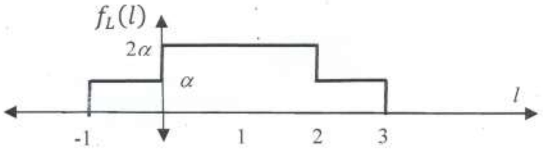
Department of Electrical Engineering
**Indian Institute of Technology, Madras**

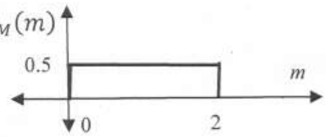
# EE 3005: Communication Systems

April 01, 2025 | **Quiz #2** | 20 Marks

**1. [2+3 = 5 marks]** Let $X$ be an RV with a two-sided exponential PDF given by $f_X(x) = 0.5e^{-|x|}$. For each of the following transformations, answer the following:
(a) For $Y = XU(x)$, where $U(.)$ is the unit step function, explicitly specify and plot the PDF, $f_Y(y)$.
(b) For $Y = (X-2)U(x-2)$, specify $f_Y(y)$, and make a rough plot of the CDF, $F_Y(y)$.

**2. [0.5+3.5 = 4 marks]** The PDFs of two independent RVs $L$ and $M$ are as below:




(a) Find $\alpha$ to make $f_L(l)$ a valid PDF.
(b) Let $N = \frac{M}{2} + 2$. Find the probability P($N$<$L$).

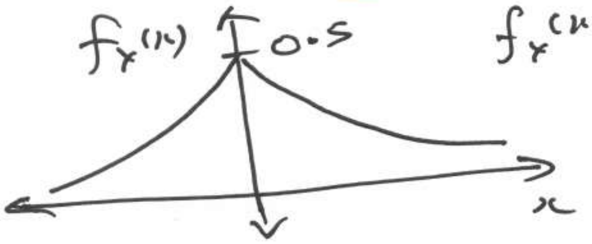
**3. [4.5+1.5 = 6 marks]** The RV $X$ has an uniform PDF between -1 and +1, i.e., $f_X(x)$ is U(-1,+1). Now consider $Y = g(X) = X^2 + X - 6$.
(a) Find and explicitly specify $f_Y(y)$. *Hint*: A plot of $g(X)$ should help you here.
(b) Make a rough plot of $f_Y(y)$.

**4. [2+3 = 5 marks]** Consider a sum of 3 statistically independent RVs, given by $W = X + Y + Z$ where the individual PDFs are $f_X(x)$ is U(-1,+1), $f_Y(y)$ is U(0,+4), and $f_Z(z) = p\delta(z-2) + q\delta(z+2)$.
(a) Find and plot $f_W(w)$ for $p = 1$ and $q = 0$.
(b) Find and plot $f_W(w)$ for $p = 0.5 = q$.

Quiz #2 Solutions

Q1. [2+3 = 5 marks]

$f_X(x) = 0.5\, e^{-|x|}$

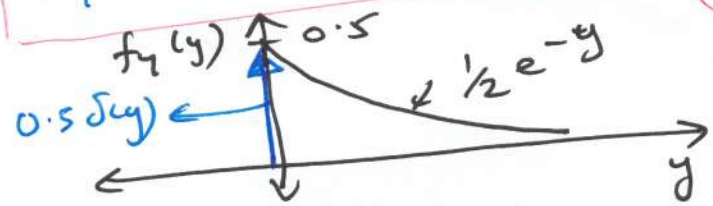



(a) For $Y = X\,u(X)$

$-\infty < x \le 0 \longrightarrow$ maps to "0" value

$$\Rightarrow \boxed{f_Y(y) = \frac{1}{2}\delta(y) + \frac{1}{2}e^{-y}}$$

-2-




(b) For $Y = (X-2)\,u(X-2)$

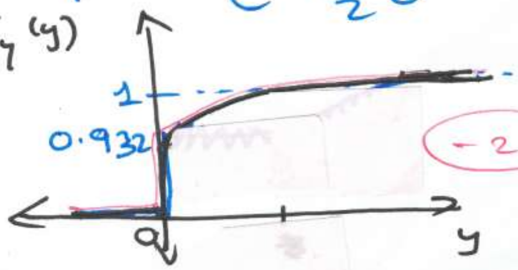
$-\infty < x \le 2 \longrightarrow$ maps to "0" value

$$\frac{1}{2}\int_2^{\infty} e^{-y}\,dy = \frac{0.1353}{2} = 0.0677$$

$$\Rightarrow f_Y(y) = \begin{cases} 0.9323\,\delta(y), & y \le 0 \\ \frac{1}{2}e^{-y-2}, & y > 0 \end{cases}$$

-1-

CDF $F_Y(y)$

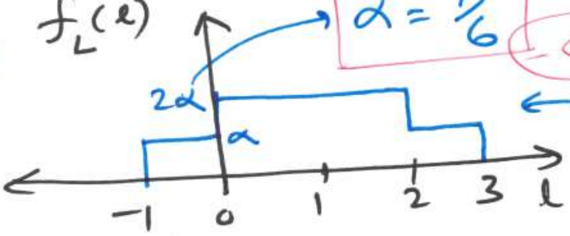



-2-

# EE-3005 Q2 Solns

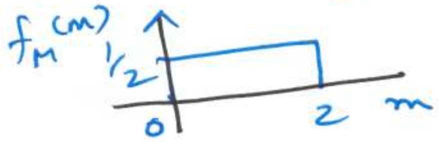
Q2. [0.5 + 3.5 = 4 marks]

(a)

