Note Title 3/20/2008

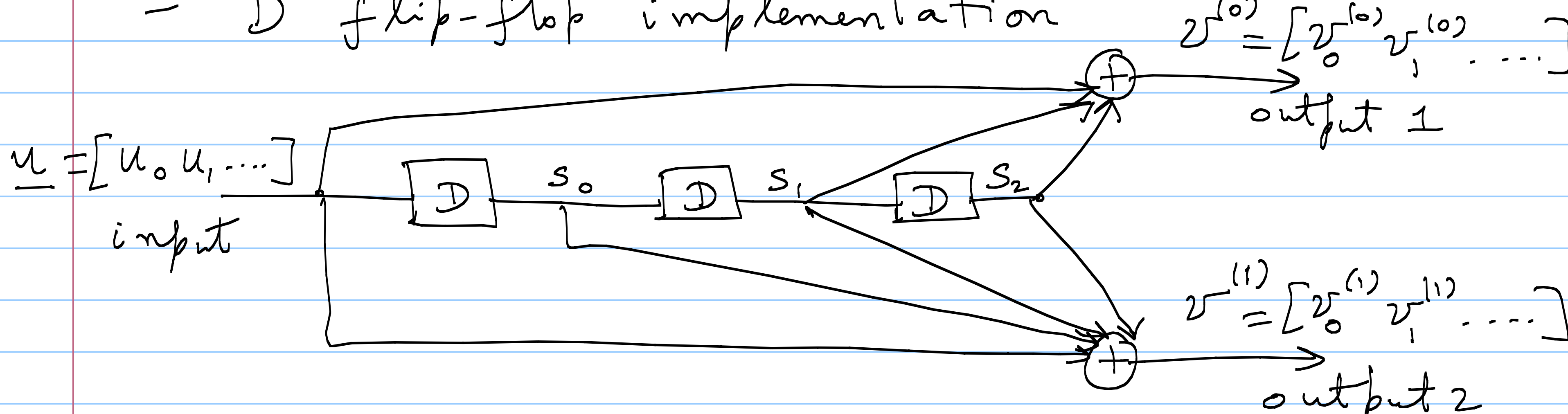
Lecture 29

Convolutional Codes

- Deterministic construction
- "Simplest" encoding
- Soft ML decoding
- Turbo codes

Example of a convolutional encoder:

- finite state machine
- D flip-flop implementation



Memory = 3

# of states = $\underline{\underline{8}}$

output $v = [v_0^{(0)}\, v_0^{(1)}\, v_1^{(0)}\, v_1^{(1)} \ldots\ldots]$

n^{th} time instant: input bit $= u_n$

$$s_0 = u_{n-1}, \quad s_1 = u_{n-2}, \quad s_2 = u_{n-3}$$

$$v_n^{(0)} = u_n + u_{n-2} + u_{n-3}, \quad v_n^{(1)} = u_n + u_{n-1} + u_{n-2} + u_{n-3}$$

Impulse responses: $g^{(0)}$, $g^{(1)}$

$$g^{(0)} = (1\ 0\ 1\ 1)$$
$$g^{(1)} = (1\ 1\ 1\ 1)$$

$$v^{(0)} = u * g^{(0)}$$
$$v^{(1)} = u * g^{(1)}$$

convolution mod 2

D-transform: $x = (x_0\ x_1\ x_2 \ldots\ldots)$

$$x(D) = x_0 + x_1 D + x_2 D^2 + \ldots$$

$$g^{(0)}(D) = 1 + D^2 + D^3, \quad g^{(1)}(D) = 1 + D + D^2 + D^3$$

$$v^{(0)}(D) = u(D)\, g^{(0)}(D), \quad v^{(1)}(D) = u(D)\, g^{(1)}(D).$$

$u = (1\ 1\ 0\ 1\ 1) \rightarrow$ length 5

$v^{(0)} = (1\ 1\ 1\ 1\ 0\ 1\ 0\ 1) \rightarrow 5 + 3 = 8$

$\downarrow$ memory

$v^{(1)} = (1\ 0\ 0\ 1\ 1\ 0\ 0\ 1)$

$v = (11\ \ 10\ \ 10\ \ 11\ \ 01\ \ 10\ \ 00\ \ 11)$

$$\text{Rate} = \frac{k}{2k+\underline{6}} \rightarrow 1/2 \text{ for large } k.$$

u	v
2^k	2^k codewords of length $2k+6$

$(2k+6, k)$-linear block code.

Generator matrix $G(D) = [\, g^{(0)}(D) \quad g^{(1)}(D) \,]$

$$[\, v^{0}(D) \quad v^{(1)}(D) \,] = u(D)\,[\, g^{(0)}(D) \quad g^{(1)}(D) \,]$$

→ rate-$1/2$ encoder

→ We will stick to rate-$1/m$ encoders.

$$G(D) = [\, g^{(0)}(D) \quad g^{(1)}(D) \; \cdots \; g^{(m-1)}(D) \,]$$

Trellis representation

$$v_n^{(0)} = u_n + s_1 + s_2$$

$$v_n^{(1)} = u_n + s_0 + s_1 + s_2$$

$u_0 / v_0^{(0)} v_0^{(1)}$ $\quad \underline{s} = [s_0 \;\; s_1 \;\; s_2]$