Electric Circuits and Networks	27-08-2018

Lecture 11: Driving Point Functions & Network Function

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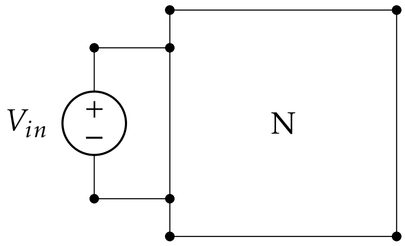
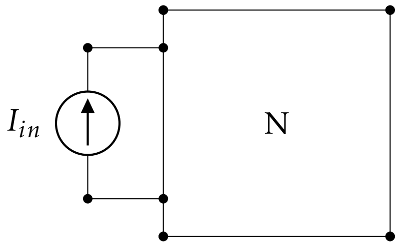
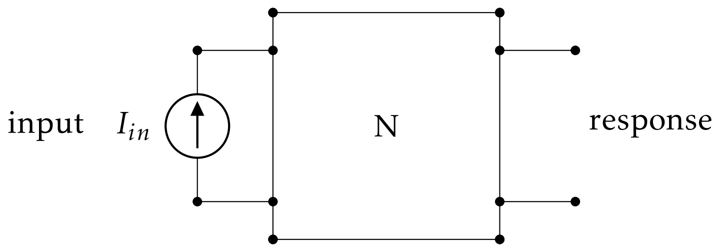
Driving Point Functions: Impedence and admittance measured at the same port of the network with assumption is LTI i.e. zero initial condition and no independent sources other than the input.



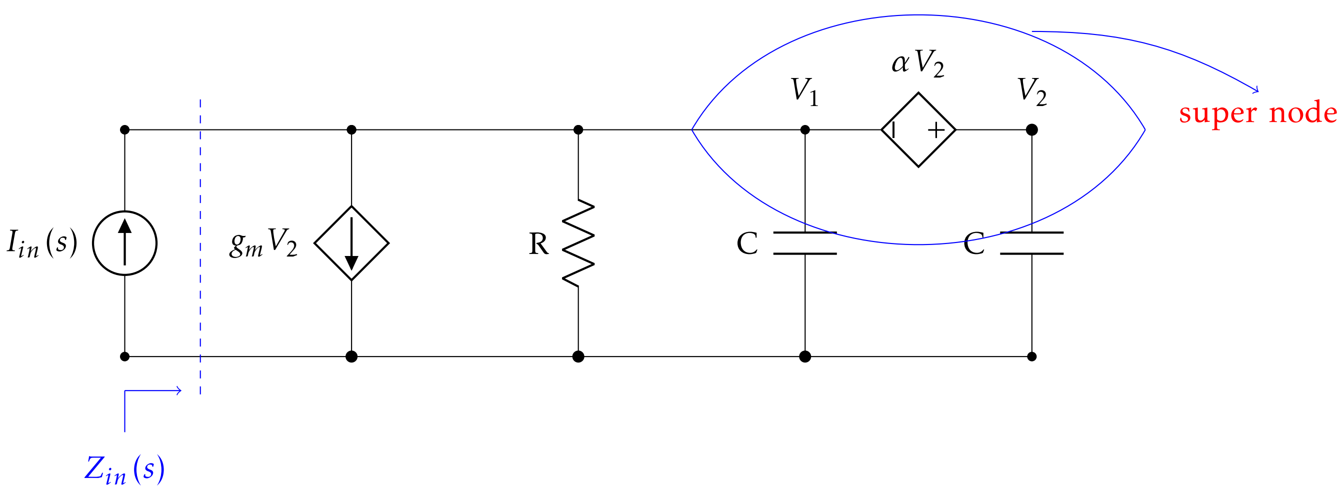


$$Y_{in}(s) = \frac{I_{in}(s)}{V_{in}(s)} \qquad\qquad Z_{in}(s) = \frac{V_{in}(s)}{I_{in}(s)}$$

Transfer Function: It is the ratio of laplace transform of input and response at different port with assumption is LTI i.e. zero initial condition and no independent sources other than the input.



Example 1:



$$V_1\left(\frac{1}{R} + sC\right) + V_2(sC) + V_2 g_m = I_{in}(s)$$

$$V_1 - V_2 = \alpha V_2$$

$$\begin{bmatrix} \frac{1+sRC}{R} & g_m + sC \\ 1 & -(1+\alpha) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{in} \\ 0 \end{bmatrix}$$

The inverse for 2×2 matrix is given as:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

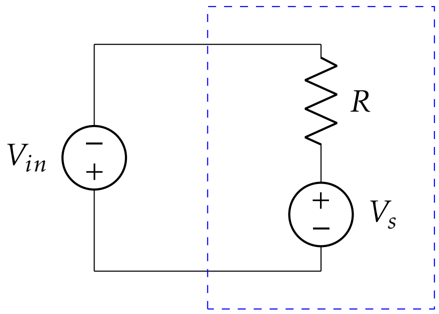
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \frac{1}{D} \begin{bmatrix} -(1+\alpha) & -(g_m + sC) \\ -1 & \frac{1+sRC}{R} \end{bmatrix} \begin{bmatrix} I_{in} \\ 0 \end{bmatrix}$$

