EE5120 Applied Linear Algebra Techniques for Electrical Engineers – Tutorial 1

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1. Let $A = \begin{bmatrix} A_1 & A_2 & \cdots A_n \end{bmatrix}$ and $B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix}$, where A_i and B_i are used to denote the columns and rows of the matrices A and B respectively. Use the fact that the columns of C are linear combinations of columns of A to show that the product $C = AB$ can be written as the sum of outer products as follows.

$$C = \sum_{i=1}^{n} A_i B_i$$

2. Using the fact that the columns of AB are linear combination of columns of A, show

$$(AB)^T = B^T A^T$$

3. Without partial pivoting, find the LU decomposition for the following matrix A

$$A = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 7 & 7 \\ 6 & 18 & 22 \end{bmatrix}$$

4. For an LU decomposition with partial pivoting, find all the permutation and lower triangular matrices for the following matrix.

$$A = \begin{bmatrix} 1 & 2 & -3 & 4 \\ 4 & 8 & 12 & -8 \\ 2 & 3 & 2 & 1 \\ -3 & -1 & 1 & -4 \end{bmatrix}$$

5. (a) Consider two vector spaces $\mathcal{V}_1$ and $\mathcal{V}_2$. Write a proof if the statement is True otherwise provide a counter example.

   1. $\mathcal{V}_1 \cap \mathcal{V}_2$ is a vector space
   2. $\mathcal{V}_1 \cup \mathcal{V}_2$ is a vector space

   (b) Are the following sets vector spaces?

   1. $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x = y, x \in \mathbb{R}, y \in \mathbb{R} \right\}$
   2. $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 = y^2, x \in \mathbb{R}, y \in \mathbb{R} \right\}$
   3. $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 > 0, x \in \mathbb{R}, y \in \mathbb{R} \right\}$
   4. $\{\underline{\boldsymbol{u}} : u_i \geq 0 \, \forall i, \underline{\boldsymbol{u}} \in \mathbb{R}^n\}$

6. For the following systems $Ax = b$, transform it to $Rx = \hat{b}$ where R is the RREF. Find solution(s) if they exist.

   a.

$$x + 2y + 3z = 4$$
$$3x + 4y + z = 5$$
$$2x + y + 3z = 6$$

b.

$$x + 2y + 3z = 4$$
$$2x + y = 2$$
$$x + 5y + 8z = 10$$

c.

$$x - 2y + z = 1$$
$$2x - 5y + 3z = 4$$
$$2x - 3y + z = 0$$