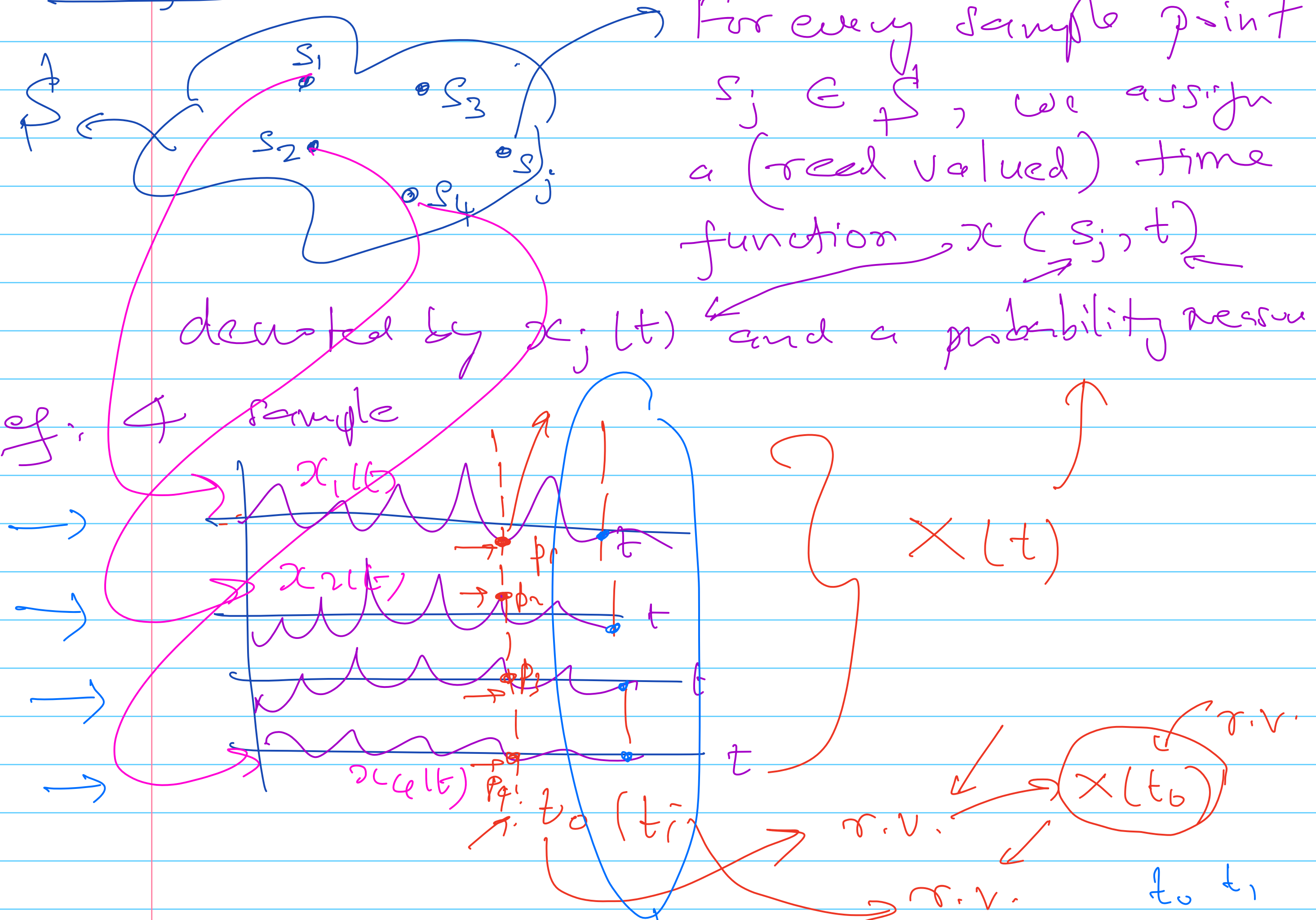
Random Processes

Definition

For every sample point $s_j \in S$, we assign a (real valued) time function $x(s_j, t)$ denoted by $x_j(t)$ and a probability measure p_j

eg: 4 sample



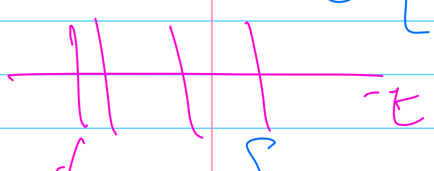
$t_0 \quad t_1$

$t_1 > t_0$

(*) $E[X(t_0)] = \sum_{j=1}^{4} p_j \, x_j(t_0)$

$E[X(t_1)] = \sum_{j=1}^{4} p_j \, x_j(t_1)$?

