A Simple Technique to Evaluate the Noise Spectral Density in Operational Amplifier based circuits using the Adjoint Network Theory

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Abstract

In this paper, we use the adjoint network theory to get quick estimates of the noise contributed by various elements in linear time-invariant opamp based circuits. The noisy operational amplifier is represented using a nullor with an input referred noise voltage and current source. The adjoint model for this representation is derived. This makes the noise analysis of operational amplifier based circuits very simple and the contribution of the operational amplifiers to the output noise can be obtained very often by inspection. The method is applicable to amplifiers, active RC and MOSFET-C filters. Examples of circuits analyzed include the universal active filter and gyrator topologies.

I. Introduction

The spectral density of noise in a linear time-invariant circuit can be written as:

$$S(\omega) = \sum_{i=1}^{M} |H_i(\omega)|^2 S_i(\omega) + \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \left(H_i(\omega)H_j^*(\omega)K_{ij} + H_i^*(\omega)H_j(\omega)K_{ij}^*\right) \sqrt{S_i(\omega)S_j(\omega)} \tag{1}$$

Here M is the total number of noise sources, including the noise sources due to resistors as well as active components, $H_i(\omega)$ is the transfer function from each noise source to the output and K_{ij} is the coherence function between noise sources i and j. In order to compute the output noise spectral density therefore, we need to find these transfer functions. In amplifiers and filters constructed using opamps, the noise sources are the resistors (or MOS transistors used as resistors) and the opamps themselves. The noisy opamp is generally represented as a noiseless amplifier, with input referred noise voltage and current sources. There is a considerable amount of published literature on analysis of noise in opamp based circuits[1-12]. In many of the works, the noise transfer functions are tabulated for specific topologies [2], [6], [5], [7]. In [1], [4], [9], [11], the input referred noise sources of the opamp are approximately referred to convenient locations to determine the noise transfer functions. In [10], [12], the focus is obtaining bounds for the dynamic range, given a power dissipation. However, for a general topology, there is no simple method to evaluate the noise contribution of the various operational amplifiers to the output noise spectral density. This issue is addressed in this paper.

The task of obtaining the transfer function from each noise source to the output can be simplified by using the technique of adjoint networks. It was applied to analysis of noise in LTI circuits in 1971 [13] and is the basis of computer-aided noise analysis in most, if not all circuit simulation routines. Surprisingly however, the concept of adjoint networks has not been used at all to get the noise transfer functions analytically. This of course, is not easy when the circuits are transisitor level circuits. But, we show in this paper, that it can be applied very efficiently to operational amplifier based circuits, if the noiseless opamp can be modelled as a nullor. We derive a methodology to quickly estimate the noise contributed by the various opamps of the circuit to the output noise spectral density. It can be used for arbitrary topologies and is very useful as a design aid.

The paper is organized as follows. Section II includes some background on using the adjoint network technique for noise analysis. In section III, the adjoint model for the nullor with input referred noise sources is derived and the steps involved in obtaining the contribution of the noise sources is outlined. Section IV contains some examples of noise analysis using the method and section V contains the conclusions.

II. Adjoint Networks for Noise Calculations - Background

Let the adjoint of the network N be denoted by $\widehat{N}$. Tellegen's theorem for the networks can be written as:

$$\sum_k (i_k \hat{v}_k - v_k \hat{i}_k) = 0 \tag{2}$$

In equation (2), the subscript k is used for the branches of the circuit in N and $\widehat{N}$. This sum is usually divided into two terms - one each for the internal branches and the ports of the network. All the branches containing the noise sources are classified as ports and the remaining branches are the internal branches of the network. Equation (2) can thus be written as:

$$\sum_b (i_b \hat{v}_b - v_b \hat{i}_b) + \sum_p (i_p \hat{v}_p - v_p \hat{i}_p) = 0 \tag{3}$$

