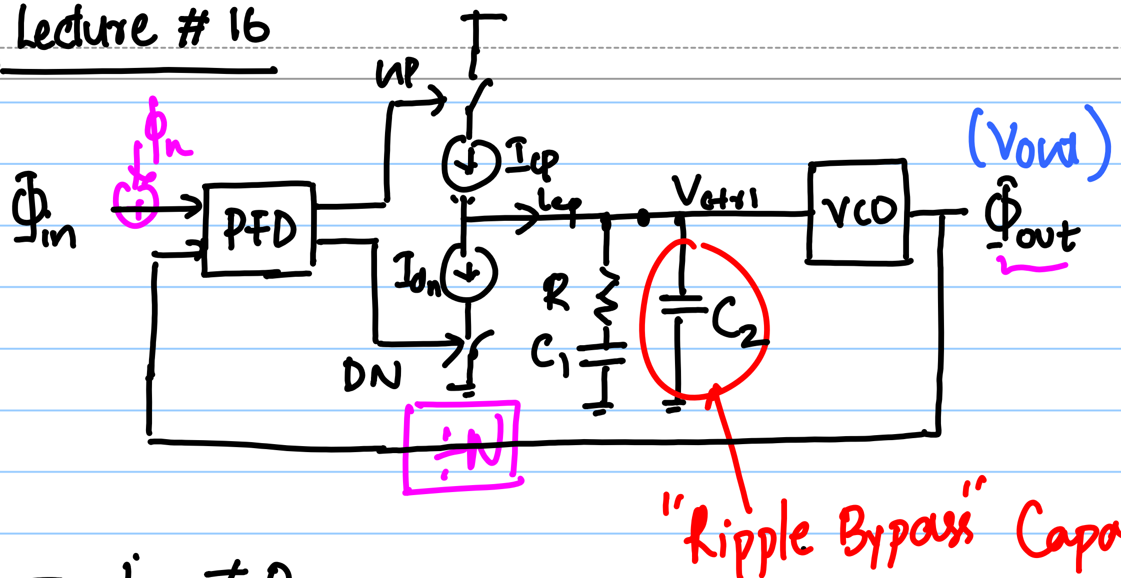
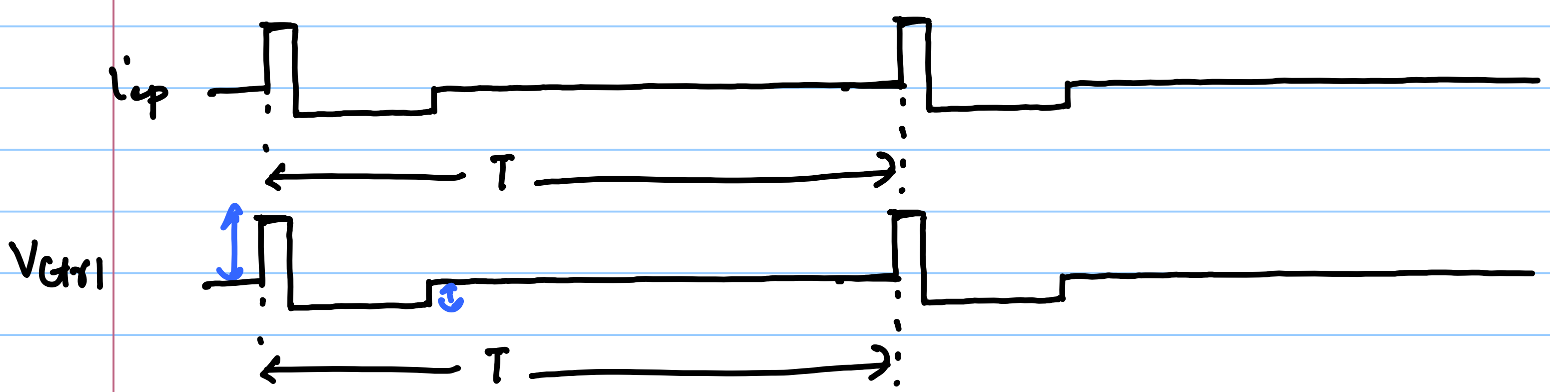
Note Title 23-06-2020

