# Nonlinear least Squares

→ <u>Gauss Newton Method</u>

Recap Newton

① Calc. $\nabla f$, $\nabla^2 f$ → check if P.D.

② Calc Newton direction $(\nabla^2 f)\, p = -\nabla f$

Solve for $p$, Newton dir

→ Gauss Newton Method

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③ Calc step length, $\alpha$, $x_{k+1} = x_k + \alpha_k p_k$.

④ Check convg, if so stop, else go to ①.

Specialise that $f(x) = \frac{1}{2} \| r(x) \|^2$ least squares problem.

Grad

$$\nabla f(x) = J^T(x)\, r(x)$$

Hessian:

$$\nabla^2 f(x) = J^T(x) J(x) + \underbrace{\sum_j r_j(x) \nabla^2 r_j(x)}$$

When can I ignore:

① If $r_j(x)$ is affine

② Close to the soln.

↳ Calc $p_N \rightarrow \quad (\nabla^2 f)\, p = -\nabla f$

$$J_k^T J_k p_k = -J_k^T r_k \quad \rightarrow \text{all are fns of } x.$$

① Solve for $p_k$, calc $\alpha_k$.

↳ $p$ is a legit descent direction.

$$p_k^T \nabla f_k < 0$$

$$p_k^T \nabla f_k = p_k^T J^T r_k = -p_k^T \left( J_k^T J_k p_k \right) = -\| J_k p_k \|^2$$

When does it go to 0? $p_k = 0$

$$\Rightarrow (\nabla^2 f)\, p = -\nabla f \Rightarrow \nabla f = 0$$

Stationary condn.

$$J_k^T J_k p_k = -J_k^T r_k$$ Solve for $p_k$.

$$J_k^T \left[ J_k p_k + r_k \right] = 0$$

eqv to $\min_p \| J_k p_k + r_k \|^2$.

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