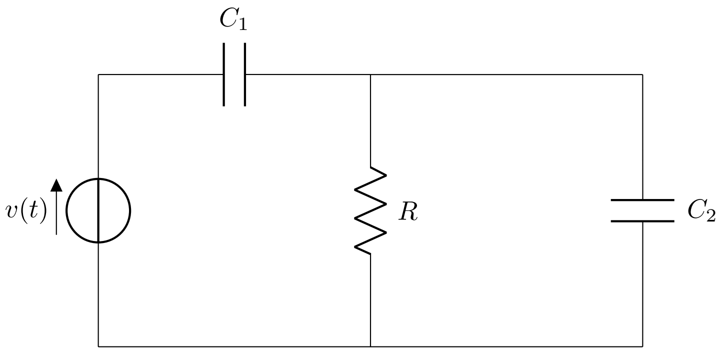
Electric Circuits and Networks	**03/09/2018**

Lecture 14: Bode plots

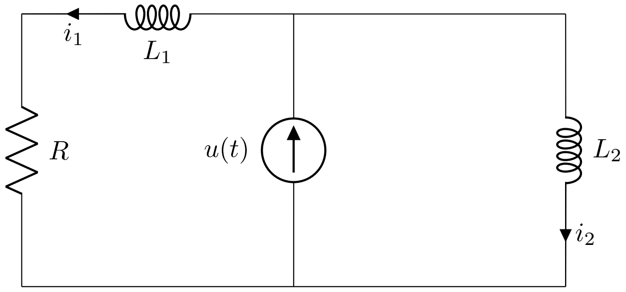
Lecturer: Dr. Vinita Vasudevan — *Scribe: Pradeep*

Loop of voltage sources and capacitor ; $V_c(0^+) \neq V_c(0^-)$



To satisfy KVL,there must be a step change in capacitor voltage. Use charge balance to find $V_c(0^+)$.

Nodes/Super Nodes with only inductors and current sources.



Assuming we To satisfy KCL, must have step changes in the inuctor current
Normally $i_l(0^+) = i_l(0^-)$, except under these conditions. At t=0 $V = K\delta(t)$, which is equal to the voltage across the inductors. This creates a flux in both inductors, which must therefore be equal. Hence,

$$L_1 i_1(0^+) = L_2 i_2(0^+)$$
$$i_1(0^+) + i_2(0^+) = 1$$

Solve these two equations to get i_1 and i_2 at (0^+)

$$V = L\frac{di}{dt}$$
$$K\delta(t) = \frac{d\phi}{dt}$$

Therefore, $\phi(0^+) - \phi(0^-) = K$ Since $\phi_1(0^-) = \phi_2(0^-) = 0$, $\phi_1(0^+) = \phi_2(0^+) = K$. Hence $L_1 i_1(0^+) = L_2 i_2(0^+)$. Substitute in KCL and find $i_1(0^+)$ and $i_2(0^+)$.

1 Bode Plots

Piecewise linear approximations

$$
\begin{aligned}
H(s) &= (1 + \frac{s}{a}) \\
H(j\omega) &= 1 + \frac{j\omega}{a} \\
|H(j\omega)| &= \sqrt[2]{1 + \frac{\omega^2}{a^2}} \\
\angle H(j\omega) &= \tan^{-1}(\frac{\omega}{a}) \\
H_{dB} &= 20 \log_{10} |H(j\omega)| \\
&= 20 \log_{10} \sqrt[2]{1 + \frac{\omega^2}{a^2}} \\
&= 10 \log_{10} 1 + \frac{\omega^2}{a^2}
\end{aligned}
$$

Let's look at two cases:

$$
\begin{aligned}
\omega << a &\implies H_{dB} = 10 \log_{10} 1 = 0 \\
\omega >> a &\implies H_{dB} = 20 \log_{10} \frac{\omega}{a}
\end{aligned}
$$