In equation (3), the subscript b stands for the internal branches and p for the ports. Generally, while choosing the elements of the adjoint network, the attempt is to make every term or at most the sum of two terms (in the case of controlled sources) zero, in the summation for the internal branches. As a result, first term in equation (3) vanishes. The choice of elements for the internal branches (including resistors, capacitors, inductors and controlled sources) of the adjoint network is discussed in [13]. Once this is done, Equation 3 can be written as:

$$\sum_p (i_p \hat{v}_p - v_p \hat{i}_p) = 0 \tag{4}$$

This gives the relationship between the various input ports and the output port. As an example, assume that there are only two ports in the networks - an input port containing a noise current source and an output port. This is shown in figure 1. Supposing we wish to find the spectral

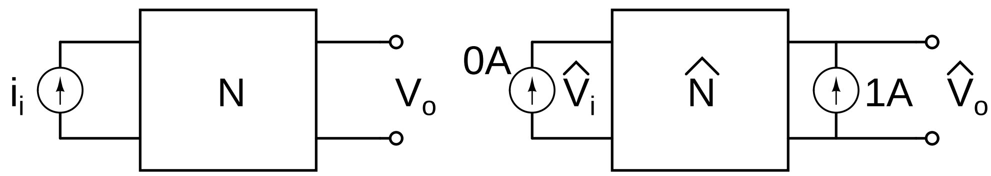

Fig. 1. A Network with two ports and its adjoint.

density of the output noise voltage due to this noise current source. In the adjoint network, it is then convenient to have a $0A$ current source in the branch (i) containing the noise current source and a $1A$ current source in the output branch (o). Equation (4) now becomes

$$i_i.\hat{v}_i - v_i.0 + 0.\hat{v}_o - v_o.1 = 0 \tag{5}$$

or

$$\begin{aligned} v_o &= i_i.\frac{\hat{v}_i}{1} \\ &= i_i T_Z \end{aligned} \tag{6}$$

In equation (6), the voltage $\hat{v}_i$ is divided by $1A$ to give T_Z. Therefore T_Z is essentially the transfer function (trans-impedance) between the two ports of the adjoint network. The value of T_Z is equal to the value of $\hat{v}_i$.

This essentially means that the contribution of the noise current source i_i to the output is given by the value of the corresponding voltage, $\hat{v}_i$, across the $0A$ current source in the adjoint network. Similarly, if instead of a noise current source, we have a noise voltage source, the value of the current through the $0V$ voltage source in the corresponding branch in the adjoint network gives the noise contribution to the output. If there is more than one input port (as is the case when there are several noise sources), each of the corresponding ports in the adjoint network is either left open or shorted. The voltage across or current through these ports of the adjoint network gives the noise contribution of the various input noise sources. Therefore, it is clear that a single analysis of the adjoint network is sufficient to obtain the noise contributions of all the noise sources to the output noise voltage. This is discussed in detail in [13].

III. Adjoint model of the Noisy Nullor

In this section, we derive the adjoint model for a nullor with input referred noise sources. We also outline the steps in the new method for noise analysis of operational amplifier based circuits.

The operational amplifier can be modelled using a nullator-norator combination as shown in figure 2. In the figure, v_n and i_n are the input referred noise voltage and current sources (over

