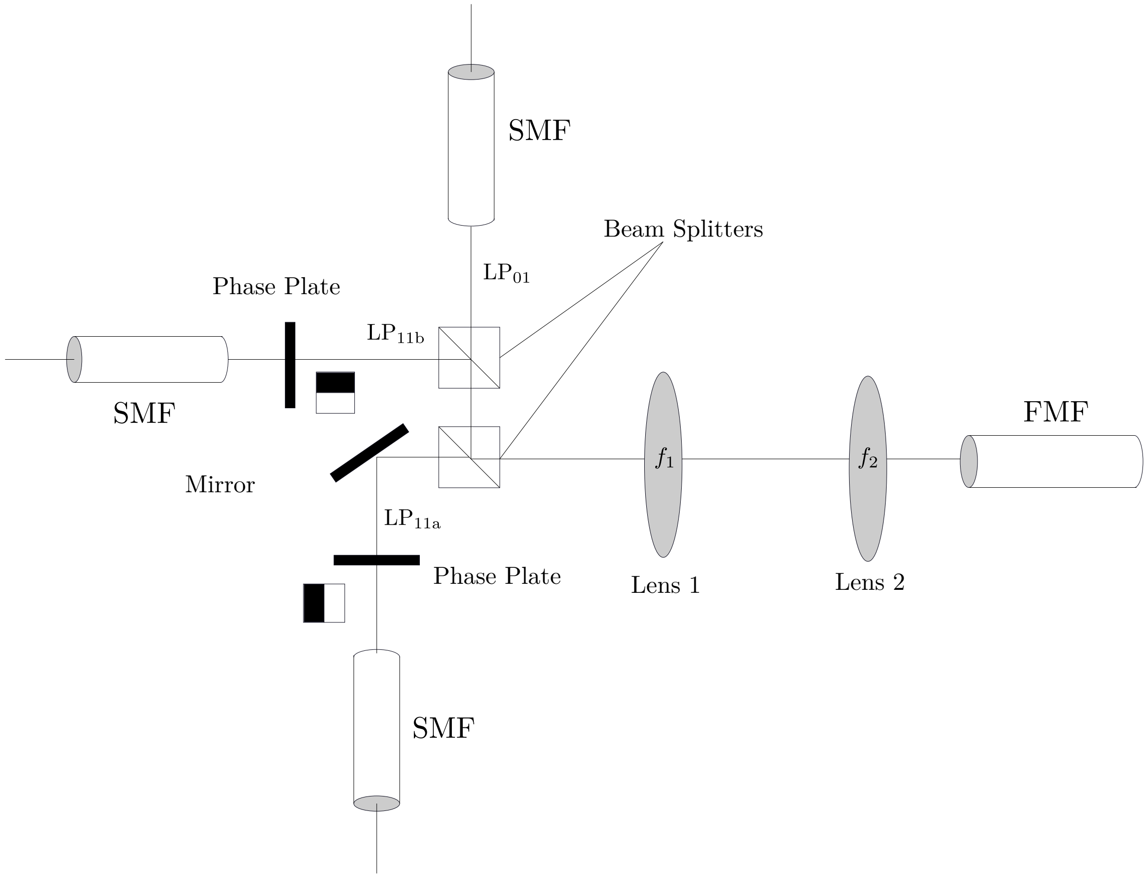


Photonic Lantern

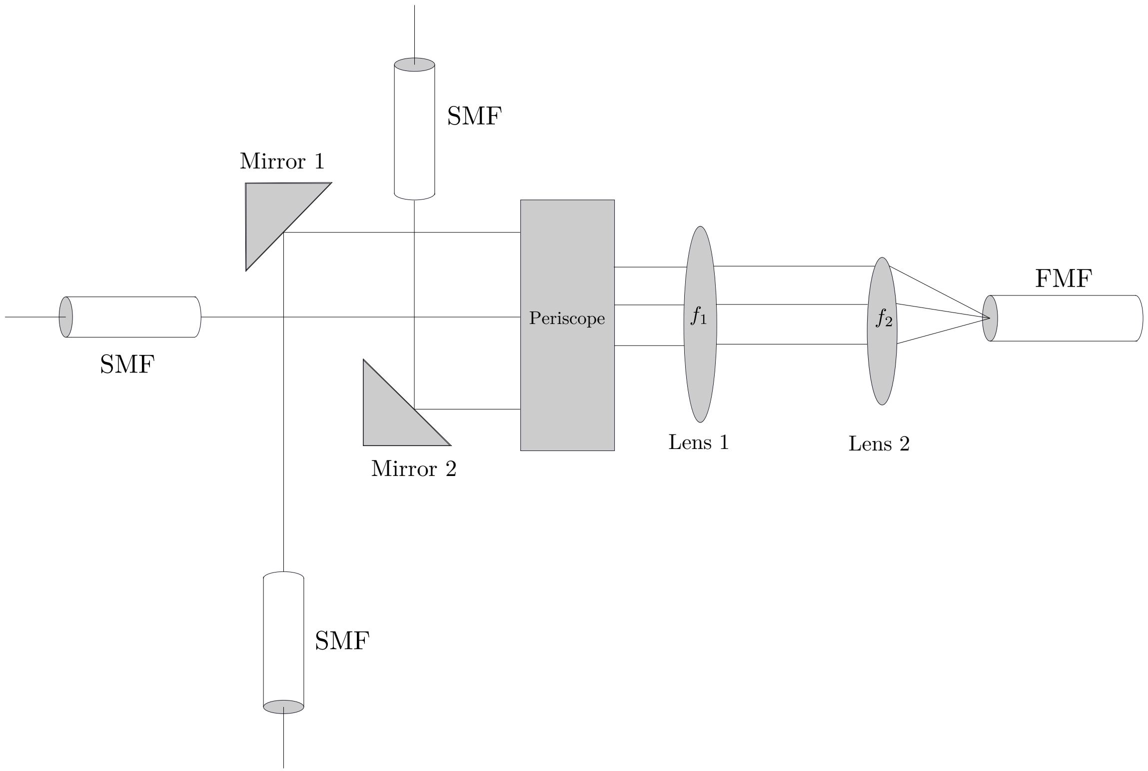


- The SMF ends act as spatial filter
- Different locations of SMFs causes coupling to different modes
- Post-processing required to separate modes

Free space coupling



Free space coupling



Latest trends

- NEC Labs: 1.05 Petabit/s with multicore fiber
- Infinera: Advanced modulation, multiplexing
- Bell Labs: Several SDM experiments
- New fibers: OM4 (100GBASE over 150 metres)
- OM5 + SWDM: Further increase in data rates