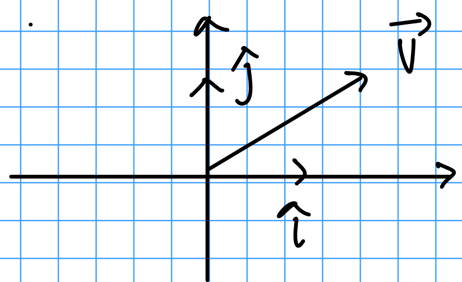
# Parsevals theorem:

The power of a periodic signal is the sum of the powers in each harmonic

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_k |a_k|^2.$$

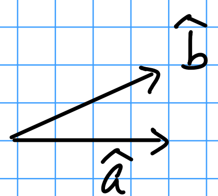
where $x(t) = \sum_k a_k e^{jk\omega_0 t}$.

It is basically because the basis functions $e^{jk\omega_0 t}$ are orthonormal.



$$\vec{V} = c_1 \hat{i} + c_2 \hat{j}$$

$$\hat{i}\cdot\hat{i} = 1 \quad , \quad \hat{i}\cdot\hat{j} = 0.$$



$$\hat{a}\cdot\hat{a} = \hat{b}\cdot\hat{b} = 1$$

$$\vec{V} = d_1 \hat{a} + d_2 \hat{b}$$

$$\vec{V}\cdot\hat{i} = c_1 \quad , \quad \vec{V}\cdot\hat{j} = c_2$$

$$\left.\begin{aligned} \vec{V}\cdot\hat{a} &= d_1 + d_2 \hat{b}\cdot\hat{a} \\ \vec{V}\cdot\hat{b} &= d_1 \hat{a}\cdot\hat{b} + d_2 . \end{aligned}\right] \text{ solve to find } d_1 \,\&\, d_2$$

$$\vec{V}\cdot\vec{V} = c_1^2 + c_2^2$$

$$\vec{V}\cdot\vec{V} = d_1^2 + d_2^2 + 2\, d_1 d_2\, \hat{a}\cdot\hat{b}$$

# Convergence:

$$x(t) = \sum a_k \underbrace{e^{jk\omega_0 t}}_{\text{All derivatives exist}}$$

but $x(t)$ could have discontinuities – possible?

Dirichlet conditions.

Uniform convergence : Given an $\epsilon$, we can find an $N$ (which depends on $\epsilon$) s.t.

$$|x(t) - x_N(t)| < \epsilon, \forall t$$

$$\text{where } x_N = \sum_{k=-N}^{N} a_k e^{jk\omega_0 t}.$$

$N$ independent of 't'.

Pointwise convergence. $N(\epsilon, t)$

For different values of 't', 'N' will be different.

Dirichlet conditions for convergence of the Fourier series.

The Fourier series converges pointwise to the function at all values of 't' where the function is continuous. At points of discontinuity, it converges to a value midway between the values to the left and right of the discontinuity. The conditions of convergence are

(a) $\int_T |x(t)|^2 < \infty$

(b) $x(t)$ should have a finite no. of maxima and minima in one period

(c) $x(t)$ should have a finite no. of

finite discontinuities in one period.

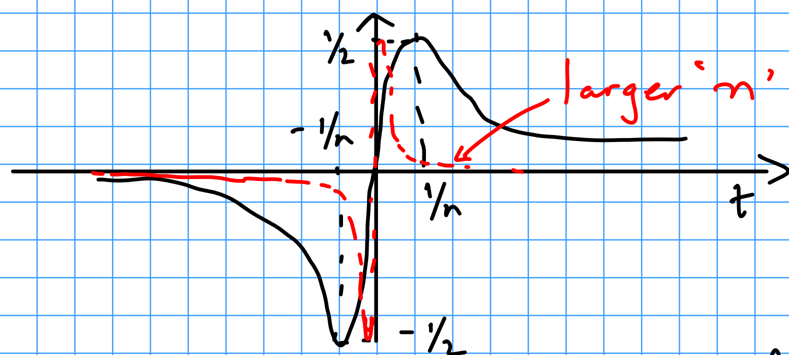
To understand consequences of pointwise convergence

Consider $S_n(t) = \dfrac{nt}{n^2 t^2 + 1}$

$$\frac{\partial S_N}{\partial t} = 0 \quad ; \quad t = \pm \frac{1}{n}$$

at $t = 1/n$ ; $S_n(t) = 1/2$

$t = -1/n$ ; $S_n(t) = -1/2$




However, $\lim_{n \to \infty} S_n(t) = 0$

Value of the maxima and minima is 1/2 and -1/2 ; independent of $n$. As '$n$' becomes larger, the points where the maxima and minima occur will shift towards '0'; but the height will remain the same. This is because for different values of '$t$', the rate of convergence to '0' is different. Need a different value of '$n$' to get the same error. (pointwise convergence)

Gibbs phenomenon; Fourier series

You get overshoots and undershoots

at both sides of the discontinuity. The over/under shoot is about 9% of the magnitude of the discontinuity.

However, Fourier series converges in the mean, i.e.

$$\int_T |x(t) - x_N(t)|^2 dt \to 0 \quad \text{as } N \to \infty$$

So even though there are overshoots and undershoots, the energy in these tends to zero as $N \to \infty$.