

To prove

$$\mathbf{C}_{\theta\theta} \mathbf{H}^T (\mathbf{H} \mathbf{C}_{\theta\theta} \mathbf{H}^T + \mathbf{C}_w)^{-1} = (\mathbf{C}_{\theta\theta}^{-1} + \mathbf{H} \mathbf{C}_w^{-1} \mathbf{H}^T)^{-1} \mathbf{H}^T \mathbf{C}_w^{-1}$$

it is the same to prove

$$(\mathbf{C}_{\theta\theta}^{-1} + \mathbf{H} \mathbf{C}_w^{-1} \mathbf{H}^T) \mathbf{C}_{\theta\theta} \mathbf{H}^T = \mathbf{H}^T \mathbf{C}_w^{-1} (\mathbf{H} \mathbf{C}_{\theta\theta} \mathbf{H}^T + \mathbf{C}_w)$$

Then it can be easily proved.