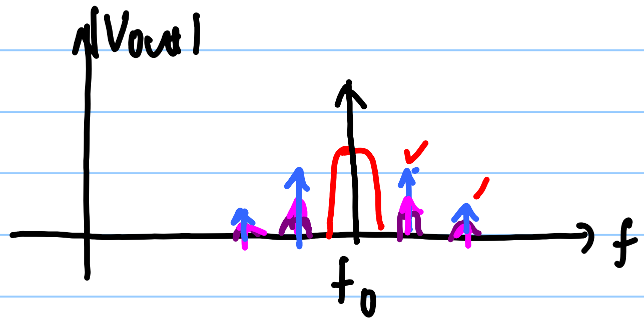
Lecture # 16

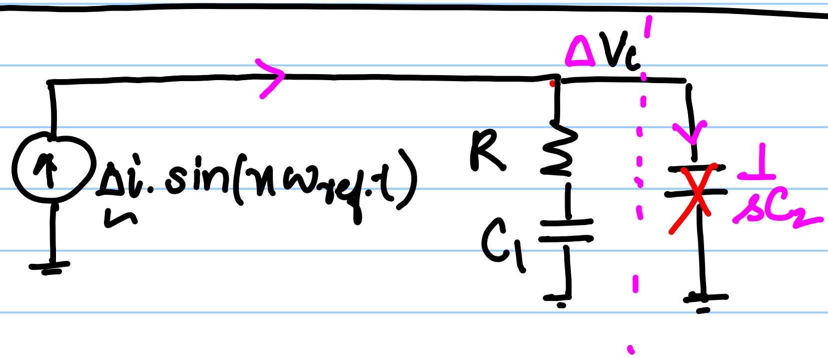


- I_{cp}, R, C_1, C_2
- ω_{-3dB}, Φ_m
- $f_{vco} = f_{free} + K_{vco} \cdot V_{ctrl}$

- $i_{cp} \neq 0$







$$Z_1 = R + \frac{1}{j\omega C_1} = \frac{1 + j\omega C_1 R}{j\omega C_1}$$

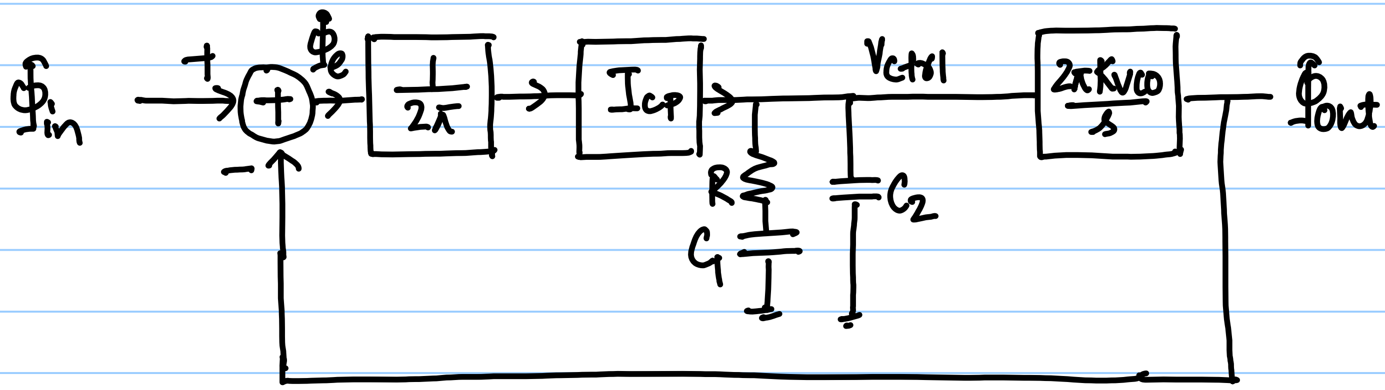
$$Z_2 = \frac{1}{j\omega C_2}$$

$$|Z_1(\omega_{ref})| \gg |Z_2(\omega_{ref})|$$

$$\left| \frac{1 + j\omega_{ref} C_1 R}{j\omega_{ref} \cdot C_1} \right| \gg \left| \frac{1}{j\omega_{ref} C_2} \right|$$

