Electric Circuits and Networks **10-09-2018**

Lecture 16: Nodal/Mesh Analysis, Source Transformation

Lecturer: Dr. Vinita Vasudevan *Scribe: Shashank Shekhar*

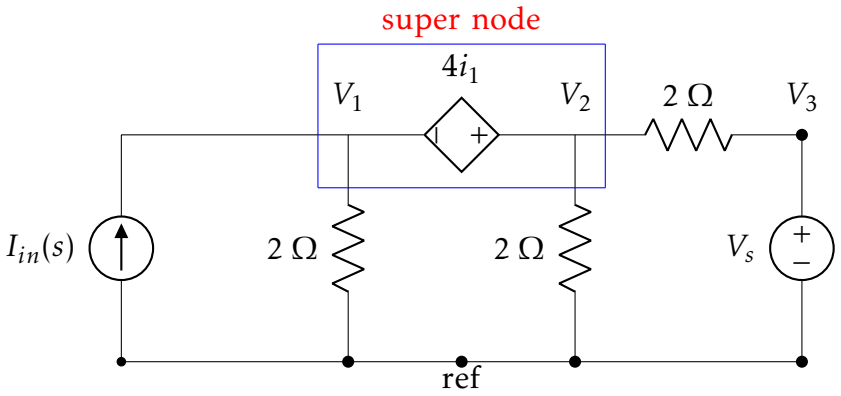
Nodal Analysis

$$\underbrace{\begin{bmatrix} G \end{bmatrix}}_{} \underbrace{\begin{bmatrix} V \end{bmatrix}}_{\text{Unknown Variables}} = \underbrace{\begin{bmatrix} I_s \end{bmatrix}}_{\text{independent sources/ initial conditions}}$$

Mesh Analysis

$$\underbrace{\begin{bmatrix} Z \end{bmatrix}}_{} \underbrace{\begin{bmatrix} I \end{bmatrix}}_{\text{Unknown Variables}} = \underbrace{\begin{bmatrix} V_s \end{bmatrix}}_{\text{independent sources/ initial conditions}}$$

Example 1: Write the nodal equations for following circuit



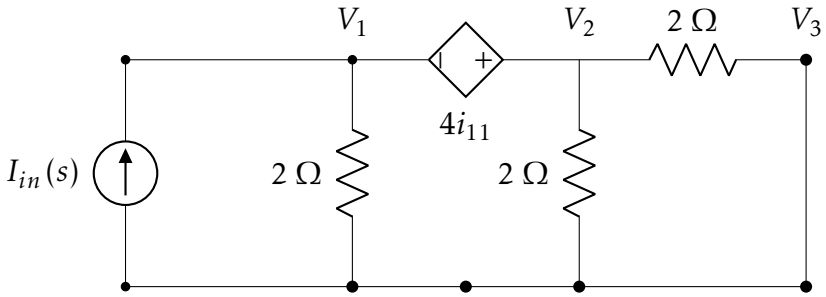
As $V_3 = V_s$ so effectively we have only two unknown V_1, V_2. By applying KCL we will get following matrix equation:

$$\begin{bmatrix} \frac{1}{2} & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_s + V_s/2 \\ 0 \end{bmatrix}$$

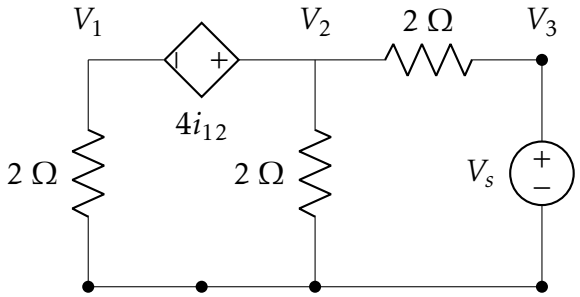
$$= \begin{bmatrix} I_s \\ 0 \end{bmatrix} + \begin{bmatrix} V_s/2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \underbrace{G^{-1} \begin{bmatrix} I_s \\ 0 \end{bmatrix}}_{S_1} + \underbrace{G^{-1} \begin{bmatrix} V_s/2 \\ 0 \end{bmatrix}}_{S_2}$$

Observation: Note that S_1 can be obtained by setting $V_s = 0$,Circuit for that will be as follows:

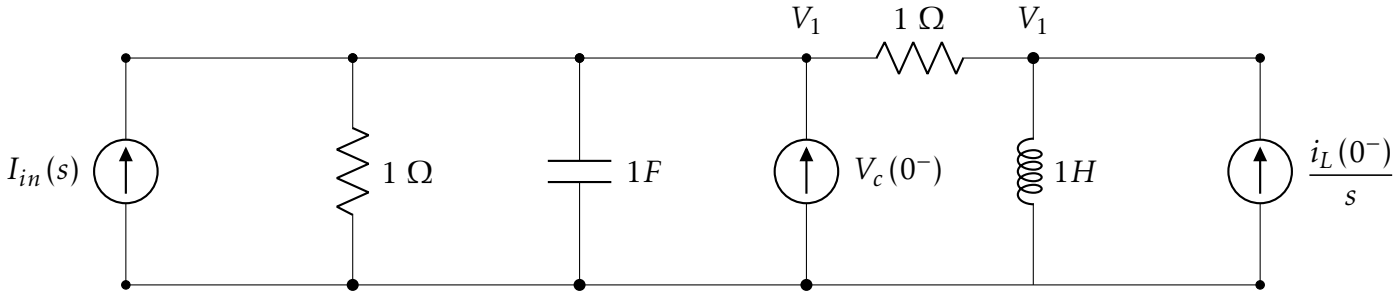


S_2 can be obtained by setting $I_s = 0$,Circuit for that will be as follows:



and the total solution is sum of the two solutions. This is expected to be happen as the system is linear w.r.t. each source, so Superposition should be applicable.