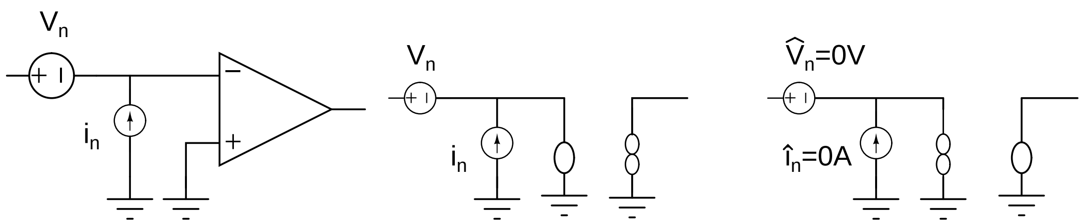

Fig. 2. Adjoint model of the noisy nullor

a band Δf). In general, these two sources are correlated. Let K be the correlation coefficient. The adjoint model for the ideal nullor can be derived as indicated in [14]. It is obtained as follows. The nullator has both $v, i = 0$, whereas the norator can accomodate any current and voltage. Therefore, the contribution of the nullator branch to equation (2) is zero. In order to make the contribution of the norator branch zero, we can interchange the nullator and norator in the adjoint network. Therefore, the adjoint model of the nullor is also a nullor, obtained by interchanging the norator and the nullator.

v_n and i_n are essentially additional "ports" in the network. Their contribution to the output noise can be obtained in a manner similar to any other noise source. Therefore, v_n is replaced by $\hat{v}_n = 0V$, and i_n is replaced $\hat{i}_n = 0A$ in the adjoint network. This is shown in figure 2. As explained in the previous section, the contribution of v_n and i_n to the output noise is given by the current through $\hat{v}_n$ and the voltage across $\hat{i}_n$ i.e, the current through and voltage across the norator in the adjoint network.

This gives a simple method to analyze noise in opamp based circuits. In most cases, it can be obtained by inspection. It involves a single analysis of the adjoint network. The steps involved in the analysis are the following.

1) To find the output noise voltage spectral density, connect a 1A current source to the output port of the adjoint network.

2) The transfer function between the noise current source of a resistor and the output is equal in value to the voltage across the resistor in the adjoint network.
3) The transfer function between the input referred noise voltage source of the opamp and the output is equal in value to the current through the corresponding norator in the adjoint network.
4) The transfer function between the input referred noise current source of the opamp and the output is equal in value to the voltage across the corresponding norator in the adjoint network.

Any correlation between the noise sources can be taken care of when adding the contributions of the various noise sources at the output.

IV. Results

The method is illustrated for the inverting amplifier, the universal active filter and various gyrator topologies.

A. Inverting amplifier

Figure 3 shows the inverting amplifier along with all the noise sources and the corresponding adjoint network. The transfer function from each noise source to the output is obtained by analyzing the adjoint network with a unit current source connected to the output. Since the nullator takes no current, the current through R_f is $1A$. Since the voltage across the nullator is zero, one end of R_f is grounded. This means that the voltage across the norator is R_f, the magnitude of the current through R_s is $\frac{R_f}{R_s}$ and the magnitude of the current through the norator is $1+\frac{R_f}{R_s}$. Assuming K is the correlation coefficient between the opamp noise sources, the output noise voltage squared (over a band Δf) can be written as:

$$\begin{aligned} v_{on}^2 = \frac{4kT}{R_f}\Delta f {R_f}^2 + \frac{4kT}{R_s}\Delta f R_f^2 + {i_n}^2 R_f^2 + \\ v_n^2(1+\frac{R_f}{R_s})^2 + 2Kv_n i_n R_f(1+\frac{R_f}{R_s}) \end{aligned} \tag{7}$$

B. Universal Active Filter

Figure 4 shows the universal active filter and the adjoint network. Here, we analyze the noise contribution of the four opamps to the noise voltage at the bandstop output at the center

