EE4140 – MidSem Solutions   October 2022

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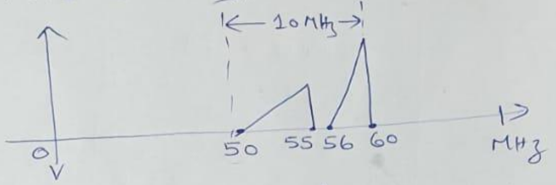
EE-4140 MidSem Solutions oct-22

1. [2+2+1 = 5 marks]



→ The 2 SSB signals can be seen as a composite ~~\#\#\#~~ (like QCM) signal of bandwidth 10 MHz $\Rightarrow 2W = 10$ & $4W = 20$ MHz

(a) * Using upper-band edge of 60 MHz

$$\left\lfloor \frac{60}{20} \right\rfloor = 3 \Rightarrow f_s = \frac{60}{3} = \boxed{20\text{ MHz } (4W')} \quad 1$$

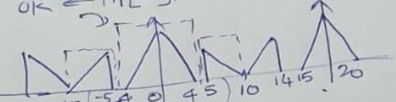
* Using lower-band edge of 50 MHz

$$\left\lfloor \frac{50}{20} \right\rfloor = 2 \Rightarrow f_s' = \frac{50}{2} = \boxed{25\text{ MHz } (25\text{ MHz}) (4W'')} \quad 1$$

Least sampling rate is $\boxed{f_s = 20\text{ MHz}}$;

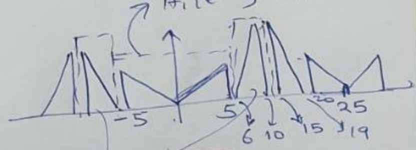
(b) $\boxed{f_s = 20\text{ MHz}}$

LPF OK ← $H_1[e^{j\omega}]$



$H_2[e^{j\omega}]$ → difficult to build (BPF)

$H_1[e^{j\omega}]$ → ok



BPF $H_2[e^{j\omega}]$ → band-gap makes it easy to build

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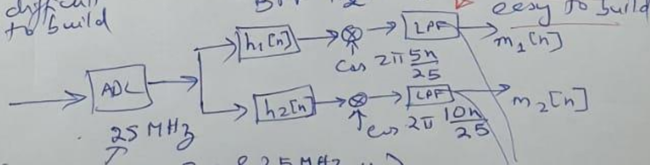

LPF to remove $2f_c$ term

(c) -1 (or, between 20 & 25 MHz any value !!)

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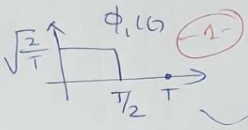
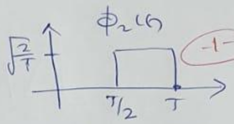
2. [2+2+1 = 5 marks]

(a)



-1-

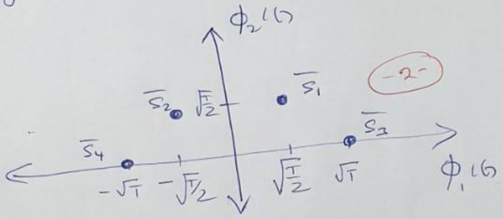
&



-1-

basis functions (note: not unique choice)

(b) Signal constellation



-2-

(c) To find d_{min}

$$d_{1\to 3} = \sqrt{\frac{T}{2} + \left(\sqrt{T} - \sqrt{T/2}\right)^2} = \sqrt{2T}\left(\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}}}\right);$$

✓

$$d_{1\to 2} = \sqrt{2T};$$

$$\therefore d_{min} = d_{1\to 3} = \sqrt{2T}\cdot\left(\sqrt{\frac{0.414}{1.414}}\right)$$

-1-

$$\approx \boxed{0.76537 \cdot \sqrt{T}}$$

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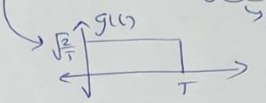
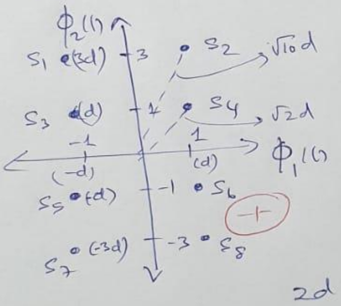
3. [1½ + 3½ = 5 marks]

$$s(t) = \underbrace{I_1(k)}_{\in\{1,-1\}} g(t) \cos 2\pi f_c t + \underbrace{I_2(k)}_{\in\{-3,-1,1,3\}} g(t) \sin 2\pi f_c t$$