$$\omega_{ref} \cdot C_2 \gg \frac{\omega_{ref} C_1}{\sqrt{1 + \omega_{ref}^2 C_1^2 R^2}}$$

$$\frac{C_2}{C_1} \gg \frac{1}{\sqrt{1 + \omega_{ref}^2 C_1^2 R^2}}$$



$$L_G(s) = \frac{1}{2\pi} I_{cp} \cdot \left(\left(R + \frac{1}{sC_1} \right) \parallel \frac{1}{sC_2} \right) \frac{2\pi K_{vco}}{s}$$

$$LG = \frac{I_{cp} \cdot K_{VCO}}{s^2(C_1+C_2)} \cdot \frac{1+sC_1R}{1+s\frac{C_1C_2R}{C_1+C_2}},$$

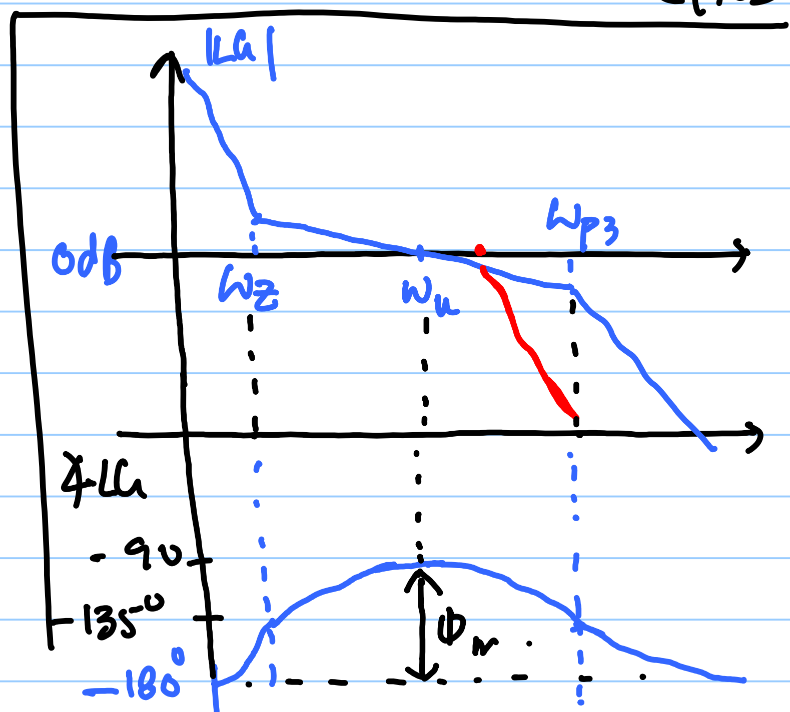
$$\frac{\frac{1+sC_1R}{sC_1} \times \frac{1}{sC_2}}{\frac{1+sC_1R}{sC_1} + \frac{1}{sC_2}} = \frac{1+sC_1R}{sC_2+s^2C_1C_2R+sC_1} = \frac{1+sC_1R}{s(C_1+C_2)\left(1+\frac{sC_1C_2R}{C_1+C_2}\right)}$$

$$LG = \frac{I_{cp}K_{VCO} \cdot C_1R}{s^2(C_1+C_2)\frac{C_1C_2R}{C_1+C_2}} \frac{\left(s+\frac{1}{C_1R}\right)}{\left(s+\frac{1}{\frac{C_1C_2}{C_1+C_2}R}\right)}$$

$$LG = \frac{I_{cp} \cdot K_{VCO}}{s^2C_2} \frac{\left(s+\frac{1}{C_1R}\right)}{\left(s+\frac{1}{\frac{C_1C_2R}{C_1+C_2}}\right)}$$

$$\omega_{p1} = \omega_{p2} = 0, \quad \omega_{p3} = \frac{1}{\frac{C_1C_2}{C_1+C_2} \cdot R}$$

$$\omega_z = \frac{1}{C_1R}$$



$$LG = \frac{I_{cp} K_{vco}}{N s^2 (C_1+C_2)} \frac{(1+s/\omega_z)}{(1+s/\omega_{p3})}$$

