Sample paper for graduate admission test for EE7 - Control and Optimization stream

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1. In the following matrix, the equality $a + b = c + d$ holds.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

   (a) Is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector?

   (b) Find both the eigenvalues.

2. Verify if the following functions are differentiable.

   - $|x|^2$
   - $|x|^3$

3. Consider two matrices $A, B \in \mathbb{R}^{n \times n}$. Analyze if the following statement are true or false. If true give reasons/proof and if false give a counter example.

   - The eigenvalues of $A + B$ is same as the sum of eigenvalues of A and B.
   - The eigenvalues of AB is same as the product of eigenvalues of A and B

   In case any of your above answer was false, can you think of some special A and B for which the above condition is true.

4. The open-loop transfer function of a unity-feedback system is given by $G(s) = \frac{K}{s(\tau s+1)}, K, \tau > 0$. By what factor should the gain K be reduced so that the peak overshoot for a unit-step response of the system is reduced from 75% to 25%?

5. Let

$$f(x) \quad = \quad \begin{cases} e^{-\frac{1}{x}} & x > 0 \\ 0 & x \leq 0 \end{cases}$$

   - Is $f(x)$ continuous at $x = 0$?.
   - Is $f(x)$ differentiable at $x = 0$?, if so, what is the derivative at $x = 0$.

6. Check if all the roots of $s^3 + 7s^2 + 25s + 39 = 0$ have real parts more negative than -1.

7. The open-loop transfer function of a system is given by

$$G(s) = \frac{K}{s(s+2)(s+4)}.$$

   What is the value of K, which will lead to sustained oscillations in the closed-loop unity feedback system? Find the corresponding frequency of oscillation at that gain.

8. The impulse response of a system is $5e^{-10t}u(t)$, where $u(t)$ is the unit step function. Find its step-response?

9. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, find $A^5 + A^3$.

10. A unity feedback system has open loop transfer function $G(s) = \frac{1}{s(3s+1)(s+2)}$. Find the phase crossover and gain crossover frequencies.

11. Determine the transfer function from the frequency response plot shown in figure 1. Assume the system is minimum phase.

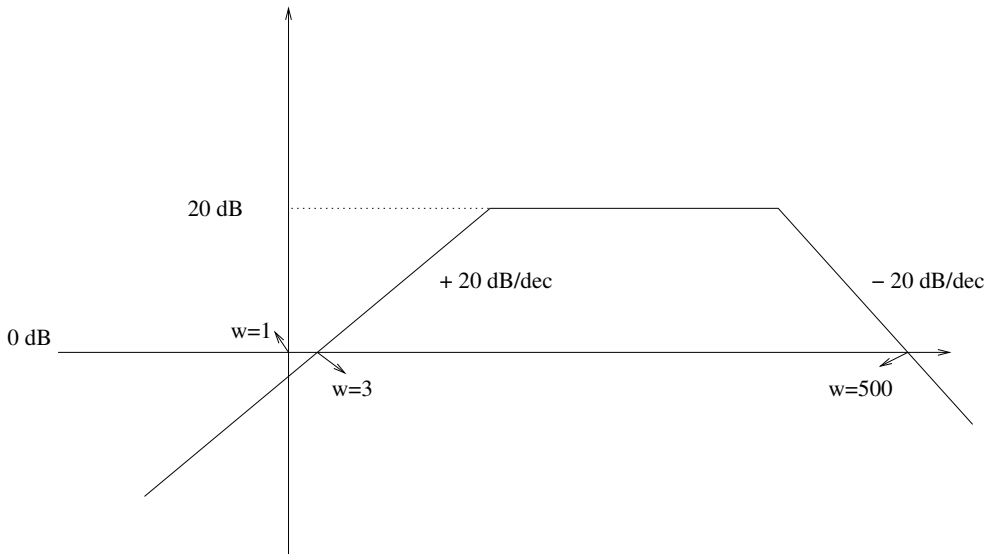

Figure 1: Magnitude plot of $G(j\omega)H(j\omega)$