$\Rightarrow$ $\phi_1(t) = g(t)\cos 2\pi f_c t$
&
$\phi_2(t) = g(t)\sin 2\pi f_c t$ 1

(a)



$$E_s = \frac{10d^2}{2} + \frac{2d^2}{2} = 6d^2;$$

Since $d = 1$ $-\frac{1}{2}$

$$\boxed{E_s = 6 \text{ Joules}}$$

$$2d = 2\sqrt{\frac{E_s}{6}} = 2;$$

(b) To find P_E, find P_c first

$P_{c1} \longrightarrow$ for s_1, s_2, s_7 & s_8

$P_{c1} = (1-q)^2$; -1

$P_{c2} \longrightarrow$ for s_3, s_4, s_5, s_6

$P_{c2} = (1-q)(1-2q)$; -1

$\therefore$ $P_c = \frac{P_{c1}}{2} + \frac{P_{c2}}{2} = \boxed{\frac{2+3q^2-5q}{2}}$ -1/2

$P_E = 1 - P_c = \boxed{\frac{5q-3q^2}{2}}$ +1

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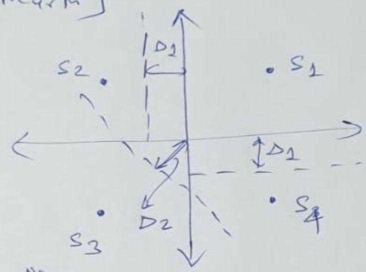
4. [5 marks]



$$\Delta_1 = \frac{d/8}{2d} \ln\left(\frac{5/8}{1/8}\right) = 0.1006 ;$$

$\frac{N_0}{2} \Rightarrow$

$s_1 - s_2$ or $s_1 - s_4$

-1

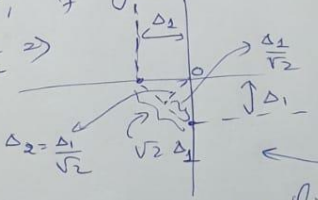
Now, $\Delta_2 = \dfrac{d/8}{\sqrt{2} \cdot 2d} \ln(5) = \dfrac{\Delta_1}{\sqrt{2}} = 0.7113 ;$

$s_1 - s_3$ (offset) note!

-1

However, if you observe carefully

$\Delta_2 = \frac{\Delta_1}{\sqrt{2}} \Rightarrow$



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$\therefore$ Decision regions are like here

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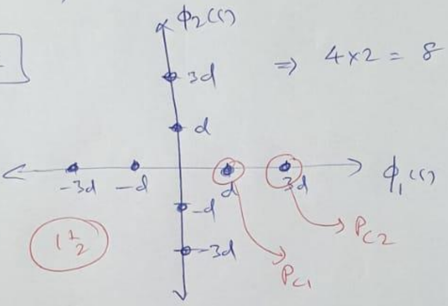
5. [1.5 + 3.5 = 5 marks]

This the extension of the bi-orthogonal constellation with 2-signal points per dimension, to 4-ary PAM type 4 signal points per dimension.

(a) $\boxed{N=2}$ $\Rightarrow 4\times 2 = 8$ signal points



($1\frac{1}{2}$)

(b) To find P_E for $N=4$, first find $P_C = \frac{1}{2}P_{c1} + \frac{1}{2}P_{c2}$ where P_{c1} is defined for "inner" points and P_{c2} is for the outer points as shown above.

for $s_i + n(t) \rightarrow f_N(\alpha) \leftarrow$ AWGN

Then,

$$P_{c2} = \int_{2d}^{\infty} f_N(\vartheta - 3d)\left(1 - 2\int_{\vartheta}^{\infty} f_N(\alpha)\,d\alpha\right)^{3} d\vartheta$$

note note $N-1$ note

($-1\frac{1}{2}-$)

IIIly

$$P_{c1} = \int_{0}^{2d} f_N(\vartheta - d)\left(1 - 2\int_{\vartheta}^{\infty} f_N(\alpha)\,d\alpha\right)^{3} d\vartheta$$

($-1\frac{1}{2}-$)

$P_c = \frac{1}{2}P_{c1} + \frac{1}{2}P_{c2}$ and $P_E = 1 - P_c$;

($-\frac{1}{2}-$)