Performance Bounds for Computational Imaging

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Abstract: A number of computational imaging techniques have been introduced to improve image quality by increasing light throughput. These techniques use optical coding to measure a stronger signal level. However, the performance of these techniques is limited by the decoding step, which amplifies noise. While it is well understood that optical coding can increase performance at low light levels, little is known about the quantitative performance advantage of computational imaging in general settings. Existing analyses are limited in two ways: (1) most analyses assume a signal independent noise model and ignore signal dependent noise and (2) most analyses neglect to model scene priors. Accurate analysis of multiplexing imaging systems requires us to explicitly consider the effect of both signal dependent photon noise and scene priors. In this work, we perform a careful analytical characterization of the effects of multiplexing under (a) a noise model incorporating both signal dependent and signal independent noise and (b) scene priors modeled both as a Gaussian and as a mixture of Gaussians (GMM). We then discuss the implications of these bounds for several real-world scenarios (illumination conditions, scene properties and sensor noise characteristics).

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1. Summary of Work

The focus of this work is on Computational Imaging (CI) techniques which are designed to improve performance in terms of image quality. These techniques use optical coding to increase light throughput and measure a stronger signal level. Examples include multiplexed 2D imaging [1–5], spectroscopy [6, 7], color imaging [8, 9], light field capture [10–12], illumination multiplexing [13–15], defocus deblurring [10, 16, 17] and motion deblurring [18]. For many CI techniques, there is a corresponding conventional imaging technique that can measure the signal directly without the need for any decoding. For example, narrow-band spectral filters can be used instead of a multiplexed spectrometer, a stopped down aperture can be used to avoid defocus blur and a shorter exposure can be used to eliminate motion blur. In this work, we refer to this class of conventional imaging methods as *impulse imaging*. The term impulse is meant to convey the small amount of light captured by these methods. Fig. 1 gives some example comparisons between CI techniques and their impulse imaging counterparts. We first address an important question in CI system design: What is the performance advantage of a CI technique with respect to the corresponding impulse camera? Since CI techniques capture more light than impulse imaging, it may appear that they must result in a higher SNR. However, CI involves a decoding step (see Fig. 1) which amplifies noise, thereby lowering the signal-to-noise-ratio (SNR). Image priors may be used to regularize the decoding step, but the same priors may also be applied to impulse imaging as well. A straightforward example is in the context of motion and defocusing deblurring cameras, where an important open question remains: Is it better to deblur or denoise?

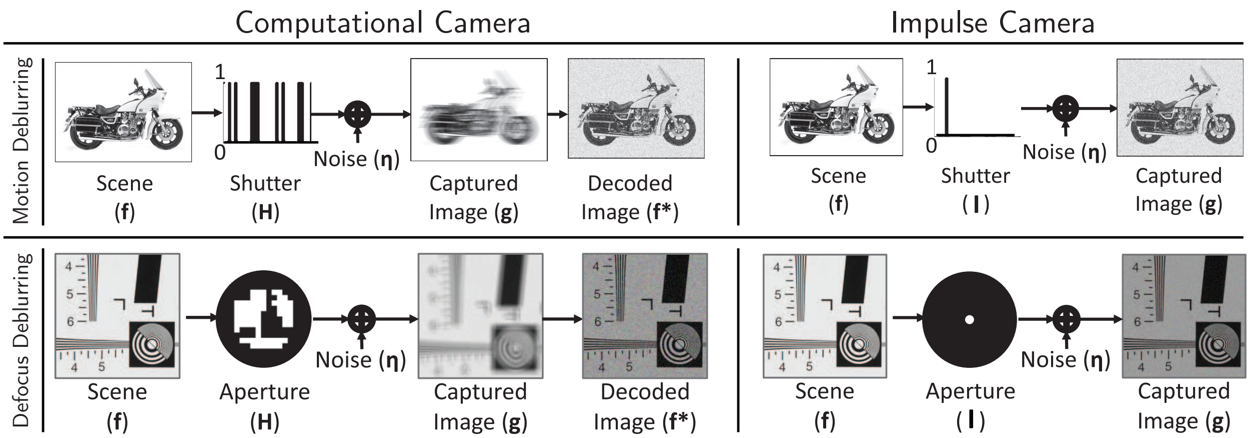

Fig. 1. **Computational versus Impulse Imaging.** (Left) CI techniques discussed in this work can be modeled using a linear image formation model. This includes defocus deblurring, motion deblurring, spectral multiplexing, and many others. In order to recover the desired image, these techniques require an additional decoding step, which amplifies noise. (Right) Impulse imaging techniques measure the signal directly without requiring any decoding. A stopped down aperture can be used to avoid defocus blur, a shorter exposure can be used to avoid motion blur.

While a number of CI systems have been proposed, their analysis has mostly been limited in two important ways: (1) they typically assume signal independent noise while many imaging systems have significant signal dependent photon noise, (2) analysis is performed without scene priors even though most state of art algorithms exploit scene priors. Our goal is to address these limitations. We model scene priors using either a Gaussian Model (GM) or a Gaussian Mixture Model (GMM) and characterize the performance of a CI system by its mean square reconstruction error (MSE). This results in an ability to compute the MSE in a tractable form. Though the prior model and the resulting analysis is quite simple, what is surprising is their ability to predict and characterize the performance of CI systems.

