# Multipliers.

1 positive integers.

$a_3 \; a_2 \; a_1 \; a_0$

$b_3 \; b_2 \; b_1 \; b_0$

$a_3b_0 \quad a_2b_0 \quad a_1b_0 \quad \boxed{a_0b_0}$

$b_1a_3 \quad b_1a_2 \quad b_1a_1 \quad b_1a_0$

.... $b_2a_1 \quad b_2a_0$

.... $b_3a_0$

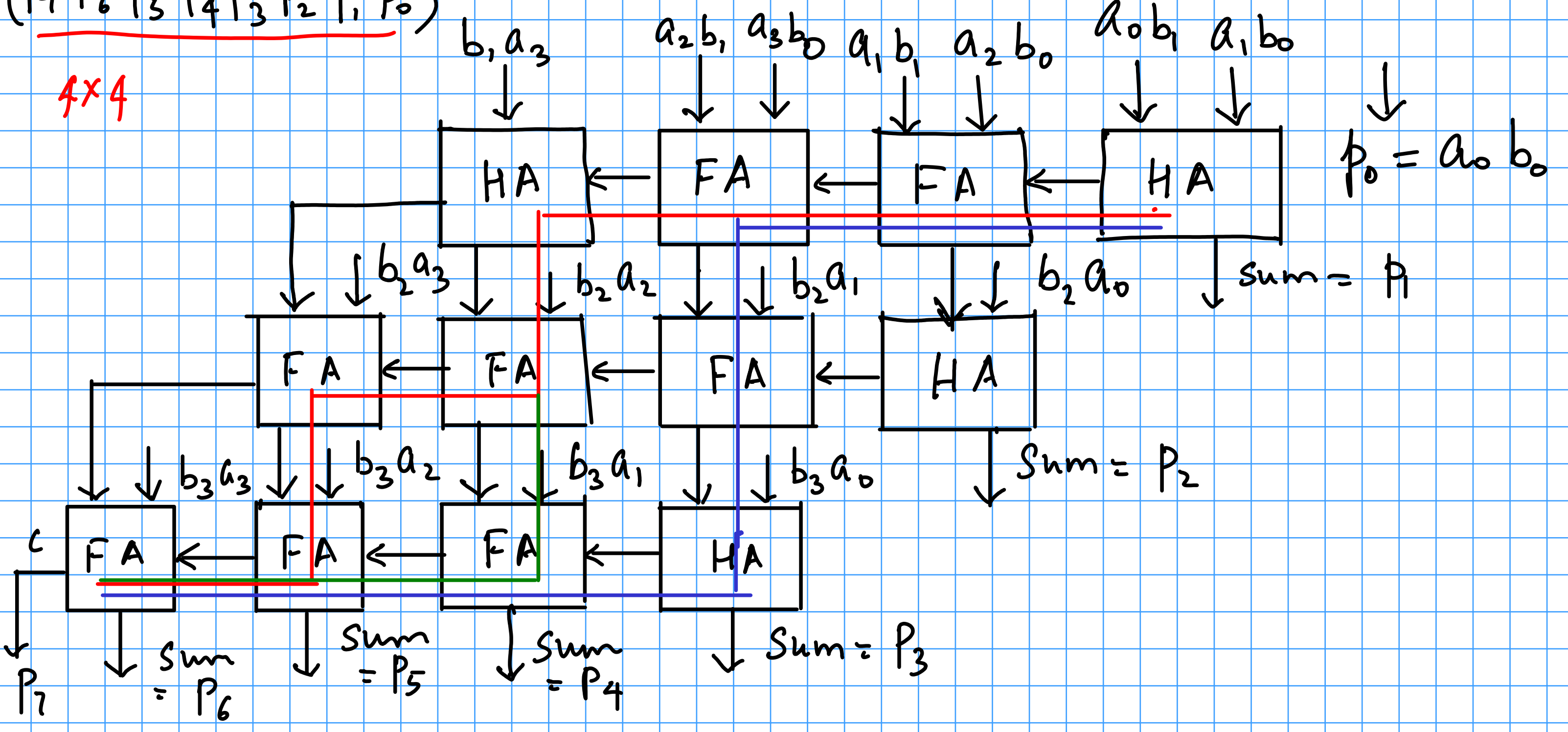
eg:

