EE5120 Linear Algebra: Tutorial Test 3, 10.09.17A

Give your answers in the space provided. No calculators or smartphones allowed.

Roll: No:_____ NAME:_____ Time: 20 mins

1. A vector space V is spanned by a set of linearly independent vectors, $\{v_i\}$, $1 \leq i \leq p$. Another set of linearly independent vectors, $\{w_i\}$, $1 \leq i \leq q$ are also found to span the same vector space V. Pick the correct option and justify with proper reasoning/derivation.
   (a) $p = q$, or
   (b) $p \neq q$ in general, or
   (c) more information is required to say anything about the relation between p, q.

   Solution: (1+3)
   Correct option is (a) because the number of basis vectors in a basis is unique, even though the basis itself need not be unique. Similar question was done in the tutorial. See it for detailed proof.

2. Given a system of linear equations represented by the matrix equation $Ax = b$ with $A \in \mathbb{R}^{m \times n}$, $x \in \mathbb{R}^n$, $b \in \mathbb{R}^m$. Given that the rows of this matrix are linearly independent and that there are more columns than row. Mention the dimensionality of each of the four fundamental subspaces of A with a one line reason for each.

   Solution: (1.5 x 4)
   $C(A^T)$: m Since rows are independent,
   $C(A)$: m Since row rank = col rank,
   $N(A)$: $n - m$ and $N(A^T)$: 0 by rank nullity theorem.