EE 613 Estimation Theory - HW 2

August 8, 2008

1. (2.2) Consider the data $\{x[0], x[1], \ldots, x[N-1]\}$, where each sample is distributed as $\mathcal{U}[0, \theta]$ and the samples are i.i.d. Can you find an unbiased estimator for θ? The range of θ is $0 < \theta < \infty$.

2. (2.5) Two samples $\{x[0], x[1]\}$ are independently observed from a $\mathcal{N}(0, \sigma^2)$ distribution. The estimator

$$\hat{\sigma}^2 = \frac{1}{2}(x^2[0] + x^2[1])$$

is unbiased. Find the PDF of $\hat{\sigma}^2$ to determine if it is symmetric about σ^2.

3. (2.6) For the estimation of a DC level in white Gaussian noise ($x[n] = A + \omega[n]$, as considered in class), consider the general estimator

$$\hat{A} = \sum_{n=0}^{N-1} \alpha_n x[n]$$

is proposed. Find the α_n's so that the estimator is unbiased and the variance is minimized.

Hint: Use Langrangian multipliers with unbiasedness as the constraint equation.

4. (2.7) Two unbiased estimators are proposed whose variances satisfy $\mathrm{var}(\hat{\theta}) < \mathrm{var}(\check{\theta})$. If both estimators are Gaussian, prove that

$$Pr\{|\hat{\theta} - \theta| > \epsilon\} < Pr\{|\check{\theta} - \theta| > \epsilon\}$$

for any $\epsilon > 0$. This says that the estimator with less variance is to be preferred since its PDF is more concentrated about the true value.

5. (2.10) Consider $x[n] = A + \omega[n]$ as discussed in class. Assume that both A and σ^2 are unknown. We wish to estimate the vector parameter

$$\theta = \begin{bmatrix} A \\ \sigma^2 \end{bmatrix}$$

Is the estimator

$$\hat{\theta} = \begin{bmatrix} \hat{A} \\ \hat{\sigma}^2 \end{bmatrix} = \begin{bmatrix} \frac{1}{N}\sum_{n=0}^{N-1} x[n] \\ \frac{1}{N-1}\sum_{n=0}^{N-1} \left(x[n] - \hat{A}\right)^2 \end{bmatrix}$$

unbiased?

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