$$|LG(\omega_u)| = 1 \Rightarrow \frac{I_{cp} \cdot K_{vco}}{\omega_u^2 (C_1+C_2)} \frac{|1+j\omega_u/\omega_z|}{|1+j\omega_u/\omega_{p3}|} = 1$$

$$\Rightarrow \boxed{\frac{I_{cp} K_{vco}}{\omega_u^2 (C_1+C_2)} \cdot \frac{\omega_u}{\omega_z} \approx 1}$$

$$\omega_u = \frac{I_{cp} \cdot K_{vco}}{C_1+C_2} \cdot R C_1$$

$$= I_{cp} K_{vco} \cdot R \times \frac{C_1}{C_1+C_2}$$

$$\measuredangle LG = -180^\circ + \tan^{-1}\left(\frac{\omega}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega}{\omega_{p3}}\right)$$

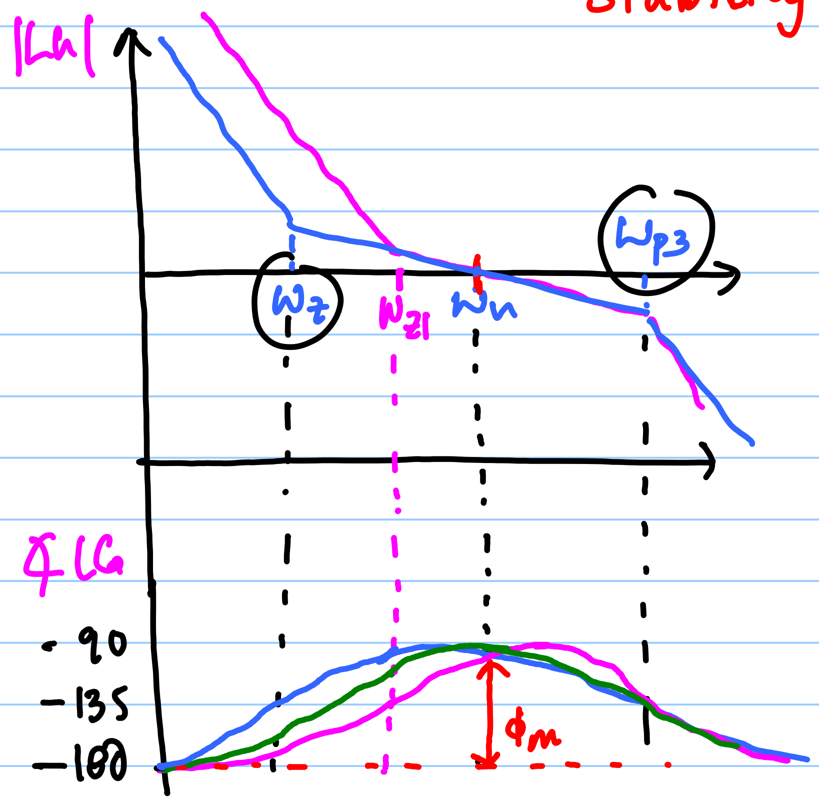
$$\phi_m = \measuredangle LG - (-180^\circ)$$