|  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|
|  |  |  |  | 1 | 0 | 1 | 0 |  |
|  |  |  |  | 1 | 0 | 1 | 1 |  |
| partial product ← | 0 | 0 | 0 | 1 | 0 | 1 | 0 | $pp_0$ |
|  | 0 | 0 | 1 | 0 | 1 | 0 |  | $pp_1$ |
|  | 0 | 0 | 0 | 0 | 0 |  |  | $pp_2$ |
|  | 1 | 0 | 1 | 0 |  |  |  | $pp_3$ |

$$\text{Product} = (((pp_0 + pp_1) + pp_2) + pp_3)$$

$(P_7 \, P_6 \, P_5 \, P_4 \, P_3 \, P_2 \, P_1 \, P_0)$

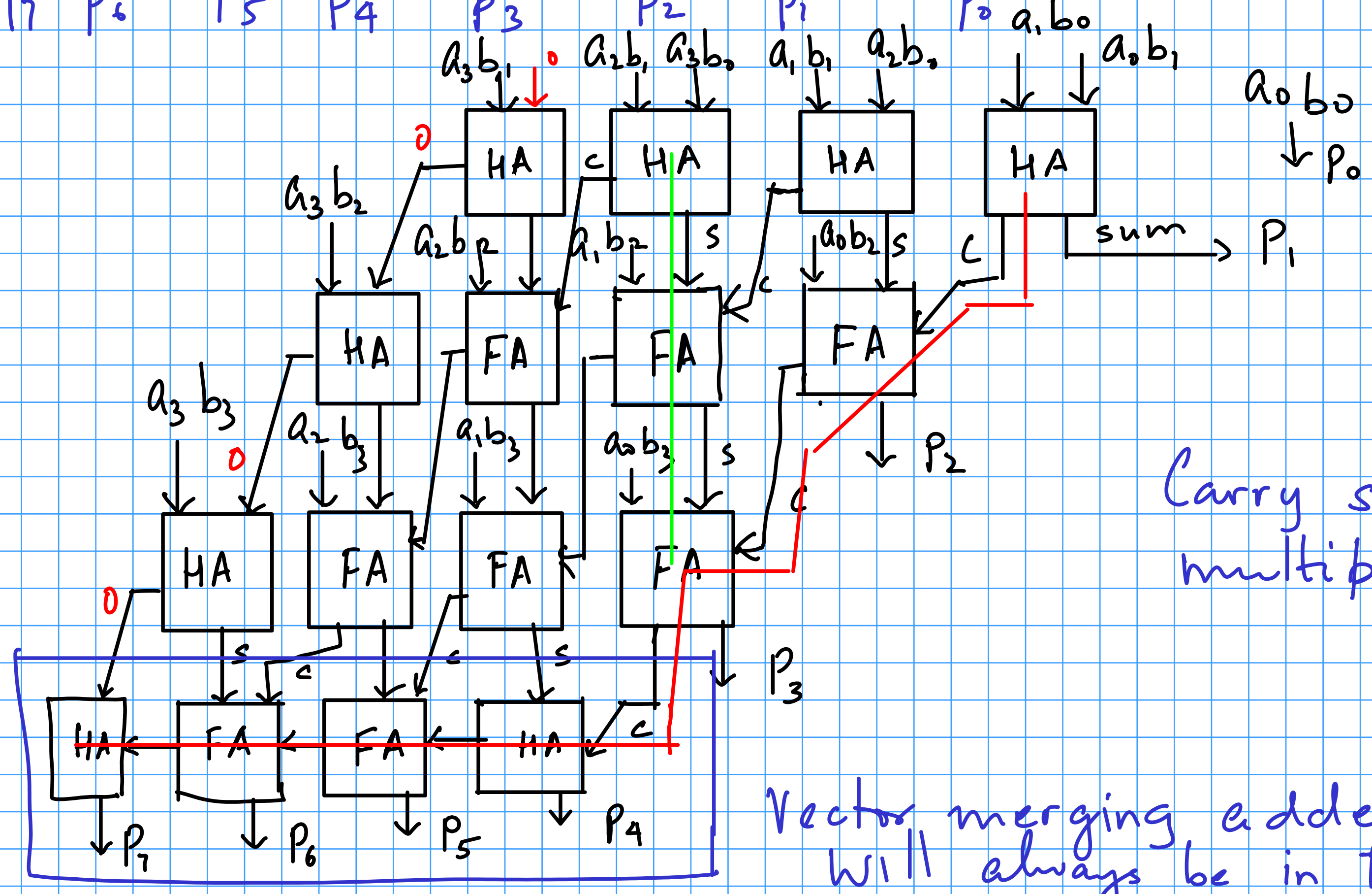
4×4



Critical path? Assume $t_{sum} = t_{carry}$.

Too many paths with almost same delay; optimisation is difficult.

$$\begin{array}{cccccccc}
 & & & & a_3 & a_2 & a_1 & a_0 \\
 & & & & b_3 & b_2 & b_1 & b_0 \\
\hline
 & & & & a_3b_0 & a_2b_0 & a_1b_0 & \underbrace{a_0b_0}_{p_0} \\
 & & & a_3b_1 & a_2b_1 & a_1b_1 & a_0 \cdot b_1 & \\
 & & & & s & c & c & \\
 & & a_3b_2 & a_2b_2 & a_1b_2 & a_0b_2 & & \\
 & a_3b_3 & a_2b_3 & a_1b_3 & a_0b_3 & & & \\
\hline
p_7 & p_6 & p_5 & p_4 & p_3 & p_2 & p_1 & p_0
\end{array}$$



Carry save multiplier.

Vector merging adder. Will always be in the critical path.

Can be any adder; need not be ripple carry.

