Department of Electrical Engineering, IIT Madras

EE5141: Fundamentals of Wireless and Cellular Communications

Marks 30 | **Simulation Assignment # 2** | **Apr. 27, 2026**

Kindly Note: This assignment sa#2 is to be submitted by email to the TA, ***Mr. M. Ravi,*** ee24m019@smail.iitm.ac.in***,*** *on or before* ***5pm on Tuesday, May 05, 2026****. Mark the name of the pdf file as* ee5141-sa2-your_rollnumber.pdf. *Your Matlab code must be included as an appendix to your report. Independent work is expected from each student, and access to your running Matlab(or Python) code may be required by us, if such a need arises.*

System Model

Sl #	Attribute	Value / Definition
1.	Subcarrier Bandwidth	f_{sub} = 10KHz = $1/T$ (T is useful symbol duration)
2.	FFT size	N = 512
3.	OFDM Signal Bandwidth	W=5.12 MHz
4.	Sampling Rate	$1/T_S = W$ = 5.12 Msps
5.	Cyclic Prefix duration	T_{CP} = 6.25 μsec
6.	Frame duration (S)	S = 5 OFDM blocks (block k =1 to S); The preamble will be the 1st block in the frame. The other 4 blocks carry QAM (and pilot) symbols.
7.	OFDM Symbol duration	$T_{OFDM} = T + T_{CP}$ = 106.25 μsec
8.	Guard Subcarrier (GS) labels	Upper Guard tones: $n \in \{256 \text{ to } 241\}$ DC subcarrier: $n = 0$ Lower Guard tones: $n \in \{-241 \text{ to } -255\}$

Channel Model

Path Gain σ_i^2(in dB scale)	-3	0	-1	-4	-9	-17
Tap Delay m (sample #)	0	7	13	18	21	26

Hint: To normalize average channel gain to unity, in each of these models, rescale the (linear value of) the path variance σ_i^2 to ensure that over the L paths, $\sum_{i=0}^{L-1} \sigma_i^2 = 1$. Each zero-mean path gain a_i, where E[$|a_i|^2$]= σ_i^2 , is a complex Gaussian random variable with each dimension having a variance of σ_i^2 /2. The impulse-response snap-shot $g[k,m]$ corresponding to a given PDP is obtained by calling L times a circular Gaussian random variable (rv), with the variance of the rv based on the power profile. The frequency response snap-shot $G[k,n]$ is obtained by zero-padding plus FFT (of typically large size to visualize shape easily).

1. [2+5=7marks] SC Frequency Sync for Freq. Selective Channel: A preamble symbol is to be designed to ensure that the *entire* frequency offset can be estimated by the Schmidl-Cox (SC) algorithm. Assume that the non-zero subcarriers in the preamble use i.i.d QPSK symbols. The maximum frequency offset seen on the received samples is determined to be Δf = ±28.65 KHz. The samples at the receiver's ADC output can be modeled by $\tilde{y}(k,m) = e^{j2\pi\Delta f m T_s}\,\tilde{r}(k,m)$, where in turn the noisy measurement $\tilde{r}(k,m) = g[k,m] * \tilde{x}(k,m) + v(m)$ ®. Here, "$*$" represents linear convolution, and $\tilde{x}(k,m)$ is obtained by adding the CP to $x(k,m)$, with $\bar{x}(k) = F\bar{d}(k)$ where F is the NxN full DFT (FFT) matrix with scaling factor $1/\sqrt{N}$ to ensure that statistically the average gain of each $\tilde{x}(k,m)$ is unity. Further, assume in ® that $g[k,m]$ is given by the PDP above, and that $v(m)$ is zero-mean, circular Gaussian with variance σ_v^2 . Therefore, the (average) received SNR based on $\tilde{y}(k,m)$ is given by SNR = $1/\sigma_v^2$, which can then be varied by varying the noise variance.

(a) Specify the preamble symbol in the frequency domain and describe this symbol's time-domain properties using a labeled simulated result. What is the maximum frequency offset that it can measure un-ambiguously?