Example 2: Write the nodal equations for following circuit



$$\begin{bmatrix} s+2 & -1 \\ -1 & 1/s+1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{in}(s) + V_c(0^-) \\ -\frac{i_L(0^-)}{s} \end{bmatrix}$$

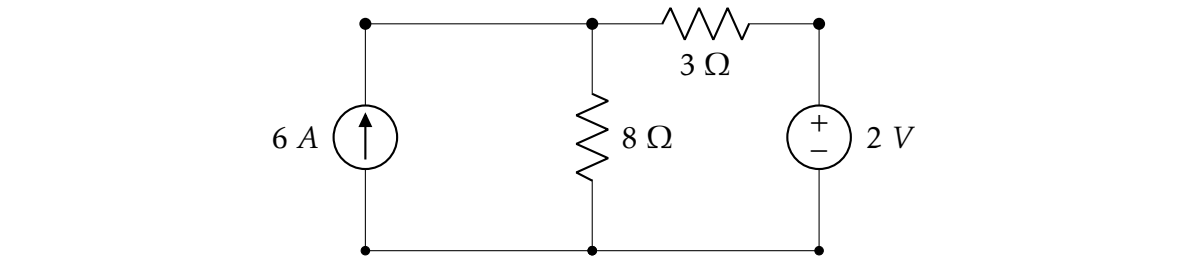
$$= \begin{bmatrix} I_{in}(s) \\ 0 \end{bmatrix} + \begin{bmatrix} V_c(0^-) \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{i_L(0^-)}{s} \end{bmatrix}$$

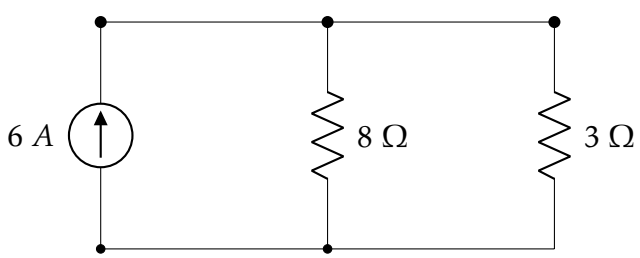
zero state

Using superposition

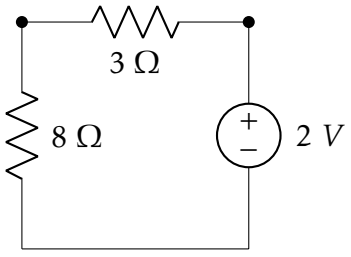
zero-input can apply one initial condition at a time

Example 3: Solve the following circuit using Superposition





$$i_1 = 6 \cdot \frac{3}{11}$$



$$i_2 = \frac{2}{11}$$

$$i = i_1 + i_2$$

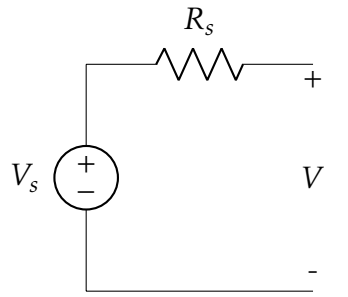
Note: Superposition can be appiled only for independent sources and sources appearing because of initial conditions

---

Setting the voltage source equal to zero ⇔ short the terminal
Setting the current source equal to zero ⇔ opencircuit the terminal

