EE 613 Estimation Theory - HW 8
October 08, 2008

1. (8.3) For the signal model
$$s[n] = \begin{cases} A & 0 \le n \le M-1 \\ -A & M \le n \le N-1 \end{cases},$$
find the LSE of A and the minimum LS error. Assume that $x[n] = s[n] + \omega[n]$ for $n = 0, 1, \ldots, N-1$ are observed. If now $\omega[n]$ is WGN with variance σ^2, find the PDF of LSE.

2. (8.4) Show that
$$(\mathbf{x} - \mathbf{H}\boldsymbol{\theta})^T(\mathbf{x} - \mathbf{H}\boldsymbol{\theta}) = (\mathbf{x} - \mathbf{H}\hat{\boldsymbol{\theta}})^T(\mathbf{x} - \mathbf{H}\hat{\boldsymbol{\theta}}) + (\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})^T\mathbf{H}^T\mathbf{H}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta})$$
where
$$\hat{\boldsymbol{\theta}} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{x}$$
Use this to argue that $\boldsymbol{\theta} = \hat{\boldsymbol{\theta}}$ is the LSE.

3. (8.11) In this problem we prove that a projection matrix $\mathbf{P}$ must be symmetric. Let $\mathbf{x} = \boldsymbol{\xi} + \boldsymbol{\xi}^\perp$ where $\boldsymbol{\xi}$ lies in a subspace which is the range of the projection matrix or $\mathbf{P}\mathbf{x} = \boldsymbol{\xi}$, and $\boldsymbol{\xi}^\perp$ lies in the orthogonal subspace of $\mathbf{P}\boldsymbol{\xi}^\perp = \mathbf{0}$. For arbitrary vectors, $\mathbf{x}_1$, $\mathbf{x}_2$ in R^N show that
$$\mathbf{x}_1^T\mathbf{P}\mathbf{x}_2 - \mathbf{x}_2^T\mathbf{P}\mathbf{x}_1 = 0$$
by decomposing $\mathbf{x}_1$ and $\mathbf{x}_2$ as discussed above. Finally, prove the desired result.

4. (8.12) Prove the following properties of the projection matrix
$$\mathbf{P} = \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T$$
   (a) $\mathbf{P}$ is idempotent.
   (b) $\mathbf{H}$ is positive semidefinite.
   (c) The eigenvalues of $\mathbf{H}$ are either 1 or 0.

5. (8.25) If the signal model is
$$s[n] = A + B(-1)^n \qquad n = 0, 1, \ldots, N-1$$
and N is even, find the LSE of $\boldsymbol{\theta} = [A\ B]^T$. Now assume that $A = B$ and repeat the problem using the constrained LS approach. Compare your results.

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