

Lecture 1: Review of Linear System

Lecturer: Dr. Vinita Vasudevan

Scribe: Shashank Shekhar

We will begin with a brief review of linear systems.

Linearity: A system is said to be linear if it follows the property of additivity *i.e.*

$$\begin{aligned}x_1(t) &\mapsto y_1(t) \\x_2(t) &\mapsto y_2(t) \\x_1(t) + x_2(t) &\mapsto y_1(t) + y_2(t)\end{aligned}$$

and homogeneity *i.e.*

$$\begin{aligned}x(t) &\mapsto y(t) \\ \alpha x(t) &\mapsto \alpha y(t)\end{aligned}$$

where α is a scalar. Linearity ensures the existence of “impulse response” $h(t)$.

Time Variance/ Time Invariance: A system is said to be time invariant if a “particular” delay/advance in input results in same delay/advance in output *i.e.*

$$\begin{aligned}x(t) &\mapsto y(t) \\x(t - \tau) &\mapsto y(t - \tau)\end{aligned}$$

By the virtue of time invariance of a system, it does not matter “at point of time” $h(t)$ is computed. Also, Let H : system operator such that $y = Hx$ and Δ_τ : Delay operator then for time invariant system these two operator are commutative *i.e.*

$$H\Delta_\tau = \Delta_\tau H$$

Causality: A linear time invariant system is said to be causal if

$$h(t) = 0 \quad \forall t < 0$$

In this course, Linear time invariant causal(LTIC) will be our prime focus.

Frequency domain representation of signal:

If $x(t)$ is periodic signal then we use Fourier series representation.

$$x(t) = \sum_{-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

If $x(t)$ is aperiodic signal then we use Fourier transform.

$$x(t) \longleftrightarrow X(j\omega)$$

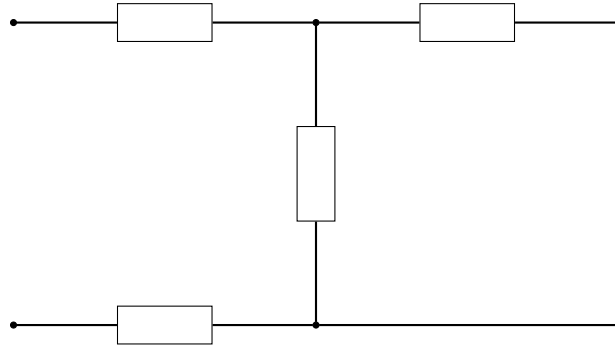
Fourier transform of the output of LTI system can be given as

$$Y(j\omega) = H(j\omega)X(j\omega)$$

where $H(j\omega)$ is fourier transform of $h(t)$. $e^{jk\omega t}$ is an eigen function for linear systems therefore the response at ω depends only on the component of the transfer function and the input at ω .

Also, We will be using Unilateral laplace transform because the systems of interest are causal.

System: it's a interconnection of various component.



In above picture “rectangles” represents the components and the “solid lines” represents the connecting wires. In order to analyse the system, we need I-V characteristics of all components. As our aim is to find out the response (voltage across the component or current through the component) using the component models for a given excitation.

Transient Analysis: It is time domain response like impluse response $h(t)$, step response $g(t)$. It can be done using differential equation or by converting the differential equation in algebric equation by using laplace transform.

Steady State Analysis: It is the response of the system corresponding to single frequency input($e^{jk\omega t}$).

Other than this, we can also calculate power and energy consumption for each component also.