Electric Circuits and Networks **04/09/2018**

Lecture 14: Parallel RLC network

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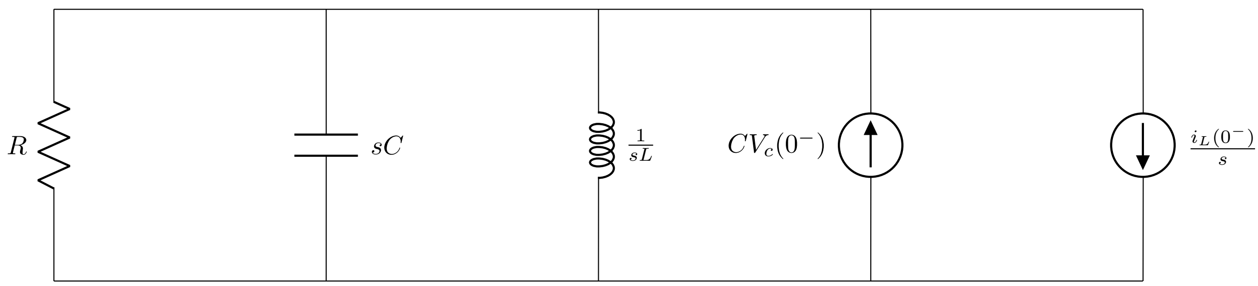
Assuming we know $V_C(0^-), I_L(0^-)$, the s-domain circuit is as follows



Applying KCL at top node we get the following equation:

$$V(s)[\frac{1}{R}+\frac{1}{sL}+sC] = \frac{-i_L(0^-)}{s}+CV_C(0^-)$$

$$V(s) = \frac{[CV_C(0^-)-\frac{i_L(0^-)}{s}]}{R+sL+s^2RLC}sLR$$

$$= \frac{\left[CV_C(0^-)-\frac{i_L(0^-)}{s}\right]}{RLC\left[s^2+\frac{s}{RC}+\frac{1}{LC}\right]}sLR$$

$$V(s) = \frac{[CV_C(0^-)-\frac{i_L(0^-)}{s}]}{C[s^2+\frac{s}{RC}+\frac{1}{LC}]}s$$

The natural frequencies are determined by the roots of the denominator polynomial

$$s_1, s_2 = \frac{-1}{2RC} \pm \sqrt[2]{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

$$\alpha = \frac{1}{2RC} \rightarrow \text{frequency; damping factor}$$
$$w_0 = \frac{1}{\sqrt{LC}} \Rightarrow \text{resonant frequency}$$
$$s_1, s_2 = -\alpha \pm \sqrt[2]{\alpha^2 - w_0^2}$$

This gives us 3 cases:

Case1: $\alpha^2 > w_0^2$
denominator has two distinct roots s_1, s_2
s_1, s_2 are real values
The solution will be of form $A_1e^{s_1t} + A_2e^{s_2t}$

since $\sqrt[2]{\alpha^2 - w_0^2} < \alpha$
The solution is a exponentially decaying function, one time constant larger than the other.
It is an over damped situation.

Case2: $\alpha^2 = w_0^2$
In this case denominator has two equal real roots.
$s_1, s_2 = -\alpha$
The solution will be of form $A_1 e^{-\alpha t} + A_2 t e^{-\alpha t}$
It is a critically damped situation.

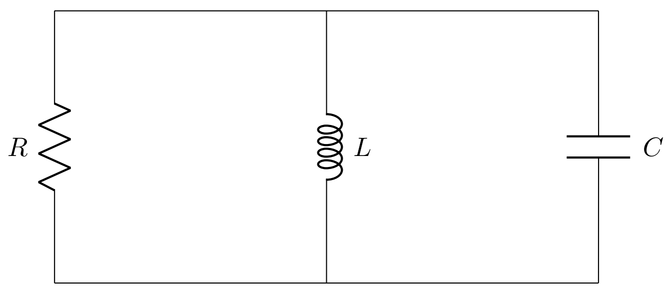
case3: $\alpha^2 < w_0^2$
In this case the denominator has complex roots.
let $w_\alpha = \sqrt[2]{w_0^2 - \alpha^2}$
$s_1, s_2 = -\alpha \pm j w_\alpha$

$$\begin{aligned} denominator &= (s + \alpha + j w_\alpha)(s + \alpha - j w_\alpha) \\ &= (S + \alpha)^2 + w_\alpha^2 \end{aligned}$$

The solution will be of form $e^{-\alpha t}[A_1 \cos(w_\alpha t) + A_2 \sin(w_\alpha t]$
This is an under damped system.

$$\text{'Q' Quality factor} = \frac{w_0}{2\alpha}$$

$$\begin{aligned} \text{over damped;} w_0 < \alpha &\implies Q < \frac{1}{2} \\ \text{critically damped;} w_0 = \alpha &\implies Q = \frac{1}{2} \\ \text{under damped;} w_0 > \alpha &\implies Q > \frac{1}{2} \end{aligned}$$



For high quality factor, R should be large