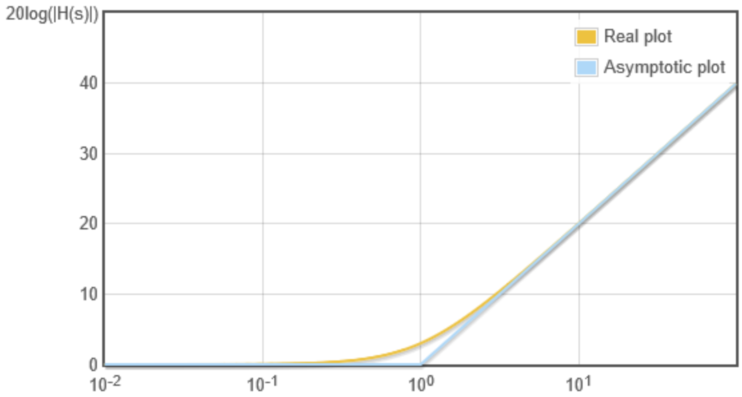


Figure 1: considering a=1

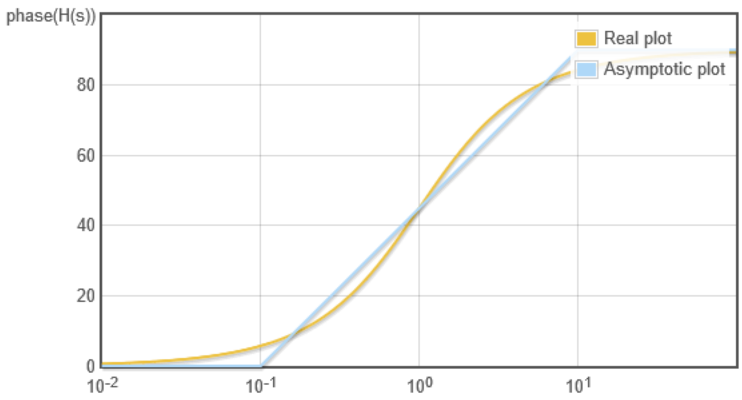


Figure 2: considering a=1

Example 2:

$$H(s) = (1 + \frac{s}{20})(1 + \frac{s}{1000})$$
$$|H(j\omega)| = |1 + \frac{j\omega}{20}||1 + \frac{j\omega}{1000}|$$
$$= \sqrt[2]{1 + \frac{\omega^2}{20^2}}\sqrt[2]{1 + \frac{\omega^2}{1000^2}}$$
$$H_{dB} = 20\log_{10}\sqrt[2]{1 + \frac{\omega^2}{20^2}} + 20\log_{10}\sqrt[2]{1 + \frac{\omega^2}{1000^2}}$$
$$\omega << 20 \implies H_{dB} = 20\log_{10} 1 + 20\log_{10} 1 = 0$$
$$20 < \omega << 1000 \implies H_{dB} = 20\log_{10}\frac{w}{20} + 20\log_{10} 1$$
$$\omega >> 1000 \implies H_{dB} = 20\log_{10}\frac{\omega}{20} + 20\log_{10}\frac{\omega}{1000}$$

The asymptote is initially zero, then rises as 20 dB/decade and finally at 40 dB/decade.

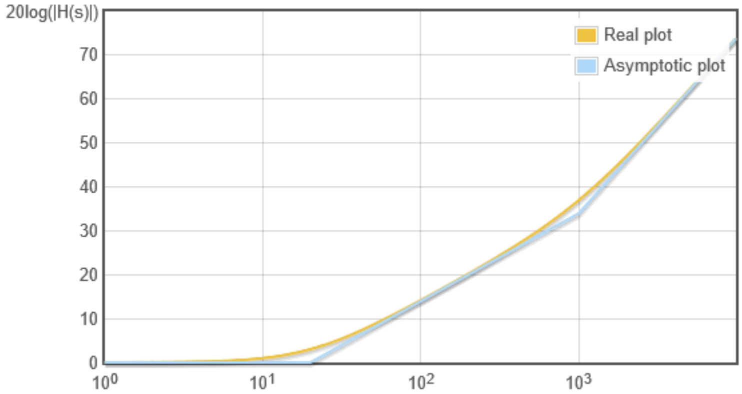


Figure 3: Magnitude plot

$$\angle H(j\omega) = \tan^{-1}(\frac{\omega}{20}) + \tan^{-1}(\frac{\omega}{1000})$$