# Two complement:

Algorithm: Invert all bits and add '1'

Why?

+ve numbers, '0' ; MSB is '0'

-ve numbers MSB is '1'

If 'a' is a +ve number, represent $-a$ as $2^n - a$ ; $a = a_{n-1} a_{n-2} \dots a_0$.

eg: 0101 → +5

$$\begin{array}{r} 10000 \\ -\ 0101 \\ \hline \underbrace{1011} \end{array} \rightarrow -5$$

Same as 1. Subtract from 1111 $(2^n - 1)$

2. Add 1

$$\left.\begin{array}{r} 1111 \\ -\ 0101 \\ \hline 1010 \end{array}\right\rangle \text{Invert bits.}$$

Add '1'.

# Deimal eq. of 2s complement

$$a_{n-1}\, a_{n-2}\, a_{n-3} \cdots a_0 = a$$

## decimal equivalent

+ve no $\Rightarrow a_{n-1} = 0$ ; decimal eq. $= \sum_{i=0}^{n-2} a_i 2^i$

−ve no; $a_{n-1} = 1$ ; no. is $-(2^n - a)$

$$= -2^n + 2^{n-1} + \sum_{i=0}^{n-2} a_i 2^i$$

$$= -2^{n-1} + \sum_{i=0}^{n-2} a_i 2^i$$

$\therefore$ In general

$$= -a_{n-1} 2^{n-1} + \sum_{i=0}^{n-2} a_i 2^i$$

## Sign extension

0000101

1010 > same decimal equivalent.

$\underbrace{1111}1010$

$$-2^7 + 2^6 + 2^5 + 2^4 + 2^2.$$

$$= -2^6 + 2^5 + 2^4 + 2^2 = -2^5 + 2^4 + 2^2 = -2^4 + 2^2$$

Sign extension does not change the number.

array multiplier with sign extension.