Electric Circuits and Networks **8-10-2018**

Lecture 25: Sinusoidal Steady State

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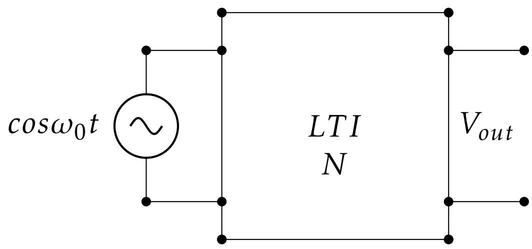
Since power supply is 50 Hz sinusoids, we are interested in steady state voltage/current.



Assume poles of $'N'$ are in LHP; no poles in $j\omega$ axis. One way to analyze this is to use Laplace transforms and let $t \to \infty$.

$$V_{out}(s) = H(s)\frac{s}{s^2+\omega_0^2} \qquad \{Effectively\ input\ is\ cos\omega_0 t u(t)\}$$

$$= \underbrace{\frac{A_1}{s+\alpha_1} + \frac{A_2}{s+\alpha_2} + \cdots}_{} + \frac{B_1 s + B_2}{s^2+\omega_0^2}$$

Natural response

Since poles are in LHP there are transient that will die out in times $\gg \max_i\{\frac{1}{\alpha_i}\}$

Steady state solution (forced solution) is due to $\frac{B_1 s + B_2}{s^2+\omega_0^2}$

Find $B_1 \& B_2$; multiply both sides by $s^2+\omega_0^2$ and substitute $s = \pm j\omega_0$

$$H(j\omega_0)\cdot j\omega_0 = B_1(j\omega_0) + B_2$$
$$H(-j\omega_0)\cdot(-j\omega_0) = B_1(-j\omega_0) + B_2$$
$$B_1 = Re\{H(j\omega_0)\}$$
$$B_2 = -\omega_0 Im\{H(j\omega_0)\}$$
$$\because \frac{B_1 s + B_2}{s^2+\omega_0^2} \longleftrightarrow Re\{H(j\omega_0)\}\cos\omega_0 t - Im\{H(j\omega_0)\}\sin\omega_0 t$$
$$= |H(j\omega_0)|\cos(\omega_0 t + \theta)$$

where

$$\theta = \tan^{-1}\left[\frac{Im\{H(j\omega_0)\}}{Re\{H(j\omega_0)\}}\right]$$

The other way to look at is the eigenfunction approach

$$e^{j\omega t} \to H(j\omega)e^{j\omega t}$$

Note that this is a steady state response after all transients have died out. If we find the response to $e^{j\omega t}u(t)$, we will also get the transient response (Not an eigenfunction)

$$e^{-j\omega t} \to H(-j\omega)\, e^{-j\omega t}$$

Superposition

$$\cos \omega t \to Re\{H(j\omega)\, e^{j\omega t}\}$$

Same as steady state response obtained using Laplace transforms. So to find response to $\cos \omega_0 t$ all we need is $H(j\omega_0)$

Input is $Ae^{j\omega t}$; where, A is complex. Linearity $\Rightarrow$ Output is $AH(j\omega)e^{j\omega t}$
Actual input is $Re\{Ae^{j\omega t}\}$ where $A = |A|e^{j\theta}$. So actual input is $= |A|\cos(\omega t + \theta)$.

Phasors: The complex coefficient multiplying $e^{j\omega t}$.

$$e^{j\omega t} \to 1\measuredangle 0$$
$$Ae^{j\theta}e^{j\omega t} \to A\measuredangle\theta$$

To get response to $\sin \omega_0 t$;

$$\begin{aligned}\frac{e^{j\omega t} - e^{-j\omega t}}{2j} &\to \frac{H(j\omega)\, e^{j\omega t} - H(-j\omega)\, e^{-j\omega t}}{2j} \\ &= Im\{H(j\omega)\, e^{j\omega t}\} \\ &= Re\{-jH(j\omega)\, e^{j\omega t}\} \\ &= Re\{H(j\omega)\, e^{-j\pi/2} e^{j\omega t}\} \\ -\sin \omega_0 t &\to 1\measuredangle -\pi/2\end{aligned}$$

All response in a circuit can be written as a (Phasor)$e^{j\omega t}$.

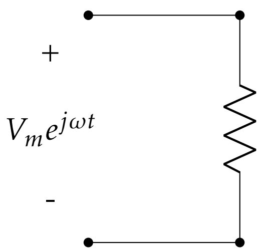
Denote currents as $Ie^{j\omega t}$; I complex, voltages as $Ve^{j\omega t}$; V complex

KCL: $\sum_k I_k e^{j\omega t} = 0$ at each node.

