EE5120 Linear Algebra: Tutorial Test 5, 13.10.17A

Give your answers in the space provided. No calculators or smartphones allowed.

Roll: No:__________________ NAME:__________________________ Time: 20 mins

[2] 1. Is it possible for a square matrix to have a zero eigenvalue and be full rank? Yes/No with one line reason. [2]

Solution: No. Zero eigenvalue means that the determinant is zero, which means that the matrix is not invertible, and hence is not full rank.

[5] 2. Is the matrix $A = \begin{bmatrix} 3 & 2 \\ 3 & 4 \end{bmatrix}$ diagonalizable? Why? Compute the eigen decomposition (i.e. $S\Lambda S^{-1}$, where S is orthogonal and Λ is diagonal) if possible.

Solution: First compute the eigenvalues; they are 1,6. [2]
Since they are distinct, there are linearly independent eigenvectors and S matrix is invertible. [1]
The eigenvectors making up S are $\begin{bmatrix} -1/\sqrt{2} & 2/\sqrt{13} \\ 1/\sqrt{2} & 3/\sqrt{13} \end{bmatrix}$ [2]

[3] 3. Find the determinant of $A + I$, where $A = \begin{bmatrix} p & p & p \\ q & q & q \\ r & r & r \end{bmatrix}$ using the properties discussed in class (i.e. no brute force computation).

Solution: With $M = A + I$ we note that $|M| = |M^T|$ and $M^T = \begin{bmatrix} 1+p & q & r \\ p & 1+q & r \\ p & q & 1+r \end{bmatrix}$.
Row or col transformations don't change the determinant, so:

$$\begin{bmatrix} 1+p & q & r \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \quad R2' = R2 - R1, R3' = R3 - R1$$

$$\begin{bmatrix} 1+p+q+r & q & r \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad C1' = C1 + C2$$

Now, M^T is upper triangular, so $|M^T| = 1 + p + q + r$. [3]