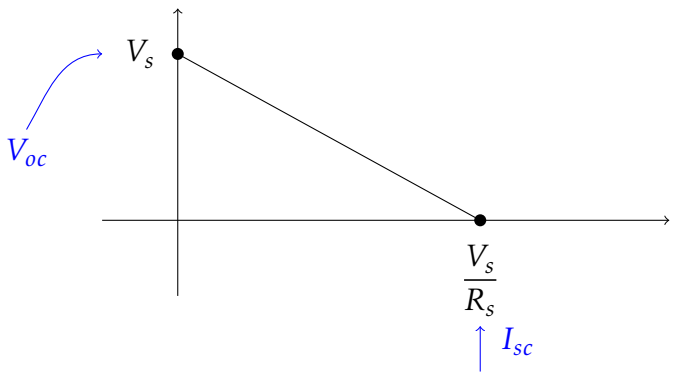
---

Source Transformation

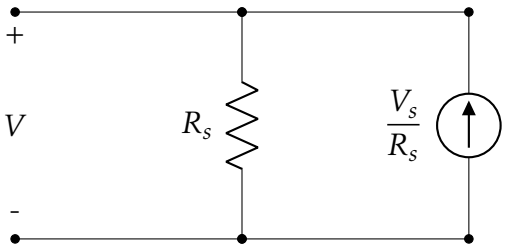


equivalent

$$V = V_s - IR_s$$

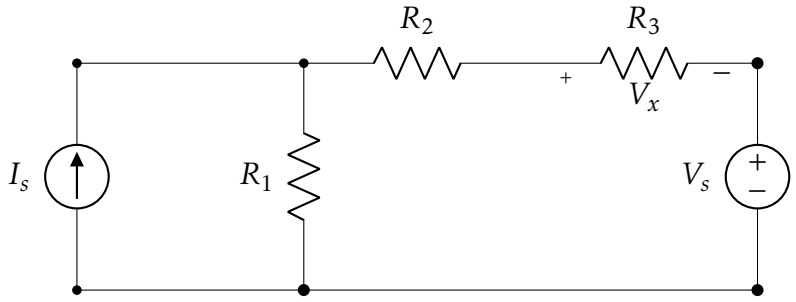


$$I = \frac{V_{in} - V_s}{R}$$

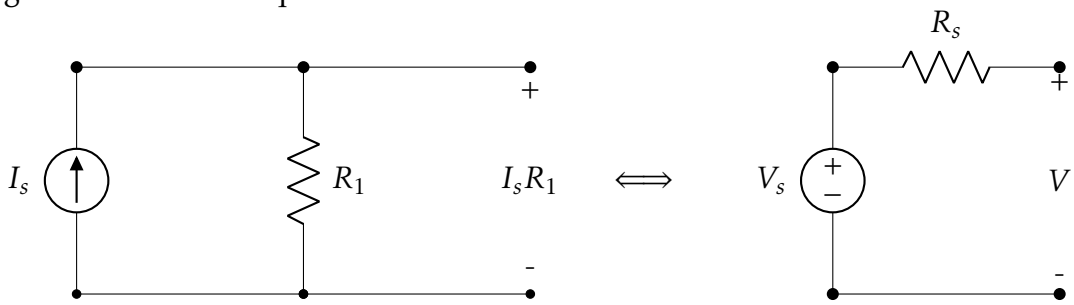


Hence we can replace voltage source with resistance in series by a current source with resistance in parallel.

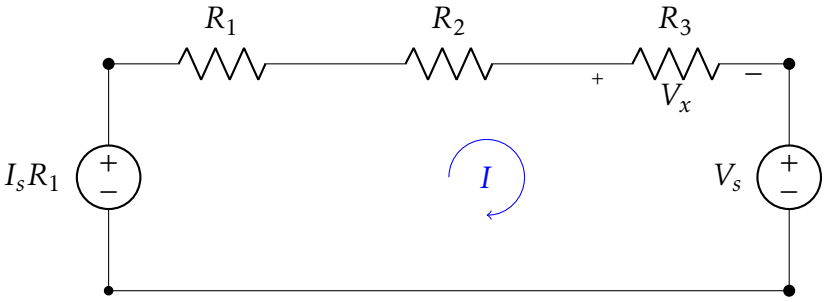
Example 4: Replace current source with resistance in parallel by voltage source with resistance in series.



Since the following two circuits are equivalent



So the given circuit can be replaced by following:

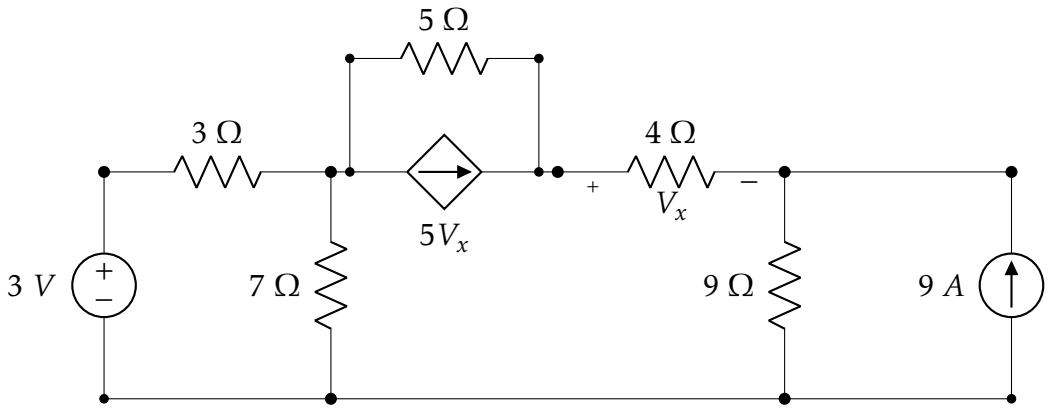


$$I = \frac{I_sR_1 - V_s}{R_1 + R_2 + R_3}$$

$$V_x = R_3 I$$

Note : Source transformation can be used for independent, dependent sources and also for initial conditions, as the I-V relationship is the same in all conditions

Exercise 1 : Apply the series of source transformations to get circuit with single mesh



Exercise 2 : Replace the dashed part of given circuit by single voltage source in series with a inpedence