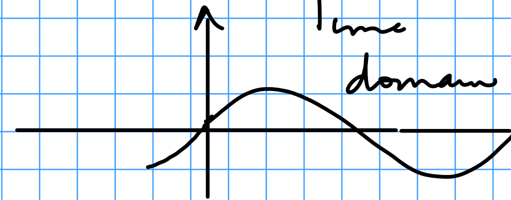
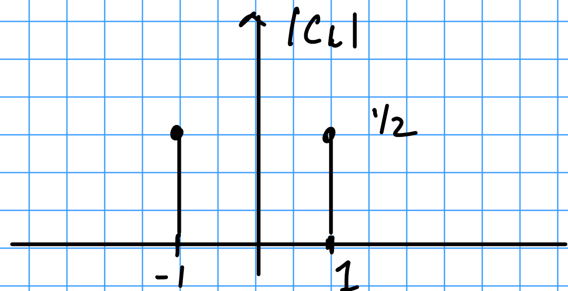
Magnitude and phase spectrum.

1. $\sin \pi t = f(t)$

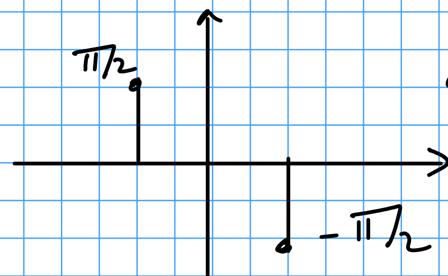


$C_1 = 1/2j \qquad |c_1| = |c_{-1}| = 1/2$

$C_{-1} = -1/2j \qquad \measuredangle c_1 = -\pi/2, \quad \measuredangle c_{-1} = \pi/2.$



magnitude spectrum



Frequency domain representation

Phase spectrum

2. $x(t) = 2 + \sin \pi t + \cos\left(2\pi t + \frac{\pi}{4}\right)$

$C_0 = 2, \quad c_1 = \frac{1}{2j}, \quad c_{-1} = -\frac{1}{2j}$

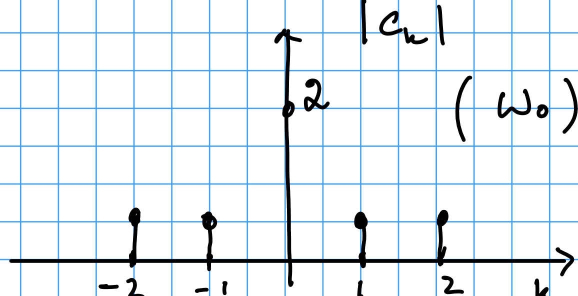
$c_2 = \frac{1}{2} e^{j\pi/4}, \quad c_{-2} = +\frac{1}{2} e^{-j\pi/4}$

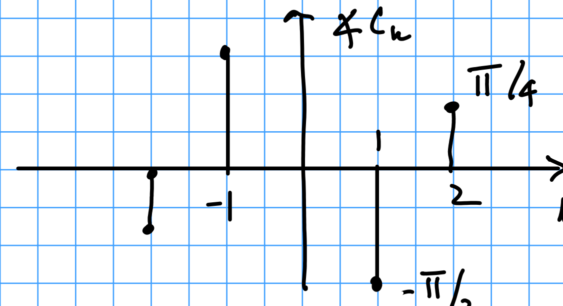
$x(t)$ real; $\quad x(t) = x^*(t)$

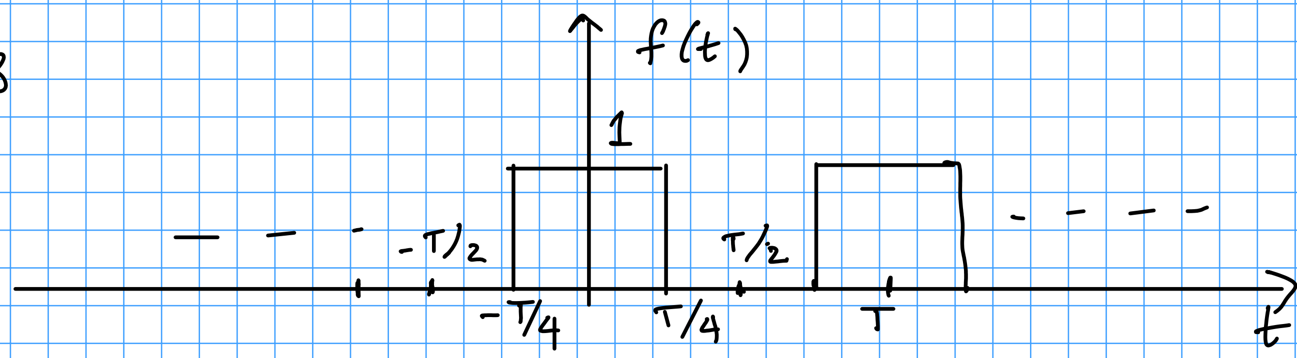
$c_{-k} = c_k^* \qquad c_k = |c_k| e^{j\theta_k}$

