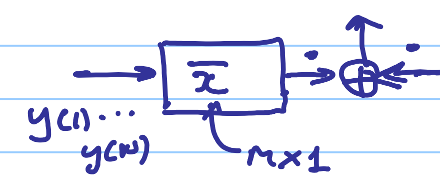
EE6110 Aug.-Nov., 2021

Class #26 (Nov. 06, 2021)

Note Title 27-03-2020

Kalman filter → Models



✓ → State Equation

✓ → Measurement Equation

Statement of the KF Problem

(*) "State" vector $\bar{x}(n)$ $\leftarrow M\times 1$ $\longrightarrow$ set of quantities that need to be estimated and/or tracked

(*) Measurement vector $\bar{y}(n)$ $\leftarrow N\times 1$ $\rightarrow$ scalar measurement model

$\Phi(1)$, $\Phi_1(2)$, …, $\Phi_1(n)$

□ State Equation (Process Equation)

$$\boxed{\underset{M\times 1}{\bar{x}(n+1)} = \underbrace{F(n+1,n)}_{M\times M \text{ "State-Transition Matrix"}}\,\bar{x}(n) + \underbrace{\underset{M\times 1}{\bar{v}_1(n)}}_{\text{(Process) State Noise}}}$$

$F = \lambda I_{M\times M}$

$0 < \lambda < 1$

$\hookrightarrow$ Doppler $\rightarrow$ Jakes' PSD $\rightarrow J_0(1)$

$\bar{v}_1$

$E[\bar{v}_1] = \bar{0}$

$E[\bar{v}_1 \bar{v}_1^H] = \Phi_1$ $(M\times M)$ $= \sigma_{v_1}^2 \cdot I_{M\times M}$

$$\bar{x}_0(n) = \begin{bmatrix} x(n) \\ y(n) \\ z(n) \\ v(n) \\ a(n) \end{bmatrix}_{5\times 1} \begin{matrix} m \\ m \\ m \\ m/s \\ m/s^2 \end{matrix}$$

$(x, y) \rightarrow$ azimuth

$z \rightarrow$ elevation

$v(n) \rightarrow$ velocity

$a(n) \rightarrow$ acceleration

$x_i(n)$

$F \rightarrow \begin{bmatrix} x_1 & & & & 0 \\ & x_2 & & & \\ & & x_3 & & \\ & & & x_4 & \\ 0 & & & & x_5 \end{bmatrix}$

"welltuned"

$F(n+1, n)$

$F(n+1, n) \rightarrow$

$Q_1(n) \rightarrow$

$Q_2 = E[\bar{v}_1 \bar{v}_1^H]$

$Q_1 = \begin{bmatrix} \sigma_1^2 & & & & 0 \\ & \sigma_2^2 & & & \\ & & \sigma_3^2 & & \\ & & & \sigma_4^2 & \\ 0 & & & & \sigma_5^2 \end{bmatrix}$

$\sigma_i \neq \sigma_{v_1}$

F, Q_1, Q_2

"Auto tuning" ← Finely Tuned

"Ill-tuned"

$\bar{x}(n) \rightarrow \hat{x}(n|n)$ ($M \times 1$)

$P(n|n-1)$ ($M \times M$)

Robust KF → bounded uncertainity

$\tilde{F}, \tilde{Q}_1, \tilde{Q}_2$

KF → Parametric Approach

$F, Q_1, Q_2, C(n)$

□ Measurement Equation:

$\Phi_1 \longrightarrow \Phi_2 \neq \Phi_1$

$$\boxed{\underset{N\times 1}{\bar{y}(n)} = \underbrace{C(n)}_{N\times M \text{ Measurement matrix} \Rightarrow \text{known for all time}} \bar{x}(n) + \underset{N\times 1}{\bar{v}_2(n)}}$$

$E(\bar{v}_2 \bar{v}_2^H) = \underset{N\times N}{\Phi_2}$

Assume also that $\bar{x}(0)$, is uncorrelated with $\bar{v}_1(n)$, $\bar{v}_2(n)$ $\forall n \geq 0$

& $E\left[\bar{v}_1(n)\,\bar{v}_2^H(k)\right] = \underset{M\times N}{0} \quad \forall\, n, k$

KF Pbm:

using $\bar{y}(1), \bar{y}(2), \bar{y}(3), \ldots, \bar{y}(n)$, for each $n \geq 1$, find the MMSE estimates of $\bar{x}(i)$

- ✓ (*) Filtering Pbm if $i = n$
- (*) Smoothing Pbm if $i < n$
- ✓ (*) Prediction Pbm if $i > n$

Outline:

$$\bar{\alpha}(n) = \bar{y}(n) - \underbrace{\hat{\bar{y}}(n \mid Y^{n-1})}_{\bar{y}(n-1),\, \bar{y}(n-2) \ldots \bar{y}(1)}$$

innovations process

$$\{\bar{y}(i)\} \rightleftharpoons \{\bar{\alpha}(i)\}$$

$$\hat{x}(n \mid Y^{n}) = \hat{x}(n \mid \alpha^{n}) \rightarrow \text{Gaussian r.v.s}$$

α^{n-1}

no Gaussian assumption

$$\underbrace{2 \text{ derivations of KF}}$$

Filtered KF Eqns: — Haykin

Predictor–Corrector KF Eqn. — J. M. Mendel's