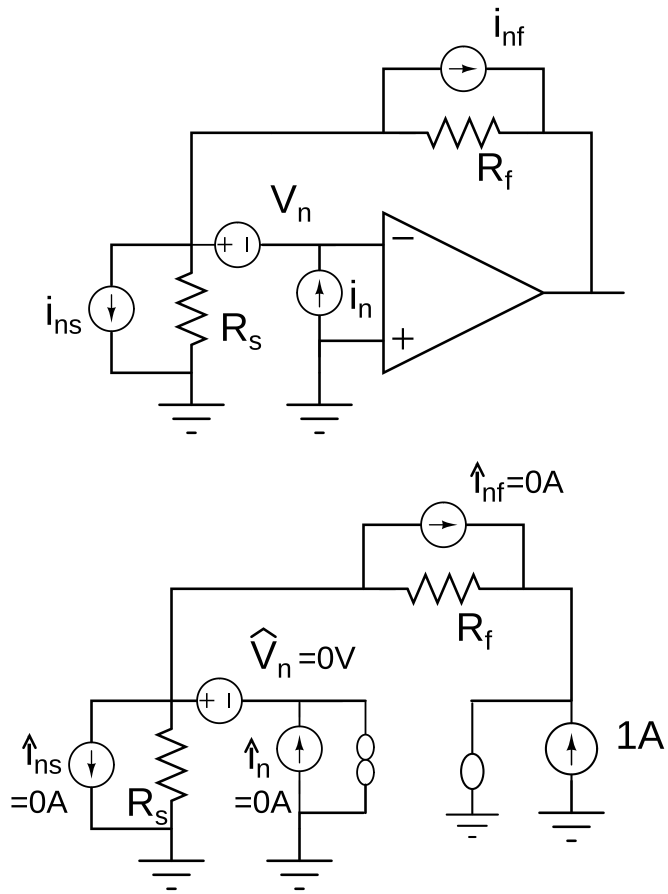

Fig. 3. Inverting amplifier and the adjoint network

frequency $\omega_o = \frac{1}{RC}$. The currents through the various resistors and capacitors is indicated in the figure. It is seen from the figure of the adjoint circuit that the current $\hat{i}_2$ is zero (since the nullator takes no current and the incoming and outgoing branch currents at the node marked 'A' in figure 4 are equal to $j\hat{i}_1$). Therefore $\hat{i}_1 = 1A$. Using this, the currents through the norators of $O1, O2, O3$ and $O4$ in the adjoint networks can be obtained as $(1+j)A, (1-j)A, -3A$ and $0A$ respectively. The corresponding voltages are -RV, jRV, RV and $0V$. The noise power (over a band Δf) at the centre frequency at the bandstop output due to the opamps alone can thus be written as:

$$v_{on}^2 = (2(v_{n1}^2 + v_{n2}^2) + 9v_{n3}^2) + (i_{n1}^2 + i_{n2}^2 + i_{n3}^2)R^2 \tag{8}$$

Here v_{n1}, v_{n2} and v_{n3} and the input referred noise voltages and i_{n1}, i_{n2} and i_{n3} are the input referred noise current sources (over a band Δf) of opamps $O1$, $O2$ and $O3$ respectively.

Interestingly therefore, the opamp $O4$, from which the bandstop output is obtained, contributes no noise at all to the bandstop output at the notch frequency. The resistors connected to this opamp also do not contribute any noise. The opamp $O3$ contributes the maximum noise.

