EE5120 Linear Algebra: Tutorial 8, July-Dec 2017-18

Covers sec 6.1,6.2 (exclude law of inertia and generalized eigenvalue problem),6.3 of GS

1. Compute the SVD of $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$.

2. (a) Prove that a symmetric matrix A is positive definite if and only if there exists a matrix B with independent columns such that $A = B^T B$.

   (b) If A is written as its eigenvalue decomposition, what will B be ?

   Hint: Substitute A in $x^T A x$ and solve.

3. Suppose A is a 2 by 2 symmetric matrix with unit eigenvectors u_1 and u_2. If its eigenvalues are $\lambda_1 = 3$ and $\lambda_2 = -2$, what are U, Σ and V^T ?

   Hint: Use property of symmetric matrices to find singular values.

4. For what range of a and b are the matrices **A**,**B** positive definite

$$\mathbf{A} = \begin{bmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 1 & 2 & 4 \\ 2 & b & 8 \\ 4 & 8 & 7 \end{bmatrix}$$

   *Hint:*Use definitions.

5. Let **A** and **B** be real square symmetric Positive semi-definite matrices. Is **AB**+**BA** positive semi-definite always

   *Hint:*Counter examples?.

6. Give a quick reason why each of these statements is true:

   (a) Every positive definite matrix is invertible.

   (b) The only positive definite projection matrix is $P = I$.

   (c) A diagonal matrix with positive diagonal entries is positive definite.

   (d) A symmetric matrix with a positive determinant might not be positive definite

7. Suppose $\mathbf{u_1}, ..., \mathbf{u_n}$ and $\mathbf{v_1}, ..., \mathbf{v_n}$ are orthonormal bases for R_n. Construct the matrix A that transforms each $\mathbf{v_j}$ into $\mathbf{u_j}$ to give $A\mathbf{v_1} = \mathbf{u_1}, ..., A\mathbf{v_n} = \mathbf{u_n}$.

   *Hint:*Write the equations in matrix form to find expression for A.

8. Let A be an $n \times n$ hermitian matrix with n distinct eigenvalues. Prove that for any n length vector $\mathbf{x}$, $\lambda_{\min}||\mathbf{x}||^2 \leq \mathbf{x}^H A \mathbf{x} \leq \lambda_{\max}||\mathbf{x}||^2$, where $||\mathbf{x}||^2 = \mathbf{x}^H \mathbf{x}$, $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum and maximum eigenvalues of A.

   Hint: Use eigen decomposition of A and its properties (refer to Q3 of prev. tutorial).

9. The graph of $F_1(x, y) = x^2 + y^2$ is a bowl opening upward. The graph of $F_3(x, y) = x^2 - y^2$ is a saddle. The graph of $F_3(x, y) = -x^2 - y^2$ is a bowl opening downward. What is a test on $F(x, y)$ for having maxima, minima or saddle point at $(0, 0)$?

   Hint: Use second derivative matrix of the function.

10. (a) If A changes to $4A$, what is the change in the SVD?

(b) What is the SVD for A^T and for A^{-1}?

(c) Why doesn't the SVD for $A + I$ just use $\Sigma + I$?

Hint: Calculate SVD of (A+I) in terms of SVD of A.