

# Problems

Note Title

(6)

GF(16)

$$(a) x + y + z = 0$$

$$(b) x^2 + y^2 + z^2 = 0$$

$$(c) x + y + z = 0$$

$$\underbrace{x^3 + y^3 + z^3}_{=} = 0$$

$$\begin{array}{l} z = 0, \\ \hline \end{array}$$

(7)

BCH code

(8)

BCH code

(9)

$n = 63, t$  BCH code

(a) Smallest  $t$  s.t.  $k \neq n - bt$

$$t = 5$$

(b)  $n = 255$ . Smallest  $t$  s.t.  $k \neq n - 8t$

$$t = 9$$

(10)  $t=1$ , RS code over GF(8)

(a) Find a nonzero codeword  $c = [c_1 c_2 \dots c_7]$

with  $c_1 = c_5 = c_6 = c_7 = 0$

$$H = \begin{bmatrix} 1 & \alpha & \alpha^2 & \alpha^3 & \alpha^4 & \alpha^5 & \alpha^6 \\ 1 & \alpha^7 & \alpha^4 & \alpha^1 & \alpha^8 & \alpha^{10} & \alpha^{12} \end{bmatrix}$$

$$H \underline{c}^T = 0 \Rightarrow \begin{bmatrix} 1 & \alpha & \alpha^7 \\ 1 & \alpha^2 & \alpha^4 \\ 1 & \alpha^3 & \alpha^9 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 0 & \alpha + \alpha^7 & \alpha^4 + \alpha^9 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = 0$$

(b) Nonzero codeword with  $c_2 = c_4 = c_5 = c_6 = 0$

(11) RS code

(12) BCH code

(13)  $n=15$ ,  $t=1$

(a)  $C_1$ : Binary BCH,  $k=11$

$C_2$ : RS code,  $t=1$ , over  $\text{GF}(8)$   
 $N=5$  over  $\text{GF}(8)$

$$K = 5 - 2t = 3 \text{ over } \text{GF}(8)$$

↓ expanded

$$k = 9$$

(b)  $C_1$ :  $\Pr(\text{block error}) = 1 - (1-p)^{15}$

$$C_2: p_s = 1 - (1-p)^3 \rightarrow 105p^2 \underset{\substack{\sim \\ K}}{\approx} p \rightarrow 0$$

$$\Pr(\text{block error}) = 1 - (1-p_s)^5 - 5p_s(1-p_s)^4 \rightarrow 90p^5 \underset{\sim}{\approx} p \rightarrow 0$$

(c)  $C_1$  over  $C_2$ :  $K$  is higher

$C_2$  over  $C_1$ : field is smaller

1A (2,1) code  $C_1 = \{00, 11\}$

(6,4) code  $C_2$  (systematic)

2 bits  $\rightarrow C_1 \rightarrow 4$  bits  $\rightarrow C_2 \rightarrow 6$  bits

Find  $C_2$  s.t. overall code  $C: (6,2,3)$   
code

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

15(a)  $(6,3)$  code  $C$  s.t.  $C \cap C^\perp = \{000000\}$

$$m \underline{G} \underline{G^T} = 0 \Rightarrow m = 0$$

(b)  $(10,2,7)$  binary code X

$$G = \left[ \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \end{array} \right] \quad \begin{array}{l} \text{wt } \geq 7 \\ \text{wt } \geq 7 \end{array}$$

16  $(7,3,4)$  linear code

$$[0011101] \in C, [0100111] \in C$$

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & a & b & c \\ 0 & 0 & 1 & 0 & - & - & - \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- (17) BCH, RS
- (18) Done in class
- (19) burst error-correcting capability
- (20) bug
- (21) bug.