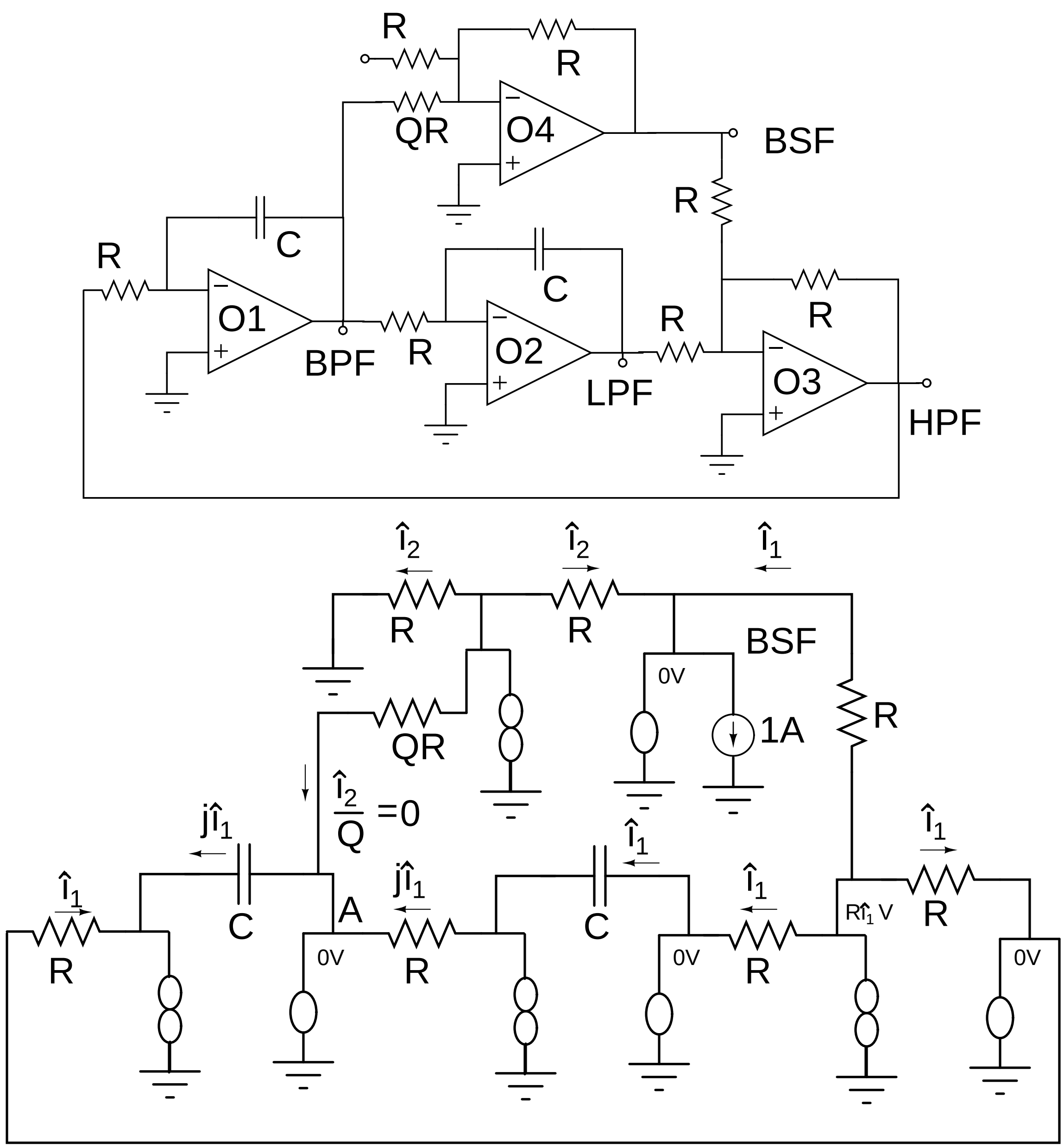

Fig. 4. Universal active filter and its adjoint network. The currents in the adjoint network are obtained at the centre frequency $\omega_o = \frac{1}{RC}$

C. Gyrators

Figure 5 shows the basic gyrator (used for grounded inductors). Depending on which nullator-norator combination is replaced by an opamp, two different gyrator topologies are possible. As usual, the adjoint network is obtained by interchanging the nullator and norator.