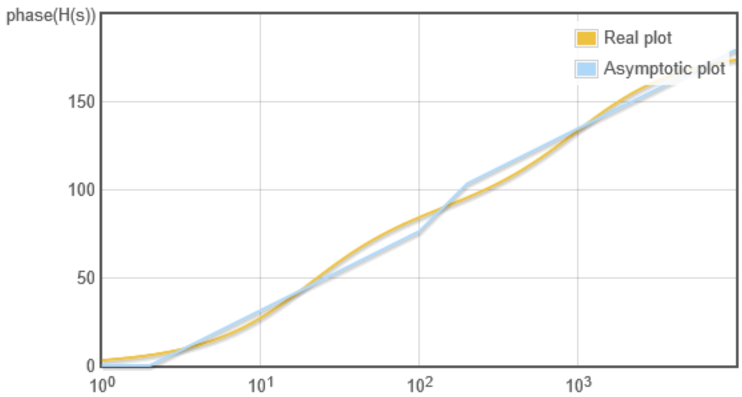


Figure 4: Phase plot

$$\angle H(j\omega) = \tan^{-1}(\frac{\omega}{10}) - \tan^{-1}(\frac{\omega}{200}) - \tan^{-1}(\frac{\omega}{10^5})$$

From $\omega = 2$ to $\omega = 100$, it rises as 45°/decade, from $\omega = 100$ to $\omega = 200$, it rises as 90°/decade, from $\omega = 200$ to $\omega = 10000$, it rises as 45°/decade. Beyond $\omega = 10000$, it is constant at 180°.

Example 3:

$$H(s) = \frac{s+10}{(s+200)(s+10^5)}$$

$$|H(j\omega)| = \frac{|1+\frac{j\omega}{10}|}{(|1+\frac{j\omega}{200}|)(|1+\frac{j\omega}{10^5}|)}$$

$$H_{dB} = 20\log_{10} k + 20\log_{10}\sqrt[2]{1+\frac{\omega^2}{10^2}} - 20\log_{10}\sqrt[2]{1+\frac{\omega^2}{200^2}} - 20\log_{10}\sqrt[2]{1+\frac{\omega^2}{10^{10}}}$$

$$H_1 = 20\log_{10} k(\text{ constant independent of } \omega)$$

$$H_2 = 20\log_{10}\sqrt[2]{1+\frac{\omega^2}{10^2}}(\text{rises as 20db/decade when } \omega\text{=10})$$

$$H_3 = -20\log_{10}\sqrt[2]{1+\frac{\omega^2}{200^2}}\ (\text{falls by 20 db/decade when } \omega = 100)$$

$$H_4 = -20\log_{10}\sqrt[2]{1+\frac{\omega^2}{10^{10}}}\ (\text{ falls by 20 db/decade at } \omega = 100000\text{ })$$

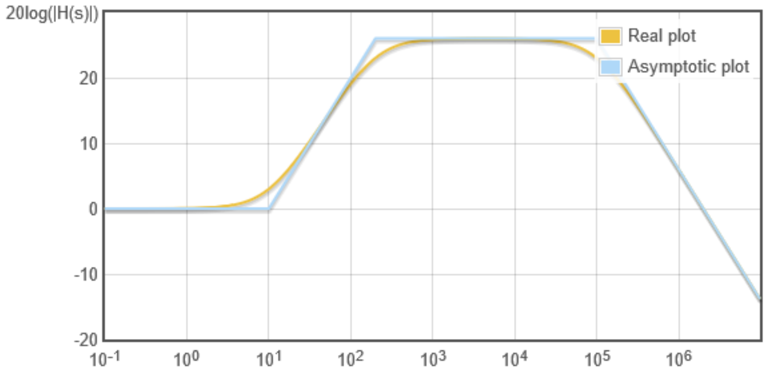


Figure 5: considering a=1

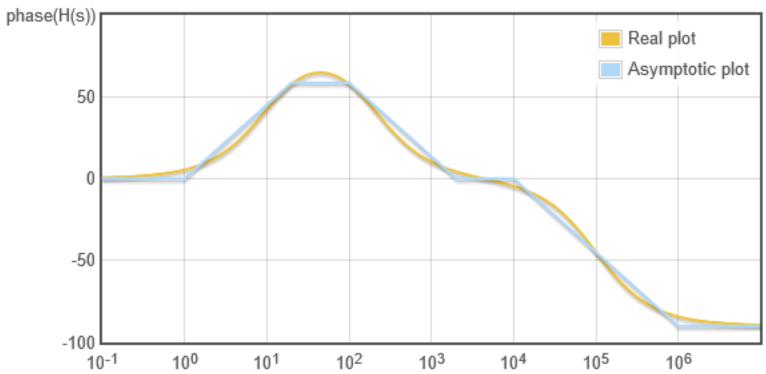


Figure 6: Phase plot