where,

$$\frac{1}{D} = \frac{-R}{(1+g_m R+\alpha)+sRC(2+\alpha)}$$

$$V_1(s) = \frac{-(1+\alpha) I_{in}}{D}$$

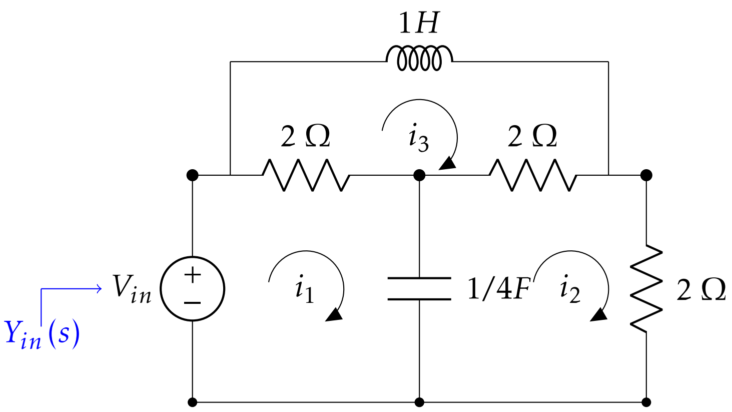
$$Z_{in}(s) = \frac{R(1+\alpha)}{(1+g_m R+\alpha)+sRC(2+\alpha)}$$



$$I = \frac{V_{in} - V_s}{R}$$

Not linear

Example 2: Find admittance $Y_{in}(s)$



$$\begin{bmatrix} 2(s+2/s) & -4/s & -2 \\ -4/s & 4\frac{s+1}{s} & -2 \\ -2 & -2 & 4+s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_{in} \\ 0 \\ 0 \end{bmatrix}$$

$$Det = \frac{8(s+2)^2}{s}$$

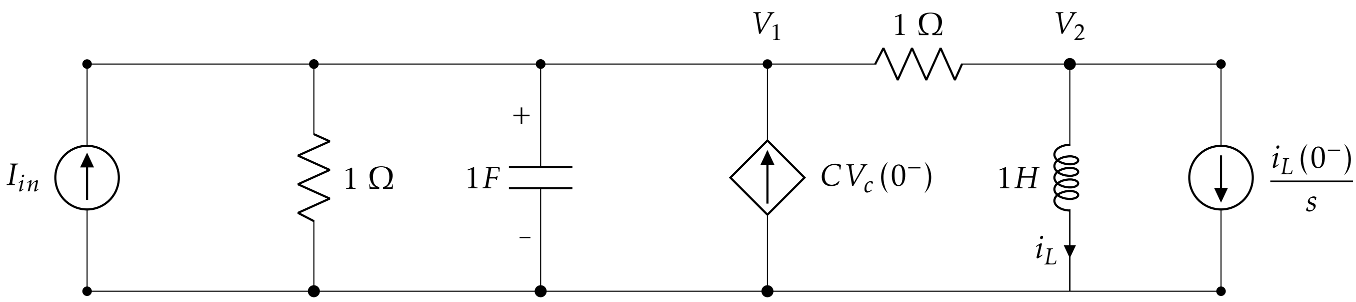
Use Cramer's rule to solve

$$I_1(s) = V_{in}(s)\frac{4\left(s^2+4s+4\right)/s}{8(s+2)^2/s}$$

$$Y_{in}(s) = 1/2$$

Note that poles and zeros cancel. But it is a second order system with repeated root $s=-2$.

Example 3:



$$\begin{bmatrix} s+2 & -1 \\ -1 & 1/s+1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_{in} + CV_c(0^-) \\ -i_L(0^-)/s \end{bmatrix}$$

Initial value and final value theorem

$$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0^-) = \int_{0^-}^{\infty} \frac{df}{dt} e^{-st} dt$$

$$\lim_{s\to\infty}\left(sF(s) - f(0^-)\right) = \lim_{s\to\infty}\int_{0^-}^{0^+} \frac{df}{dt} e^{-st} dt + \lim_{s\to\infty}\int_{0^-}^{\infty} \frac{df}{dt} e^{-st} dt$$

$$RHS = \lim_{s\to\infty}\int_{0^-}^{0^+} \frac{df}{=} f(0^+) - f(0^-)$$

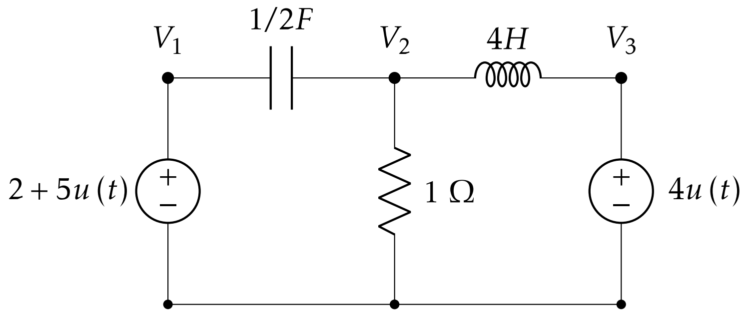
$$\lim_{s\to\infty} sF(s) = f(0^+)$$

Final Value

$$\lim_{s\to 0}\left(sF(s)-f(0^-)\right)=\int_{0^-}^{\infty}\frac{df}{dt}dt$$
$$=\lim_{t\to\infty}\int_{0^-}^{t}\frac{df}{d\tau}d\tau$$
$$=\lim_{t\to\infty}f(t)-f(0^-)$$
$$\lim_{s\to 0}sF(s)=\lim_{t\to\infty}f(t)$$

Note: Final value theorem will give steady state solution only if there is a steady state. It works only if all poles are in the LHP and you have at most one simple pole at the origin e.g. $\lim_{s\to 0}\frac{s\cdot s}{s^2+\omega^2}=0$. Not correct as $\lim_{t\to\infty}cos\omega t\neq 0$

Exercise : Find $V_2(s), V_2(t)$ for circuit given below



Use initial value theorem and find $v_2(0^+)$ from $V_2(s)$ and final value theorem to find $\lim_{t\to\infty}v_2(t)$