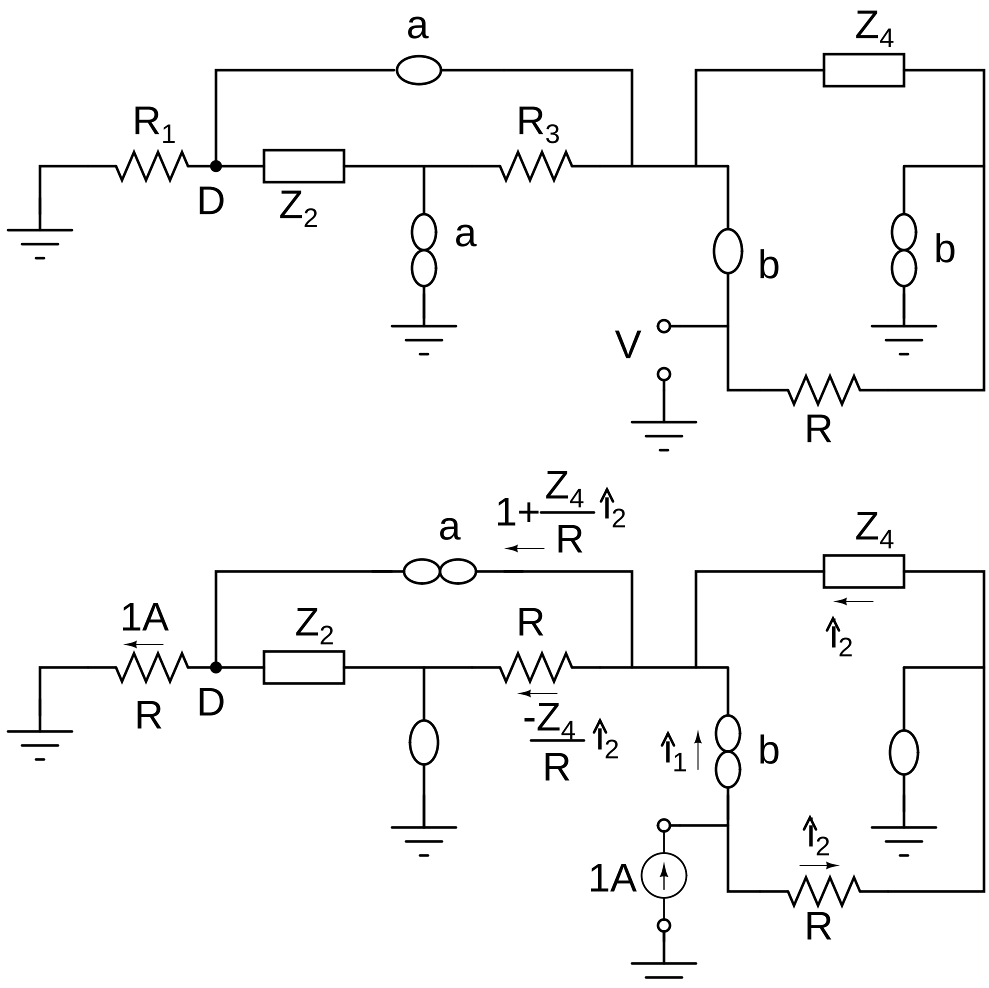

Fig. 5. Gyrator topology and its adjoint

From the figure, the voltage at node D is equal to the voltage across the resistor R which in turn is equal to the voltage across Z_2. Therefore,

$$R.1 = \frac{Z_2 Z_4}{R}\hat{i}_2 \tag{9}$$

Therefore, the currents through the norators $\hat{i}_{n_a}$ and $\hat{i}_{n_b}$ are given by:

$$\begin{aligned} \hat{i}_{n_a} &= 1 + \frac{R}{Z_2}.1A \\ \hat{i}_{n_b} &= 1 - \frac{R^2}{Z_2 Z_4}.1A \end{aligned} \tag{10}$$

The voltages across the two norators are identical. It is given by:

$$\hat{v}_{na} = \hat{v}_{nb} = \left(R + \frac{R^2}{Z_2}\right).1V \tag{11}$$

There are two possibilities, corresponding to $Z_2 = R, Z_4 = 1/sC$ and $Z_2 = 1/sC, Z_4 = R$, giving a total of four different gyrator topologies. Since the voltages across the two norators are equal, the opamp input referred noise current sources contribute equally to the noise spectral density at the output. Clearly, if $Z_2 = R$, and $Z_4 = 1/sC$, the noise due to the opamp input referred voltage source corresponding to norator a at $\omega = \frac{1}{RC}$ is larger. In terms of noise therefore, it is not as good as the two topologies that have $Z_2 = 1/sC$ and $Z_4 = R$.

