**Electric Circuits and Networks** **06-08-2018**

# Lecture 4: Power and Energy

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## Linearity

Consider current $i_L(t)$ flowing through an inductor of inductance $L$, of the form $i_L(t) = k_1 i_1(t) + k_2 i_2(t)$. Voltage $v_L(t)$ across the inductor is given by

$$v_L(t) = L\frac{\mathrm{d}i_L(t)}{\mathrm{d}t} = k_1 v_1(t) + k_2 v_2(t)$$

where $v_1(t) = L\frac{\mathrm{d}i_i(t)}{\mathrm{d}t}$ and $v_2(t) = L\frac{\mathrm{d}i_2(r)}{\mathrm{d}t}$. So it seems linear. However if the inductor has a nonzero inital current,

$$i_L(t) = i_L(0) + \frac{1}{L}\int_0^t v_L(\tau)\mathrm{d}\tau$$

Consider $v_L^{'}(t) = kv_L(t)$, then corresponding current

$$i_L^{'}(t) = i_L(0) + \frac{k}{L}\int_0^t v_L(\tau)\mathrm{d}\tau \neq ki_L(t)$$

. Therefore it is not linear. In a similar way, if we write capacitor equation in integral form then corresponding $v_c(t)$ is not linear w.r.t $i_C(t)$. Therefore a capacitor with a non-zero initial voltage and an inductor with an non-zero initial current are not linear components.

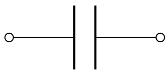
For the next few classes, unless mentioned, we will deal with capacitors and inductors with zero initial conditions. Also whenever transfer functions, impulse responses etc are used to study the circuit, zero initial conditions are implicitly assumed.

## Units and Ideal Elements

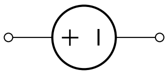
| **Element** | **Units** | **Typical range** |
|---|---|---|
| Resistor | Ohms (Ω) | Ω - $M$Ω |
| Capacitor | Farad (F) | $p$F - $\mu$F |
| Inductor | Henry (H) | $\mu$H - H |

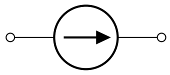
For ease of calculations, we might use non-typical range values (eg: 1F capacitor). All the elements that we use for calculations are assumed to be ideal.


Resistor


Capacitor

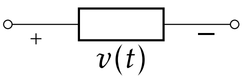

Inductor


Voltage Source

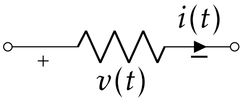

Current Source

But in a practical scenario, there will be leakages across capacitors, voltage drops across voltage source as the time passes etc. When we build models, we also need to worry about range of validity of ideal approximation in practical situations.

**Power and Energy**



Current direction tells us about the direction in which $+ve$ charge moves, but actually $e^-$ moves in the opposite direction. Whenever $+ve$ charge moves from point of higher potential to lower potential, it looses energy.



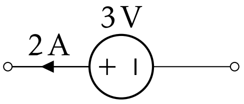
In the above case, energy is absorbed by the resistor and and dissipated in the form of thermal energy.

$$\underbrace{\mathrm{d}W}_{\substack{\text{Energy absorbed by the resistor} \\ \text{(lost by charge d}q\text{)}}} = \underbrace{V}_{\text{Potential drop across resistor}} .\mathrm{d}q$$

$$V = \frac{\mathrm{d}W}{\mathrm{d}q}, I = \frac{\mathrm{d}q}{\mathrm{d}t} \implies v(t).i(t) = \frac{\mathrm{d}W}{\mathrm{d}t} = p(t)$$

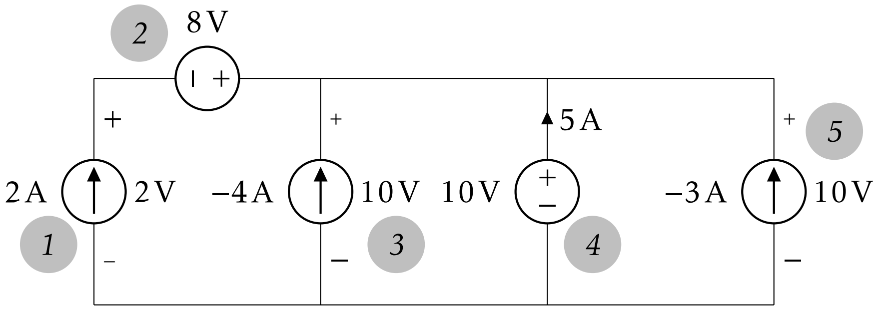
Rate of change of energy absorbed by the resistor, $\frac{\mathrm{d}W}{\mathrm{d}t}$, also know as power absorbed by the resistor $p(t)$. Convention for a generic element is to take potential drop across the element in the direction of current. With this convention, if an element has $p(t)$ as $+ve$ then it is absorbing energy and if it has $p(t)$ as $-ve$ then it is generating energy.