$\bar{X}_{n\times 1} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$



$x(t_i)$

$\{X(t_0), X(t_1) \ldots X(t_n)\}$ n RVs

→ time-ordered sequence of RVs

$\rightarrow$ If $E[X(t_0)] = E[X(t_1)]$ rv

Yes $\rightarrow$ "<u>stationary</u>" r.p.

<u>Defn</u> (of a r.p.): In general, a probability system composed of an (infinite point) sample space S, an <u>ensemble</u> (or collection) of time functions, and a probability measure, is called a Random Process and is denoted by $X(s,t)$ where $s \in S$ and $-\infty < t < \infty$

<u>Notation</u>:

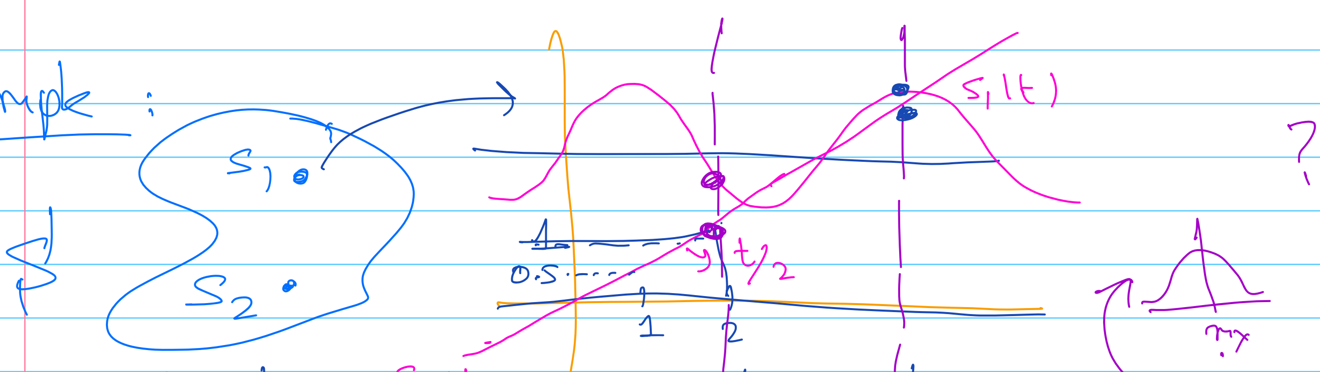
(a) $X(s,t) \longrightarrow X(t)$ r.p. $s_j \to x_j(t)$

(b) $x_j(t) \longrightarrow$ sample function

(c) $X(t_i) \longrightarrow$ rv obtained by sampling the rp at time $t = t_i$

(d) $x_j(t_i) \longrightarrow$ value of the jth sample function at time $t = t_i$

<u>Example</u>:



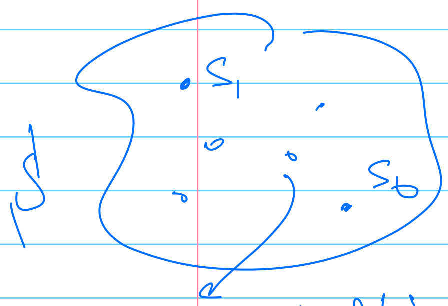
$P(S_1) = P(S_2) = \frac{1}{2}$

$X(t_1)$ $X(t_2)$

$\rightarrow E[X(t_1)] \neq E[X(t_2)]$

$f_{X_1}(x_1) \neq f_{X_2}(x_2)$

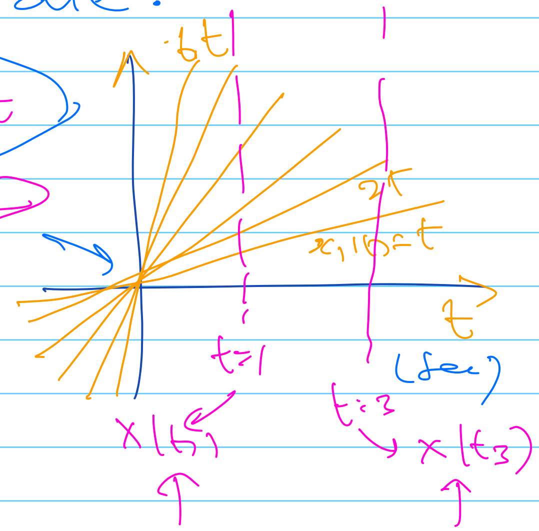
Exercise :



$p(s_i) = \frac{1}{6} \; \forall i$

Pick a waveform (sample function) $x_i(t)$, $i = 1 \ldots 6$, based on a throw ~~of~~ of a die.

$$x_i(t) = i \cdot t$$



(a) Find $f_{X(t_j)}(x)$ of $X(t_j)$

for (i) $t_j = 1$ sec

(ii) $t_j = 3$ sec

(b) Find $E[X(t_j)]$ for both of the above

Stationarity

Fully specified rp

(*) A rp $X(t) \longrightarrow n$ time instants $t_1, \ldots t_n$

$$\underbrace{\{X(t_1), X(t_2) \ldots X(t_n)\}}_{\text{joint distribution function}}$$

(*) joint cdf

$$F_{X(t_1) \ldots X(t_n)}(x_1, x_2 \ldots x_n) \triangleq P\{X(t_1) \le x_1, X(t_2) \le x_2, \ldots X(t_n) \le x_n\}$$

$$F_{\vec{X}(t)}(\vec{x}) \qquad \vec{X}(t) = \begin{bmatrix} X(t_1) \\ \vdots \\ X(t_n) \end{bmatrix}^T_{n \times 1} \quad \& \quad \vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}^T$$

joint pdf (*) $\quad f_{\vec{X}}(\vec{x}; t) \triangleq \frac{\partial^n}{\partial x_1 \partial x_2 \cdots \partial x_n} \cdot F_{\vec{X}}(\vec{x}; t)$

(*) If $F_{\vec{X}}(\vec{x})$ (or $f_{\vec{X}}(\vec{x})$) is known for all finite set of time instances $\{t_1, t_2 \cdots t_n\}$, $n < \infty$, then the rp is said to be fully specified.

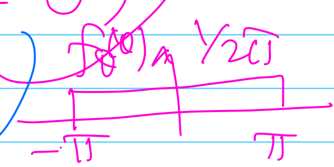
3 ways to specify an rp

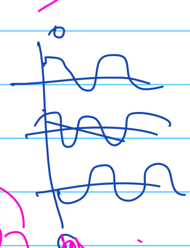
1. For a finite sample space, S, with sample waveforms (which are specified for $\forall t$) $\longrightarrow$ the joint pdf depends in a known way on t

2. The rp is a time function of one or more rvs

$$X(t) = A \cos(2\pi f_c t + \Theta)$$

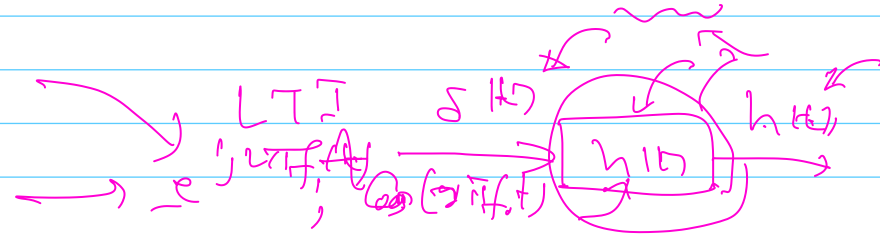
$\Theta \in S$





3. $x(t_1)$ $-A$ $+A$ t_1

$H(f_1) e^{j2\pi f_1 (t - t_1)}$



3

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) \; ?$$

white Gaussian r.p ← "known" r.p

rp is characterized in terms of $x(t)$ & $h(t)$

deterministic transformation / filter

"Model-based Signal Processing"