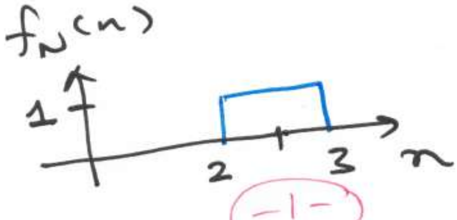



$\boxed{\alpha = 1/6}$ (0.5) ← to make this a valid pdf

(b) $N = \frac{M}{2} + 2$




→




(1)

Now, $P(N<L)$

$$= \underbrace{P(N<L \mid -1<L\le 2)}_{0}\, P(-1<L\le 2)$$

$$+ \underbrace{P(N<L \mid 2<L\le 3)\, P(2<L\le 3)}$$

$$= 0 + P(N<L,\ 2<L\le 3)$$

how?

$$= P(N<L,\ 2<L\le 3,\ 2<N\le 3)$$

$$= \overbrace{P(N<L \mid 2<L\le 3,\ 2<N\le 3)}^{=1/2}$$

$$\underbrace{P(2<L\le 3,\ 2<N\le 3)}$$

independent events (rvs)

$$\underbrace{P(2<L\le 3)}_{=1/6} \cdot \underbrace{P(2<N\le 3)}_{=1}$$

$$\therefore P(N<L) = \tfrac{1}{2}\cdot\tfrac{1}{6} = \boxed{\frac{1}{12}}$$

(2.5)

Q.3 [4.5 + 1.5 = 6 marks]

(a) $g(x) = y = x^2 + x - 6$

$(x+3)(x-2)$

$g'(x) = 2x + 1$

$\Downarrow$

$|g'(x_1)| \,\&\, |g'(x_2)| = \sqrt{4y+25}$

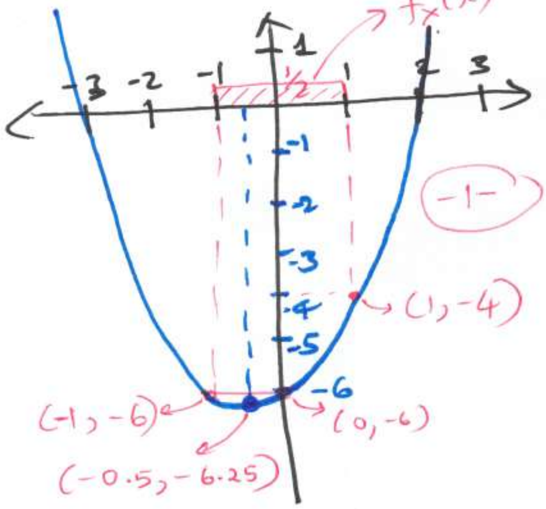
(Solutions) Roots are

$$x_1 = \frac{-1 + \sqrt{4y+25}}{2} \quad \& \quad x_2 = \frac{-1 - \sqrt{4y+25}}{2}$$

and

$$f_y(y) = \frac{f_x(x_1)}{\sqrt{4y+25}} + \frac{f_x(x_2)}{\sqrt{4y+25}}$$ -1-

Now $4y + 25 > 0 \Rightarrow y > -6.25$



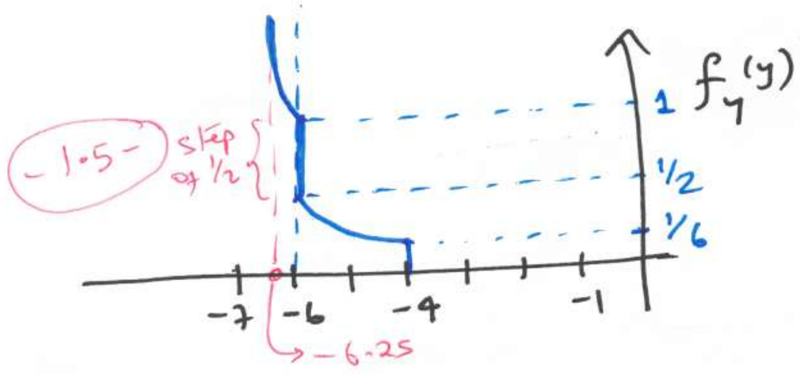

both roots contribute

only 1 contributes

$$f_y(y) = \begin{cases} \dfrac{1}{\sqrt{4y+25}}, & -6.25 < y \le -6 \\ \dfrac{1/2}{\sqrt{4y+25}}, & -6 < y \le -4 \\ 0, & y > 4 \end{cases}$$

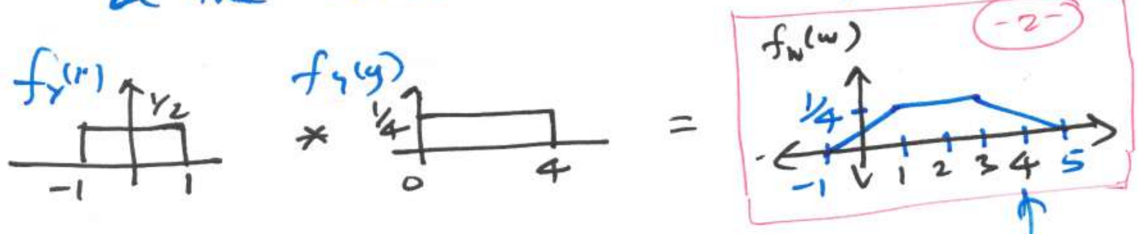
-2.5-

(b)



Q. 4 [2+3 = 5 marks]

(a) Since the 3 rv's are statistically independent, the pdf of W would be the linear convolution of all 3




→ For $p = 1$ & $q = 0$ $f_Z(z) = 1 \cdot \delta(z-2)$ and the effective pdf of $f_W(w)$ is this

(b) For $p = 1/2$ & $q = 1/2$