(b) We define the ergodic Mean Square Error (MSE) in the frequency offset estimate by MSE $\triangleq \frac{1}{J}\sum_{j=1}^{J}\left(\Delta f_j - \Delta\hat{f}_j\right)^2$, where $\Delta\hat{f}_j$ is the estimate from the "j^{th}" trial of the SC algorithm with independent signal and noise samples in each trial. Use J =10 for your MSE simulations. Vary the SNR between 0dB and 20dB in steps of 2dB, and measure the MSE in each case. Plot the MSE in the dB scale on the Y-axis, and SNR (also in the dB scale) on the X-axis.

2. [2+4=6marks] SC Timing Sync: For the preamble designed for Δf = 28.65KHz in Q.**1** above:
(a) Given that it is a single-tap channel, i.e., h_m =1 in equation ®, provide the plot of the SC correlation output from which the FFT window (timing instant) is derived. Plot this for SNR=6dB, over 2 consecutive frames of size S = 5 OFDM symbols where the first symbol is the preamble in each frame. Use random QPSK data for the other 4 symbols.
(b) Now, replace the h_m in equation ® with the channel model used in Q.**1**. Provide the plot of the SC correlation output from which the FFT window (timing instant) is derived. Plot this for SNR=6 dB, over 2 consecutive frames of size S = 5 OFDM symbols where the first symbol is the preamble in each frame. Comment on your result.

3. [2+2+5+4+4=17 marks] OFDM Channel Estimation: The OFDM data symbols following the preamble symbol carry embedded pilots. The NP pilot sub-carriers carry (known) QPSK symbols, and have a frequency spacing of 8 subcarriers (n, n-8, n-16,…) starting from uppermost used subcarrier, i.e., subcarrier # 240. Recall that the DC-subcarrier must not be used in any case, whether to carry pilots or data symbols. Assuming perfect timing and frequency synchronization, we have IBI and ICI free measurements which translate after FFT at the receiver to the scalar measurement model given by

$Y[k,n] = X[k,n]G[k,n] + V[k,n]$.

The corresponding vector measurement model on the NP equi-spaced pilot subcarriers is given by

$\mathbf{Y}[n] = \mathbf{X}[n]\mathbf{G}[n] + \mathbf{V}[n]$

where $\mathbf{X}[n]$ is NP x NP matrix and has all the pilot symbols placed on the diagonal. Simulate J=100 independent channel realizations (Monte-Carlo trials) using the above PDP. For each realization of CIR $\mathbf{g}[k]$, let the corresponding CFR be $\mathbf{G}[k]$. The aim is to estimate this CFR using some of the well known channel estimation algorithms, and compare their performance.

Given the N_Ux1 error vector $\mathbf{e}[k] = \mathbf{G}[k] - \widehat{\mathbf{G}}[k]$, (where N_U is the number of useful "data-carrying" subcarriers), the Mean Square Error (MSE) using the 100 Monte-Carlo runs is approximated as follows:

$$MSE \triangleq \frac{1}{J}\sum_{k=1}^{J} \mathbf{e}[k]^H\,\mathbf{e}[k]$$

The MSE performance curve is obtained by plotting 10 $\log_{10}(MSE)$ in the Y-axis versus 10 $\log_{10}$(SNR) on the X-axis, where both are in dB scale. Let the SNR vary in 3 dB steps from 0dB to 30dB. *All the below MSE curves must be plotted on the* ***same*** *graph, for the same set of channel realizations.*

(a) What is the MSE curve of the Zero-Forcing CFR estimate $\widehat{\mathbf{G}}_{ZF}$ measured only on the N_P pilot locations?

(b) If the ZF estimate is linearly interpolated to the remaining subcarriers, what is the MSE performance which is now measured on all the N_U locations?

(c) Instead of linear interpolation, use the FFT-based interpolator discussed in the class. *Bonus Question*: Use any other method including windowing to enhance the MSE performance of this $\widehat{\mathbf{G}}_{FFT}$, and plot the same.

(d) Now, develop the modified Least Squares (mLS) estimator, where the estimator assumes only that the CIR is confined to be less that NCP. Plot the MSE of this $\widehat{\mathbf{G}}_{mLS}$.

(e) Now if the receiver has accurate knowledge of the delay profile (i.e., all the multipath delay locations are known), can the estimation of the corresponding $\widehat{\mathbf{G}}_{mLS}$ be improved? Describe your method clearly and provide this performance curve too. Comment on your result.