V. Conclusions

In this paper, we have outlined a simple technique to evaluate analytically the noise spectral density in operational amplifer based circuits. The technique was illustrated for the inverting amplifier, the universal active filter and gyrators. It is a general method and can be used for any active RC or MOSFET-C filter topologies.

References

[1] L.T.Bruton, F.N.Trofimenkoff, and DH.Treleaven, “Noise performance of low sensitivity active filters,” *IEEE J. Solid State Circuits*, vol. SC-8, pp. 85–91, Feb 1973.

[2] F.N.Trofimenkoff, D.H.Treleaven, and L.T.Bruton, “Noise performance of rc active quadratic filter sections,” *IEEE Trans. Circuit Theory*, vol. CT-20, pp. 524–532, Sept. 1973.

[3] J.W.Haslett, “Noise performance limitations of single amplifier rc active filters,” *IEEE Trans. Circuits Syst.*, vol. CAS-22, Oct. 1975.

[4] J.Zurada and M.Bialko, “Noise and dynamic range of active filters with operational amplifiers,” *IEEE Trans Circuitsand Syst.*, vol. CAS-22, pp. 805–809, Oct. 1975.

[5] H.J.Bachler and W.Guggenbuhl, “Noise analysis and comparison of second order networks containing a single amplifier,” *IEEE Trans Circuits Syst.*, vol. CAS-27, pp. 85–91, Feb. 1980.

[6] P.J.Billam, “Practical optimization of noise and distortion in sallen and key filter sections,” *IEEE J.Solid State Circuits*, vol. SC-14, pp. 768–771, Aug. 1979.

[7] F.N.Trofimenkoff and O.A.Onwuachi, “Noise performance of operational amplifier based circuits,” *IEEE Trans. Education*, vol. 32, pp. 12–17, Feb. 1989.

[8] J. A.M.Leach, “Fundamentals of low-noise analog circuit design,” *Proc. IEEE*, vol. 82, pp. 1515–1538, Oct. 1994.

[9] P.Bowron and K.A.Mezher, “Noise analysis of second order analogue active filters,” *IEE Proc. Circuits Dev. Syst.*, vol. 141, no. 5, pp. 350–356, 1994.

[10] L.Toth, G.Efthivoulidis, V.Gopinathan, and Y.Tsividis, “General results for resistive noise in active rc and mosfet-c filters,” *IEEE Trans Circuits Syst -II*, vol. 42, pp. 785–793, Dec. 1995.

[11] K.A.Mezher and B.Bowron, “Noise analysis of higher order active filters,” *IEE Proc. Circuits Dev. Syst*, vol. 144, pp. 11–16, Feb. 1997.

[12] L.Toth, G.Efthivoulidis, and Y.Tsividis, “Further results for noise in active rc and mosfet-c filters,” *IEEE Trans Circuits Syst -II*, vol. 45, pp. 1311–1315, Sept. 1998.

[13] R.Rohrer, L.Nagel, R.Meyer, and L.Weber, “Computationally efficient electronic-circuit noise calculations,” *IEEE J.Solid-State Circuits*, vol. SC-6, pp. 204–213, 1971.

[14] L.T.Pillage, R.A.Rohrer, and C.Visweswariah, *Electronic Circuit and System Simulation Methods*. McGraw-Hill, Inc, 1995.

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Figure Captions

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Fig.2. Adjoint model of the noisy nullor.

Fig.3. Inverting amplifier and the adjoint network.

Fig.4. Universal active filter and its adjoint network. The currents in the adjoint network are obtained at the centre frequency $\omega_o = \frac{1}{RC}$

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