$|c_{-k}| = |c_k| \qquad c_k^* = |c_k| e^{-j\theta_k}$

$\measuredangle c_{-k} = -\measuredangle c_k \qquad c_{-k} = c_k^*$







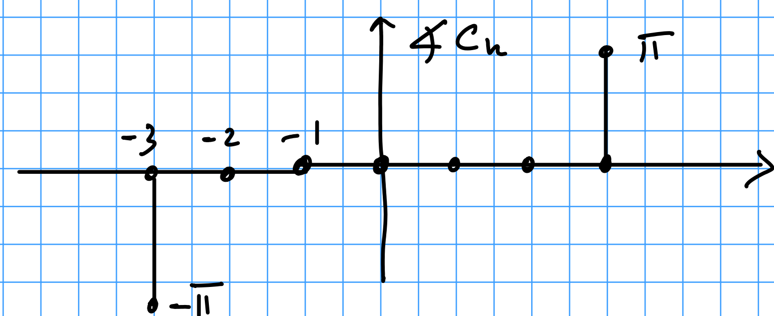
$$C_0 = \tfrac{1}{2}, \quad C_n = \frac{\sin k\pi/2}{k\pi}$$

$$C_1 = C_{-1} = 1/\pi$$

$$C_2 = C_{-2} = 0$$

$$C_3 = C_{-3} = -1/3\pi \quad \ldots$$

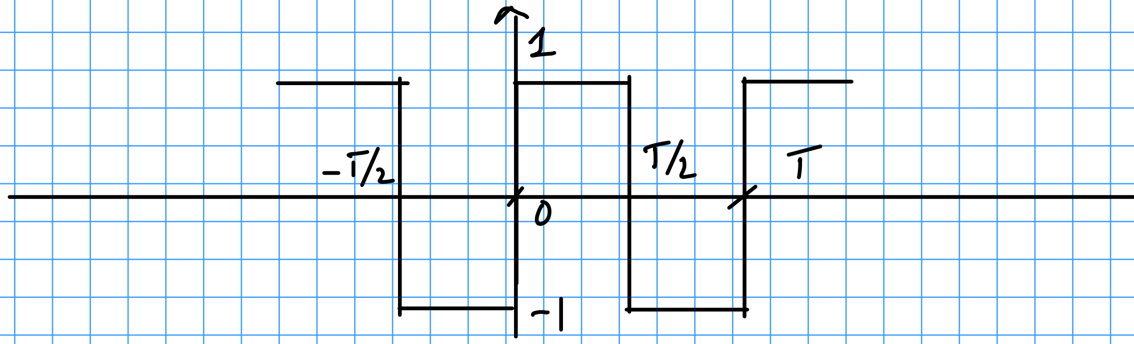
For real signals magnitude spectrum is even; phase spectrum is odd.



Use $-\pi$ to retain the fact that the phase spectrum is an odd function.

Fourier series representation is a frequency domain representation of the periodic signal. If we know what the signal contains we can selectively amplify/remove frequencies.

4.



$C_0 = 0$

$C_0 = 0$, $\quad C_n = 2\sin^2 k\pi/2 \,/\, jk\pi = \dfrac{2}{jk\pi}$;

$c_1 = \dfrac{2}{j\pi}$; $\quad c_{-1} = \dfrac{-2}{j\pi}$ $\quad$ (for odd 'k')

$|c_1| = |c_{-1}| = \dfrac{2}{\pi}$

$\measuredangle c_1 = e^{-j\pi/2}$; $\quad \measuredangle c_{-1} = e^{j\pi/2}$.

$g(t) = 2f(t - T/4) - 1$ $\quad$ [$f(t)$ defined in example 3]

$f(t) \longrightarrow \sum_k a_k e^{jk\omega_0 t}$.

$f(t - T/4) \longrightarrow \sum_k \big(\underbrace{a_k e^{-jk\omega_0 T/4}}\big) e^{jk\omega_0 t}$.

$2f(t - T/4) \longrightarrow \sum_k \left(2a_k e^{-jk\omega_0 T/4}\right) e^{jk\omega_0 t}$

DC shift changes C_0, leaves all other coeffs the same.

$\Rightarrow \; C_k = 2 \cdot \dfrac{\sin k\pi/2}{k\pi} \, e^{-jk\omega_0 T/4}$.

$= 2\,\dfrac{\sin k\pi/2}{k\pi}\left(\cos k\pi/2 - j\sin k\pi/2\right)$

$C_n = 0$ for all even harmonics.

$\Rightarrow C_n = \dfrac{2\sin^2 k\pi/2}{jk\pi} = \dfrac{2}{jk\pi}$.

Properties

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$$f(t) = \sum a_k e^{jk\omega_0 t}.$$

$$f(t)\, e^{jM\omega_0 t} = \sum_k a_k e^{j(k+M)\omega_0 t}.$$

coefficient of the k^{th} harmonic is a_{k-M}.

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$$f(t) = \sum_k a_k e^{jk\omega_0 t}.$$

$$g(t) = \sum_k b_k e^{jk\omega_0 t}.$$

$$f(t)\, g(t) = \sum_l \sum_m a_l b_m e^{j(l+m)\omega_0 t}$$

$l+m = k.$
$l = k-m$ coeffs.

$$= \sum_k \underbrace{\left(\sum_m a_{k-m} b_m\right)}\, e^{jk\omega_0 t}$$

$C_k = \sum_m a_{k-m} b_m.$ discrete time convolution of functions containing the Fourier series coefficients.