**Example 1.** *Consider the following voltage source*



*Potential drop across the voltage source in the direction of current source is* $-3V$*. Therefore power is* $-6$ *Watts. Since the power is* $-ve$*, device is generating energy.*

**Example 2.** *Consider the following circuit*



$p_1 = -4$ *W (generating power),* $p_2 = -16$ *W (generating power),* $p_3 = 40$ *W (absorbing power),* $p_4 = -50$ *W (generating power),* $p_5 = 30$ *W (absorbing power)*

Note: Net power absorbed in a circuit = Net power generated (Consequence of Kirchhoff law. Will discuss in later lectures)

For a capacitor, power is

$$p(t) = i(t).v(t) = C.v(t).\frac{\mathrm{d}v(t)}{\mathrm{d}t}$$

Energy absorbed between $t = 0$ and $t = t$ is

$$W_{absorbed}(t) = \int_0^t p(\tau)\mathrm{d}\tau = C\int_0^t v(\tau)\mathrm{d}v(\tau) = \frac{C}{2}\Big(v^2(t) - v^2(0)\Big), \quad W_{stored}(t) = \frac{C}{2}v^2(t)$$

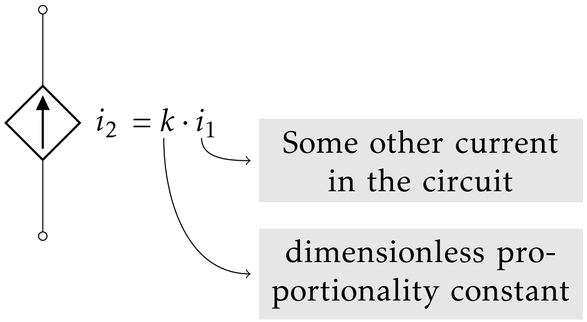
From the above equation we can see that energy stored in a capacitor depends only on the initial and final voltages and not the intermediate voltages. For an Inductor $p(t) = Li(t)\frac{\mathrm{d}i(t)}{\mathrm{d}t}$. Energy absorbed is

$$W_{absorbed}(t) = \int_0^t p(\tau)\mathrm{d}\tau = \frac{L}{2}\Big(i^2(t) - i^2(0)\Big), \quad W_{stored}(t) = \frac{L}{2}i^2(t)$$

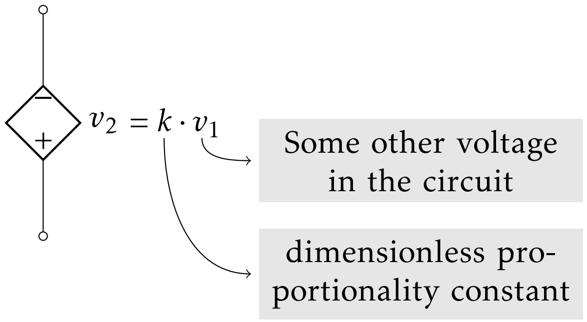
Since $p(t)$ for a capacitor (and Inductor) can be both $-ve$ and $+ve$, it can both absorb energy from the source and give back stored energy. This is not same as voltage or current source that can generate electrical energy.

**Controlled Sources (Dependent Sources)**

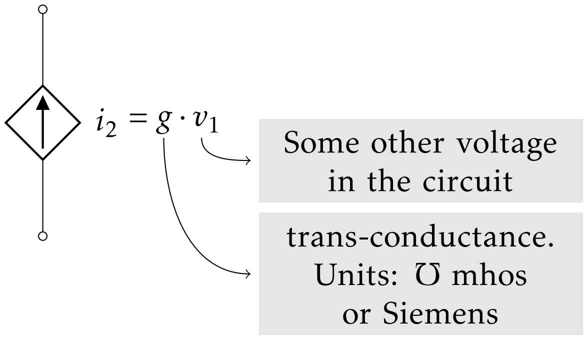
There are four types controlled sources



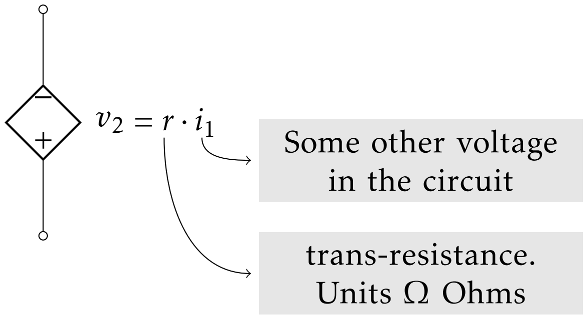
Current Controlled Current Source (**CCCS**)



Voltage Controlled Voltage Source (**VCVS**)



Voltage Controlled Current Source (**VCCS**)



Current Controlled Voltage Source (**CCVS**)