EE6110 Aug.-Nov., 2021

Class #7 (Sep. 09, 2021)

$\underline{\text{SNR}} \left(= \frac{h^2 \sigma_x^2}{\sigma_n^2} \right)$

#7 : Linear MMSE

$\underline{\text{Recall}}$: $y(k) = h x(k) + n(k)$

$\min_{w} E\left[(x(k) - \hat{x}(k))^2 \right] \rightarrow$ c.c.f

$\Rightarrow \hat{x}(k) = \underline{\underline{w\, y(k)}} \dots \rightarrow$

$\underbrace{E[x(k)\, y(k)]}_{h \underbrace{E[x^2(k)]}_{\sigma_x^2}} = E[\hat{x}(k)\, y(k)]$ q.c.f.

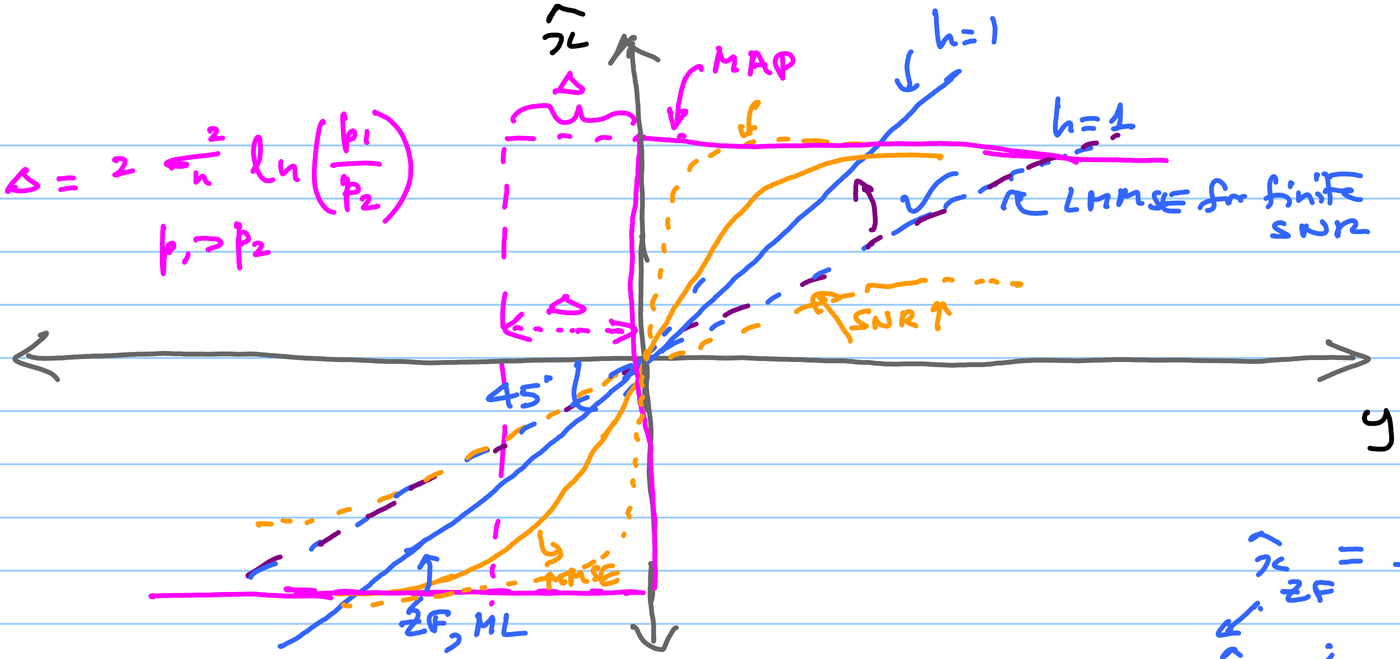
$= w \underbrace{E[y^2(k)]}_{w(h^2\sigma_x^2 + \sigma_n^2)}$

$$w = \frac{h \sigma_x^2}{(h^2 \sigma_x^2 + \sigma_n^2)}$$

$$\boxed{w_{LMMSE} = \frac{1}{h + \left(\frac{\sigma_n^2}{h \sigma_x^2} \right)}} \; ; \; \frac{1}{SNR}$$

SNR↑, $\frac{1}{SNR} \rightarrow 0$

$\Rightarrow w_{LMMSE} \longrightarrow \frac{1}{h}$; $\boxed{\hat{x} = \frac{y}{h}}$ for SNR→∞



$\Delta = 2\sigma_n^2 \ln\left(\frac{p_1}{p_2}\right)$

$p_1 > p_2$

$y = hx + n$

$p_1 = p_2 = \frac{1}{2}$

$x \in \{\underset{p_1}{1}, \underset{p_2}{-1}\}$

$\boxed{h=1}$

$p_1 > p_2$

$\hat{x}_{ZF} = \frac{y}{h} = y$

$\hat{x}_{ML}$; $\hat{x}_{LMMSE}$

Recall . → Complex Differentiation $\quad y = h^{*}x + n$

| Appendix B in Haykin

weight → $w = a + jb$ $\quad j = \text{sqrt}(-1) ;= i.$ $\quad \sqrt{-1}$;

Cost fn. ↳ $J(w)$

Definition :

$$\frac{\partial J}{\partial w} \triangleq \frac{1}{2}\left(\frac{\partial}{\partial a} - j\frac{\partial}{\partial b}\right) ; \longrightarrow \textcircled{1a}$$

$J(w)=\alpha W$

$$\frac{\partial J}{\partial w^{*}} \triangleq \frac{1}{2}\left(\frac{\partial}{\partial a} + j\frac{\partial}{\partial b}\right) ; \longrightarrow \textcircled{1b}$$

$$* \; \frac{\partial w}{\partial w} = 1 \; ; \quad \frac{\partial w}{\partial w^{*}} = 0 = \frac{\partial w^{*}}{\partial w} \; ;$$

Note : For Analytic functions $f(w)$ ← C R

$$\frac{\partial f}{\partial w^{*}} = 0 \quad \text{everywhere !}$$

Vector Case

$$J(\bar{w}) \qquad \bar{w} = \begin{bmatrix} w_0 \\ w_2 \\ \vdots \\ w_{L-1} \end{bmatrix};$$

complex gradient

$$\frac{\partial}{\partial \bar{w}} = \frac{1}{2} \begin{bmatrix} \frac{\partial}{\partial a_0} - j\frac{\partial}{\partial b_0} \\ \frac{\partial}{\partial a_1} - j\frac{\partial}{\partial b_1} \\ \vdots \\ \frac{\partial}{\partial a_{L-1}} - j\frac{\partial}{\partial b_{L-1}} \end{bmatrix}$$

eg: $\frac{\partial}{\partial \bar{w}^*}\left(\underbrace{\bar{w}^H \bar{p}}_{\text{not analytic}}\right) = \bar{p}$; $\qquad \frac{\partial}{\partial \bar{w}}\left(\bar{w}^H \bar{p}\right) = \bar{0} = \frac{\partial}{\partial \bar{w}^*}\left(\bar{p}^H \bar{w}\right)$;