EE 613 Estimation Theory - HW 1

August 1, 2008

1. (1.3) Let $x = \theta + \omega$, where ω is a random variable with PDF $f_\omega(\omega)$. If θ is a deterministic parameter, find the PDF of x in terms of f_ω and denote it by $f(x;\theta)$. Next, assume that θ is a random variable independent of ω and find the conditional PDF $f(x|\theta)$. Finally, do not assume that θ and ω are independent and determine $f(x|\theta)$. What can you say about $f(x;\theta)$ versus $f(x|\theta)$?

2. (2.1) The data $\{x[0], x[1], \ldots, x[N-1]\}$ are observed where $x[n]$'s are independent and identically distributed (i.i.d.) as $\mathcal{N}(0, \sigma^2)$. We wish to estimate the variance σ^2 as

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{n=0}^{N-1} x^2[n]$$

Is this an unbiased estimator? Find the variance of $\hat{\sigma}^2$ and examine what happens as $N \to \infty$.

3. (2.9) This problem illustrates what happens to an unbiased estimator when it undergoes a nonlinear transformation. Consider the example discussed in class: $x[n] = A + \omega[n]$, where $\omega[n]$ is zero-mean, white, Gaussian noise. If we choose to estimate the unknown parameter $\theta = A^2$ by

$$\hat{\theta} = \left(\frac{1}{N}\sum_{n=0}^{N-1} x[n]\right)^2$$

can we say that the estimator is unbiased? What happens as $N \to \infty$?

4. (2.11) Given a single observation $x[0]$ from the distribution $\mathcal{U}[0, 1/\theta]$, it is desired to estimate θ. It is assumed that $\theta > 0$. Show that for an estimator $\hat{\theta} = g(x[0])$ to be unbiased we must have

$$\int_0^{\frac{1}{\theta}} g(u)\, du = 1$$

Next prove that a function g cannot be found to satisfy this condition for all $\theta > 0$.

—– END —–