EE5120 Linear Algebra: Tutorial Test 4, 18.09.17A

Give your answers in the space provided. No calculators or smartphones allowed.

Roll: No:__________________ NAME:___________________________ Time: 20 mins

1. Given a vector space V and the basis vectors, $\{v_i\}_{i=1}^n$, which under a linear transformation $T : V \to V$ obey the following relation: $T(v_i) = \lambda_i v_{i+1}$ (for convenience, assume that $v_{n+1} \equiv v_1$). Find the matrix corresponding to this linear transformation. You may assume the same basis for the input and output of T.

> **Solution:** (3 pts)
> Write $T([v_1 \ \ldots v_n]) = [v_1 \ \ldots v_n]A$ and identify the columns of A as the linear combinations of the basis vectors required to reproduce the effect of T on each the basis vectors. $A = \begin{bmatrix} 0 & 0 & \ldots & \lambda_n \\ \lambda_1 & 0 & \ldots & 0 \\ 0 & \lambda_2 & \ldots & 0 \\ \vdots & \vdots & \ddots & 0 \end{bmatrix}$

2. Given that M is the vector space of all (real valued) upper triangular 2×2 matrices, and N is the space of all real valued 2×2 matrices.
(a) List a set of basis vectors for both spaces.
(b) A linear transformation $T : M \to N$ acts in the following way: $T(C) = C^T + C$. Compute the matrix corresponding to T.

> **Solution:** ((a):1.5 for M and 1.5 for N, (b):4)
> (a)For M : $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and for N : $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$
> (b)Transform each basis element of M and write as linear combination of elements of N: $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$