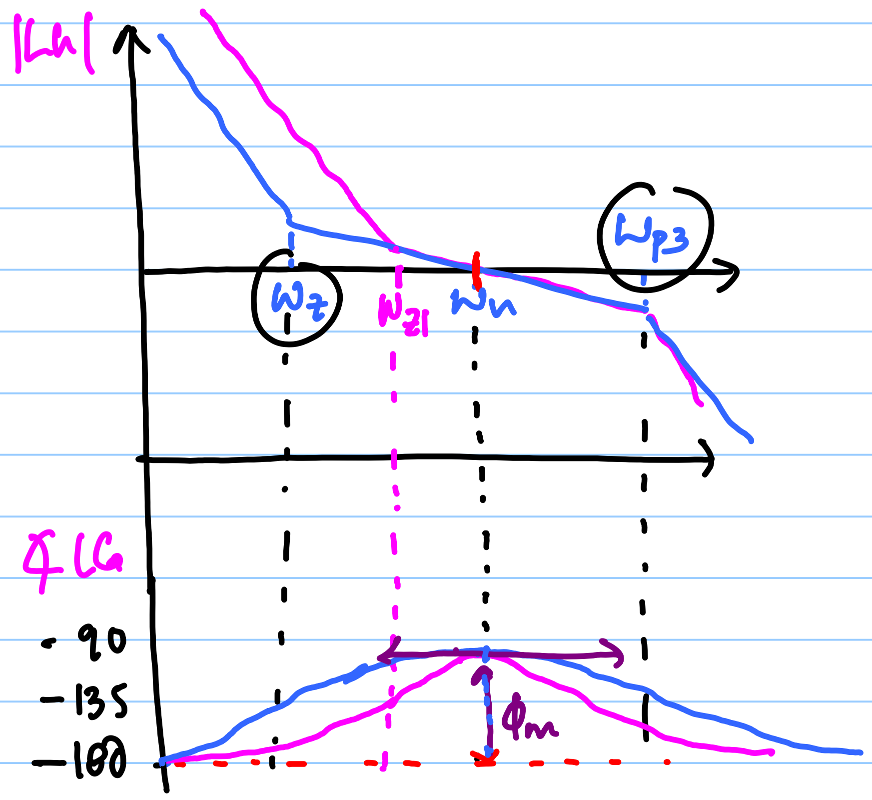
$$= \tan^{-1}\left(\frac{\omega_u}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega_u}{\omega_{p3}}\right)$$

$$\phi_m = \tan^{-1}\left(\omega_u \cdot R C_1\right) - \tan^{-1}\left(\omega_u \frac{R C_1 C_2}{C_1 + C_2}\right)$$

Given $\underbrace{\omega_u \, (\omega_{-3dB} \text{ BW in closed}) \ \& \ \phi_m}$ $\longrightarrow I_{cp}, R, C_1, C_2$

- Noise
- Settling of PLL
- Stability





$$\measuredangle L(\omega) = -180^\circ + \tan^{-1}\left(\frac{\omega}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega}{\omega_{p3}}\right)$$

$$\left.\frac{d(\measuredangle L(\omega))}{d\omega}\right|_{\omega=\omega_u} = \left.\frac{1}{1+\left(\frac{\omega}{\omega_z}\right)^2} \times \frac{1}{\omega_z} - \frac{1}{1+\left(\frac{\omega}{\omega_{p3}}\right)^2} \times \frac{1}{\omega_{p3}}\right|_{\omega=\omega_n} = 0$$

$$\boxed{\omega_u^2 = \omega_z \cdot \omega_{p3}}$$

$$\omega_u^2 = \frac{1}{RC_1} \times \frac{1}{\frac{RC_1 C_2}{C_1+C_2}} = \omega_z^2 \left(\frac{C_1+C_2}{C_2}\right)$$

$$\omega_u = \omega_z \sqrt{1+\frac{C_1}{C_2}} \checkmark$$

$$\Phi_m = \tan^{-1}\left(\frac{\omega_u}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega_n}{\omega_{p3}}\right)$$

$$= \tan^{-1}\left(\frac{\omega_n}{\omega_z}\right) - \tan^{-1}\left(\frac{\omega_n}{\omega_z\left(\frac{C_1+C_2}{C_2}\right)}\right)$$

$$= \tan^{-1}\left(\sqrt{1+\frac{C_1}{C_2}}\right) - \tan^{-1}\left(\frac{1}{\sqrt{1+\frac{C_1}{C_2}}}\right)$$