Using a GMM to model prior distributions in image processing problems like denoising and super-resolution has led to impressive results [19], leading us to believe that such priors are indeed state of the art. In addition, there are three significant advantages with such a choice. First, GMM is a well characterized prior model and allows tractable analytical characterization of the MSE [20]. Secondly, GMM is a universal prior in the sense that any prior distribution can be approximated to any desired fidelity using a large number of Gaussian mixtures. This implies that if we could learn very large number of mixture components, then the results of our analysis will apply to the state of the art reconstruction techniques such as those based on sparse regularization, dictionary learning, non local means, etc. Further, we also note that even the (single) Gaussian prior, which is a reduced form of the GMM, works quite well in practice for various applications. Finally, GMM models can be used to analyze both fully-determined and under-determined (compressive) multiplexing systems. Thus, in addition to the aforementioned CI systems, our framework can be used to analyze compressive acquisition systems such as the single pixel camera [21], compressive video acquisition [22–24], compressive hyper-spectral imagers [25, 26] and many others.

References

1. G. Skinner, “X-ray imaging with coded masks,” Scientific American;(USA) **259** (1988).
2. E. Caroli, J. Stephen, G. Cocco, L. Natalucci, and A. Spizzichino, “Coded aperture imaging in x-and gamma-ray astronomy,” Space Science Reviews **45** (1987).
3. M. Ben-Ezra, A. Zomet, and S. Nayar, “Jitter camera: high resolution video from a low resolution detector,” in “CVPR,” (2004).
4. M. Ben-Ezra, A. Zomet, and S. Nayar, “Video super-resolution using controlled subpixel detector shifts,” PAMI (2005).
5. J. Tanida, T. Kumagai, K. Yamada, S. Miyatake, K. Ishida, T. Morimoto, N. Kondou, D. Miyazaki, and Y. Ichioka, “Thin observation module by bound optics (tombo): concept and experimental verification,” Ap-

plied Optics **40**, 1806–1813 (2001).
6. M. Harwit and N. Sloane, "Hadamard transform optics," New York: Academic Press, 1979 **1** (1979).
7. Q. Hanley, P. Verveer, and T. Jovin, "Spectral imaging in a programmable array microscope by hadamard transform fluorescence spectroscopy," Applied Spectroscopy **53** (1999).
8. R. Baer, W. Holland, J. Holm, and P. Vora, "A comparison of primary and complementary color filters for ccd-based digital photography," in "SPIE Electronic Imaging Conference," (Citeseer, 1999).
9. I. Ihrke, G. Wetzstein, and W. Heidrich, "A Theory of Plenoptic Multiplexing," in "CVPR," (2010).
10. A. Veeraraghavan, R. Raskar, A. Agrawal, A. Mohan, and J. Tumblin, "Dappled photography: mask enhanced cameras for heterodyned light fields and coded aperture refocusing," in "SIGGRAPH," (2007).
11. D. Lanman, R. Raskar, A. Agrawal, and G. Taubin, "Shield fields: modeling and capturing 3d occluders," in "SIGGRAPH," (2008).
12. C. Liang, T. Lin, B. Wong, C. Liu, and H. Chen, "Programmable aperture photography: multiplexed light field acquisition," in "ACM SIGGRAPH 2008 papers," (ACM, 2008), pp. 1–10.
13. Y. Schechner, S. Nayar, and P. Belhumeur, "A theory of multiplexed illumination," in "ICCV," (2003).
14. Y. Schechner, S. Nayar, and P. Belhumeur, "Multiplexing for optimal lighting," Pattern Analysis and Machine Intelligence, IEEE Transactions on **29**, 1339–1354 (2007).
15. N. Ratner and Y. Schechner, "Illumination multiplexing within fundamental limits," in "CVPR," (2007).
16. A. Levin, R. Fergus, F. Durand, and W. T. Freeman, "Image and depth from a conventional camera with a coded aperture," in "SIGGRAPH," (2007).
17. C. Zhou and S. Nayar, "What are Good Apertures for Defocus Deblurring?" in "ICCP," (2009).
18. R. Raskar, A. Agrawal, and J. Tumblin, "Coded exposure photography: motion deblurring using fluttered shutter," in "ACM Transactions on Graphics (TOG)," , vol. 25 (ACM, 2006), vol. 25, pp. 795–804.
19. G. Yu, G. Sapiro, and S. Mallat, "Solving inverse problems with piecewise linear estimators: From gaussian mixture models to structured sparsity," arXiv:1006.3056 (2010).
20. J. T. Flam, S. Chatterjee, K. Kansanen, and T. Ekman, "Minimum mean square error estimation under gaussian mixture statistics," arXiv:1108.3410 (2011).
21. M. Duarte, M. Davenport, D. Takhar, J. Laska, T. Sun, K. Kelly, and R. Baraniuk, "Single-pixel imaging via compressive sampling," Signal Processing Magazine, IEEE **25**, 83–91 (2008).
22. D. Reddy, A. Veeraraghavan, and R. Chellappa, "P2C2: programmable pixel compressive camera for high speed imaging," in "Proceedings of IEEE International Conference On Computer Vision and Pattern Recognition," (2011), pp. 329–336.
23. Y. Hitomi, J. Gu, M. Gupta, T. Mitsunaga, S. Nayar, T. Kobayashi, O. Cossairt, D. Miau, S. Nayar, C. Zhou *et al.*, "Video from a single coded exposure photograph using a learned over-complete dictionary," in "ACM Trans. on Graphics (also Proc. of ACM SIGGRAPH)," (2010).
24. M. Gupta, A. Agrawal, A. Veeraraghavan, and S. Narasimhan, "Flexible voxels for motion-aware videography," Proceedings of European Conference On Computer Vision pp. 100–114 (2010).
25. M. Gehm, R. John, D. Brady, R. Willett, and T. Schulz, "Single-shot compressive spectral imaging with a dual-disperser architecture," Optics Express **15**, 14,013–14,027 (2007).
26. A. Wagadarikar, R. John, R. Willett, and D. Brady, "Single disperser design for coded aperture snapshot spectral imaging," Applied optics **47**, B44–B51 (2008).