Admission Test

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Important Instructions

- The question paper consists of two sections. Answer both sections.
- Clearly show the steps involved in arriving at the solutions.

Section I: Control and Digital Systems

1) Find the gain and phase margin of the transfer function $G(s) = \frac{(s-a)}{(s+a)}, a > 0$? (2marks)

Determine which of the following signals are periodic. If so, find the period T.
a.) x(t) = 2 sin(2t/3) + 3 cos(2πt/5).

b.) $y(t) = 3\sin(t) + 5\cos(4t/3)$.

(2 marks)

- 3) Consider a non-trivial matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{bmatrix}$. If A has the characteristic equation $\lambda^2 + a_1\lambda + a_2 = 0$, and I is the identity matrix, then i) Under what condition on A, it satisfies $a_1A + a_2I = 0$?
 - ii) Under what condition on A, it satisfies $(a_1 + 1)A + a_2I = 0$?

Give one example of A in each of the above cases.

4) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & -2 & 3\\ 2 & -4 & 6\\ 4 & -8 & 12 \end{bmatrix}?$$

Further, find all $x \in \mathbb{R}^3$ such that Ax = 0.

(2marks)

The frequency response of a certain plant plotted on Bode magnitude diagram and asymptotically approximated is as shown in Figure 1. Determine the transfer function of the system. Assume that the plant is minimum-phase. (3marks)

(2 marks)



Fig. 1. Magnitude plot

6) Solve the differential equation $\dot{x} = -x^{1/3}, x(0) = x_0$.

7) Find a state-space description of the following system?

$$\ddot{y} + \dot{y} + y = \dot{u} + u.$$

(2 marks)

8) Let 10011 and 01011 be the 2's complement representation of two numbers. Determine the product. (2 marks)

9) Realize the Boolean function F(x, y) = (xy) using the smallest number of 2-to-1 multiplexers (and no other logic elements). Draw the circuit. (2 marks)

10) Realize a counter that counts from 0 to 3 (and back to 0) as per the table below. Your realization should use only 2 D flip-flops and have the smallest number of external gates. Note that Q0 and Q1 are outputs of flip-flops whose inputs are D0 and D1 respectively. Draw the arrangement with the flip-flops and the external gates

Q0	Q1	D0	D1
0	0	0	1
0	1	1	0
1	0	1	1
1	1	0	0

(3 marks)

- A permanent magnet moving coil type voltmeter is used for measurement of voltage in a circuit. If the voltage is v(t) = 10 + 10 sin 314 t + 10 cos 314 t, reading of the meter will be ______ [1.5 marks]
- A second order Sallen-Key Low-Pass filter of cut-off frequency 10 kHz will act as an integrator from the frequency ______ Hz (approx). The roll-off rate of the filter just after 10 kHz is ______ dB/decade. [1.5 marks]
- 3. Calculate the gain (v_o/v_s) of the amplifier shown below. Assume the NMOS is in saturation region and it has a very large Transconductance (g_m) . [2.5 marks]



4. Calculate the current *I* flowing through *R* in the circuit given below. [2.5 marks]



Ans: *I* =_____

- 5. In the circuit given below, switch S_1 will be in position-1 when clock (CLK) is low. It will be in position-2 when CLK is high. Output voltage v_0 is given to an oscilloscope.
 - (a) Draw the waveform $v_0(t)$ that will be displayed in the oscilloscope.
 - (b) Indicate value of v_0 at the end of 10 clock cycles.

Assume that the opamp is ideal. Also, consider that Y- sensitivity of this oscilloscope is **0.5V/div**. CLK is indicated in the oscilloscope display. [3 marks]



6. Consider, in the circuit given below, that the transistors Q_1 and Q_2 are matched and isothermal. What will be the output voltage V_{OUT} , if $I_1 = 10\mu A$, $I_2 = 1\mu A$, thermal voltage of the transistors $V_T = 25mV$, $R_1 = 1 k\Omega$ and $R_2 = 3.3 k\Omega$. Indicate all relevant intermediate steps and equations derived. [0.5+0.5+1+1+1=4 marks]





V_L =____(expression)

V_{OUT} =____(expression)

V_{OUT} =____(value)