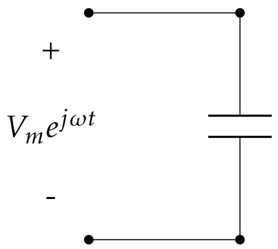
$$e^{j\omega t} \neq 0$$
$$\Rightarrow \sum_k I_k = 0 \qquad \{\text{can be written directly using phasors}\}$$

KVL: $\sum_k V_k = 0$

Branch constitutive relationship Initial conditions are of no consequence as we are looking at steady state solutions after all transients have died out. System is stable; all transients due to initial conditions have also died out.



$$I = \frac{V_m}{R} e^{j\omega t}$$
$$I = \frac{V_m}{R} \text{ (Phasor Current)}$$

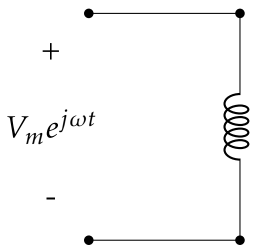


$$I = \underbrace{j\omega C V_m}_{I_m} e^{j\omega t}$$

$$V_m = |V_m| e^{j\theta}$$

$$\Rightarrow I_m = \omega C |V_m| e^{j(\theta+\pi/2)}$$

Current leads voltage by $\pi/2$



$$I = \frac{1}{L}\int V_m \; e^{j\omega t} dt$$

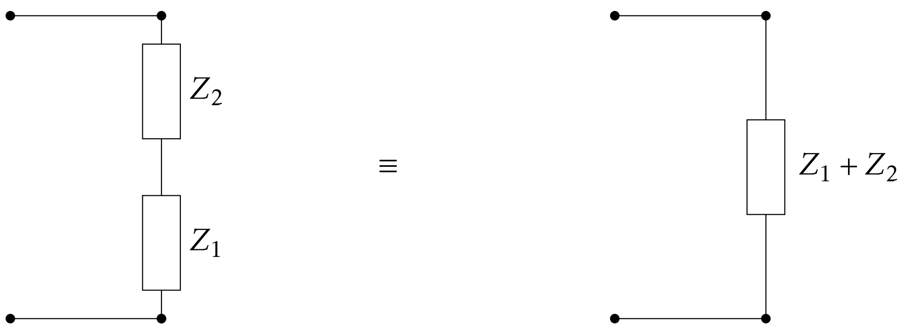
$$= \frac{1}{j\omega L} V_m \cdot e^{j\omega t} dt$$

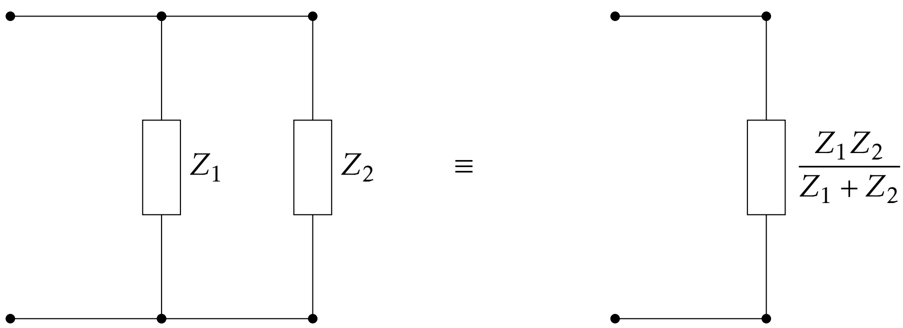
$$\Rightarrow I_m = \frac{|V_m|}{\omega L} e^{-j\pi/2}$$

Current lags voltage by $\pi/2$

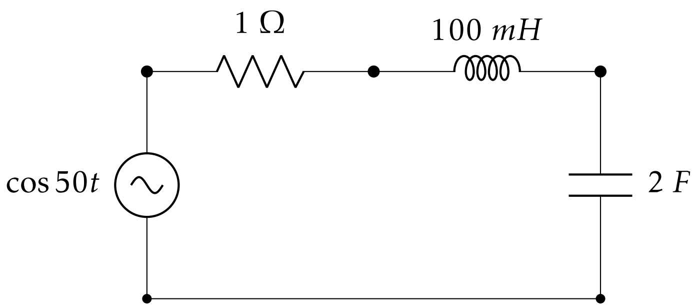
Impedance : $\dfrac{V_m}{I_m} = Z$ **Admittance :** $\dfrac{I_m}{V_m} = Y$ Both Z and Y are complex numbers.

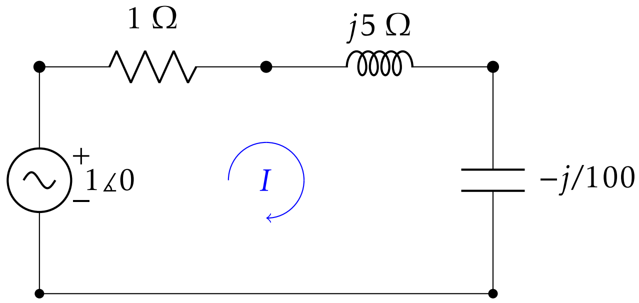
Series and Parallel connections of impedance





Example find steady state response of the following circuit





$$I\left(1 + j5 - \frac{j}{100}\right) = 1 + j0$$