## Admission Test

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## Important Instructions

- The question paper consists of two sections. Answer both sections.
- Clearly show the steps involved in arriving at the solutions.


## Section I: Control and Digital Systems

1) Find the gain and phase margin of the transfer function $G(s)=\frac{(s-a)}{(s+a)}, a>0$ ? (2marks)
2) Determine which of the following signals are periodic. If so, find the period $T$.
a.) $x(t)=2 \sin (2 t / 3)+3 \cos (2 \pi t / 5)$.
b.) $y(t)=3 \sin (t)+5 \cos (4 t / 3)$.
3) Consider a non-trivial matrix $A=\left[\begin{array}{ll}a_{11} & a_{12} \\ a_{12} & a_{22}\end{array}\right]$. If $A$ has the characteristic equation $\lambda^{2}+a_{1} \lambda+a_{2}=0$, and $I$ is the identity matrix, then
i) Under what condition on $A$, it satisfies $a_{1} A+a_{2} I=0$ ?
ii) Under what condition on $A$, it satisfies $\left(a_{1}+1\right) A+a_{2} I=0$ ?

Give one example of $A$ in each of the above cases.
(2 marks)
4) Find the rank of the matrix

$$
A=\left[\begin{array}{ccc}
1 & -2 & 3 \\
2 & -4 & 6 \\
4 & -8 & 12
\end{array}\right] ?
$$

Further, find all $x \in \mathbb{R}^{3}$ such that $A x=0$.
(2marks)
5) The frequency response of a certain plant plotted on Bode magnitude diagram and asymptotically approximated is as shown in Figure 1. Determine the transfer function of the system. Assume that the plant is minimum-phase.


Fig. 1. Magnitude plot
6) Solve the differential equation $\dot{x}=-x^{1 / 3}, x(0)=x_{0}$.
7) Find a state-space description of the following system?

$$
\dddot{y}+\dot{y}+y=\dot{u}+u
$$

(2 marks)
8) Let 10011 and 01011 be the 2's complement representation of two numbers. Determine the product.
9) Realize the Boolean function $F(x, y)=(x y)$ using the smallest number of 2 -to-1 multiplexers (and no other logic elements). Draw the circuit.
(2 marks)
10) Realize a counter that counts from 0 to 3 (and back to 0 ) as per the table below. Your realization should use only 2 D flip-flops and have the smallest number of external gates. Note that $Q 0$ and $Q 1$ are outputs of flip-flops whose inputs are $D 0$ and $D 1$ respectively. Draw the arrangement with the flip-flops and the external gates

| Q0 | Q1 | D0 | D1 |
| :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |

## Sample Question Paper-Instrumentation

1. A permanent magnet moving coil type voltmeter is used for measurement of voltage in a circuit. If the voltage is $v(t)=10+10 \sin 314 t+10 \cos 314 t$, reading of the meter will be $\qquad$ [1.5 marks]
2. A second order Sallen-Key Low-Pass filter of cut-off frequency 10 kHz will act as an integrator from the frequency $\qquad$ Hz (approx). The roll-off rate of the filter just after 10 kHz is $\qquad$ $\mathrm{dB} /$ decade.
3. Calculate the gain $\left(v_{d} / v_{s}\right)$ of the amplifier shown below. Assume the NMOS is in saturation region and it has a very large Transconductance $\left(g_{m}\right)$.

4. Calculate the current $I$ flowing through $R$ in the circuit given below.


Ans: $I=$
5. In the circuit given below, switch $\mathrm{S}_{1}$ will be in position-1 when clock (CLK) is low. It will be in position- 2 when CLK is high. Output voltage $v_{0}$ is given to an oscilloscope.
(a) Draw the waveform $v_{o}(t)$ that will be displayed in the oscilloscope.
(b) Indicate value of $v_{0}$ at the end of 10 clock cycles.

Assume that the opamp is ideal. Also, consider that Y- sensitivity of this oscilloscope is $0.5 \mathrm{~V} / \mathbf{d i v}$. CLK is indicated in the oscilloscope display.
[3 marks]

6. Consider, in the circuit given below, that the transistors $\mathrm{Q}_{1}$ and $\mathrm{Q}_{2}$ are matched and isothermal. What will be the output voltage $\mathrm{V}_{\text {OUT }}$, if $\mathrm{I}_{1}=10 \mu \mathrm{~A}, \mathrm{I}_{2}=1 \mu \mathrm{~A}$, thermal voltage of the transistors $\mathrm{V}_{\mathrm{T}}=25 \mathrm{mV}, \mathrm{R}_{1}=1 \mathrm{k} \Omega$ and $\mathrm{R}_{2}=3.3 \mathrm{k} \Omega$. Indicate all relevant intermediate steps and equations derived. $[0.5+0.5+1+1+1=4$ marks $]$

$\mathrm{V}_{\text {BEI }}=$ $\qquad$ (expression)
$\mathrm{V}_{\mathrm{BE} 2}=$ $\qquad$ (expression)
$\mathrm{V}_{\mathrm{L}}=$ $\qquad$ (expression)
$\mathrm{V}_{\text {out }}=$ $\qquad$ (expression)
$\mathrm{V}_{\text {OUT }}=$ $\qquad$ (value)

