Computational Electromagnetics : An Overview

Uday Khankhoje

Electrical Engineering, IIT Madras

Topics in this module

1. Mathematical History
2. Different regimes of Maxwell's equations
3. Different ways of solving them
4. Where CEM is useful

Table of Contents

From 1600s to today

Mathematics has been used to solve problems in physics for many centuries.

- 1600s – Kepler used precalculus – predicting celestial events
- 1600-1700s – Leibniz, Newton developed early calculus for mechanics
- 1800s – Theory of differential equations – first for fluid problems (Navier-Stokes equation)
- 1900s – Mathematicians looking to solve partial differential equations – first applied to comp. fluid dyanmics. Why?

  no analytical solns.
- 1960s onwards – CEM for problems that can't be solved analytically

The equations for Electromagentics

"Maxwell's" equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \text{ Faraday, 1843} \quad (1)$$

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}, \text{ Ampere, 1823} \quad (2)$$

$$\nabla \cdot \vec{D} = \rho, \text{ Coulomb, 1785} \quad (3)$$

$$\nabla \cdot \vec{B} = 0, \text{ Gauss, 1841} \quad (4)$$

$$\frac{\partial \vec{D}}{\partial t} \rightarrow \text{ Maxwell, 1864}$$

Maxwell's time $\rightarrow$ 20 eqns (1873)
Heaviside $\rightarrow$ distilled into 4 eqns(1888)

- Equations are *linear*
- Optics $\leftrightarrow$ Electromagnetics
- Fourier techniques

Table of Contents

Derivatives in space and time $\to e^{j\omega t}$

$$\nabla\times\bar{E} \to \frac{\partial}{\partial x} = -\frac{\partial \bar{B}}{\partial t}$$

Boundary conditions apply → object boundaries

$$\frac{\partial}{\partial x} \sim \frac{1}{L} \leftrightarrow \frac{c}{\lambda} - \omega \leftarrow \frac{\partial}{\partial t}$$

(field contorts around obj) (time harmonic field)

- Low frequency, $L \ll \lambda$ Statics. ↳ uncoupled.
- Mid frequency, $L \approx \lambda$ ← coupled eqns. wave like
- High frequency, $L \gg \lambda$ → optics, ray like solns.

Table of Contents

$e^{j\omega t}$

Techniques

Time domain v/s Frequency ($\frac{\partial}{\partial t} \to j\omega$)

transient responses.

single freq

- IEM – integral eqn
- FEM – finite element
- FDTD – finite difference time domain

Time domain, diff form

Differential v/s Integral methods

$\frac{\partial}{\partial t}$ $\frac{\partial}{\partial x}$

$$\boxed{\frac{f(t+\Delta t) - f(t)}{\Delta t}}$$

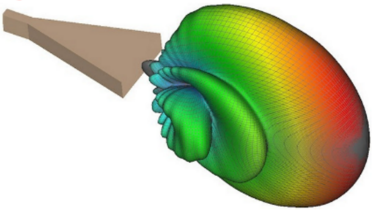
$$\underbrace{\oint_S \vec{A}\cdot\vec{ds}}_{2D} = \underbrace{\int_V \nabla\cdot\vec{A}\, dv}_{3D}$$

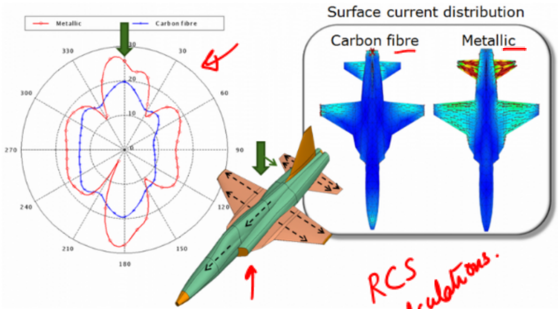
$$\frac{v}{s} \propto \frac{r^3}{r^2} = r$$

Table of Contents

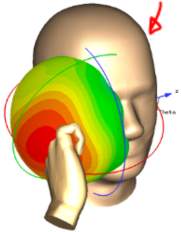
Some applications that need CEM

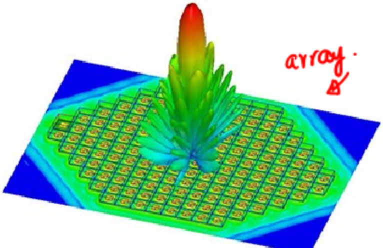
Horn





RCS calculations.





array.

Topics that were covered in this module

1. Mathematical History
2. Different regimes of Maxwell's equations
3. Different ways of solving them
4. Where CEM is useful

Reference: Ch. 1 of Integral Equation Methods for Electromagnetic and Elastic Waves - by Chew, Tong, Hu