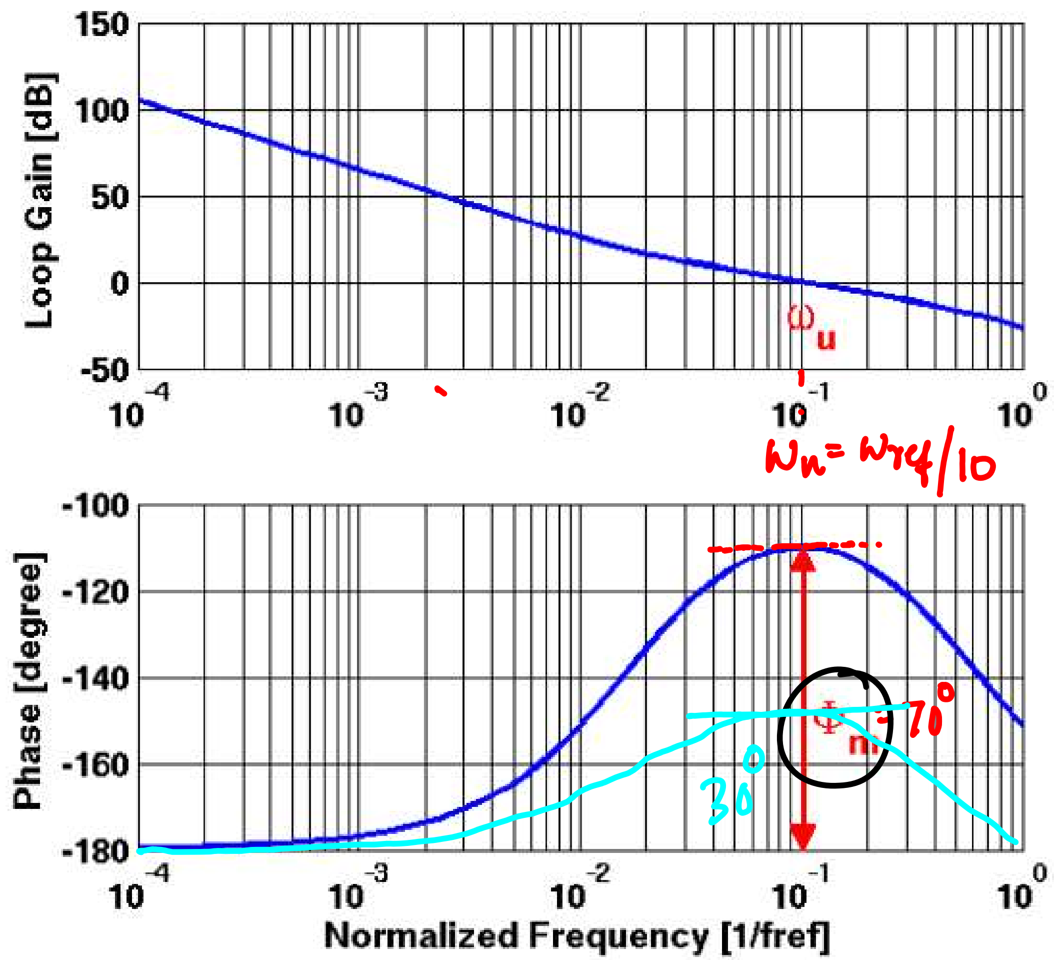
$$\Rightarrow \frac{C_1}{C_2} = 2\left(\tan^2\phi_m + \tan\phi_m\sqrt{1+\tan^2\phi_m}\right) = K_c$$

- ω_u, ϕ_m
- $\underbrace{\frac{C_1}{C_2}}$ = , $\underbrace{\omega_z}$ = $\frac{\omega_u}{\sqrt{1+\frac{C_1}{C_2}}}$
- $\omega_z = \frac{1}{RC_1}$: Choose Low R for lesser noise.

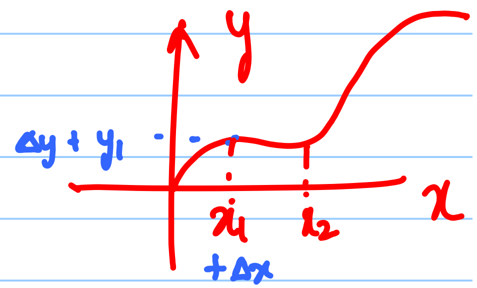
  $C_1 = \frac{1}{\omega_z R}$
- $\frac{C_1}{C_2} = K_c$
- $\omega_{p3} = \frac{(C_1+C_2)}{RC_1C_2}$

$$- \quad |LG(\omega_u)| = 1$$

$$I_{cp} = \frac{N C_2}{K_{VCO}} \omega_n^2 \sqrt{\frac{\omega_{p3}^2 + \omega_u^2}{\omega_z^2 + \omega_u^2}}$$



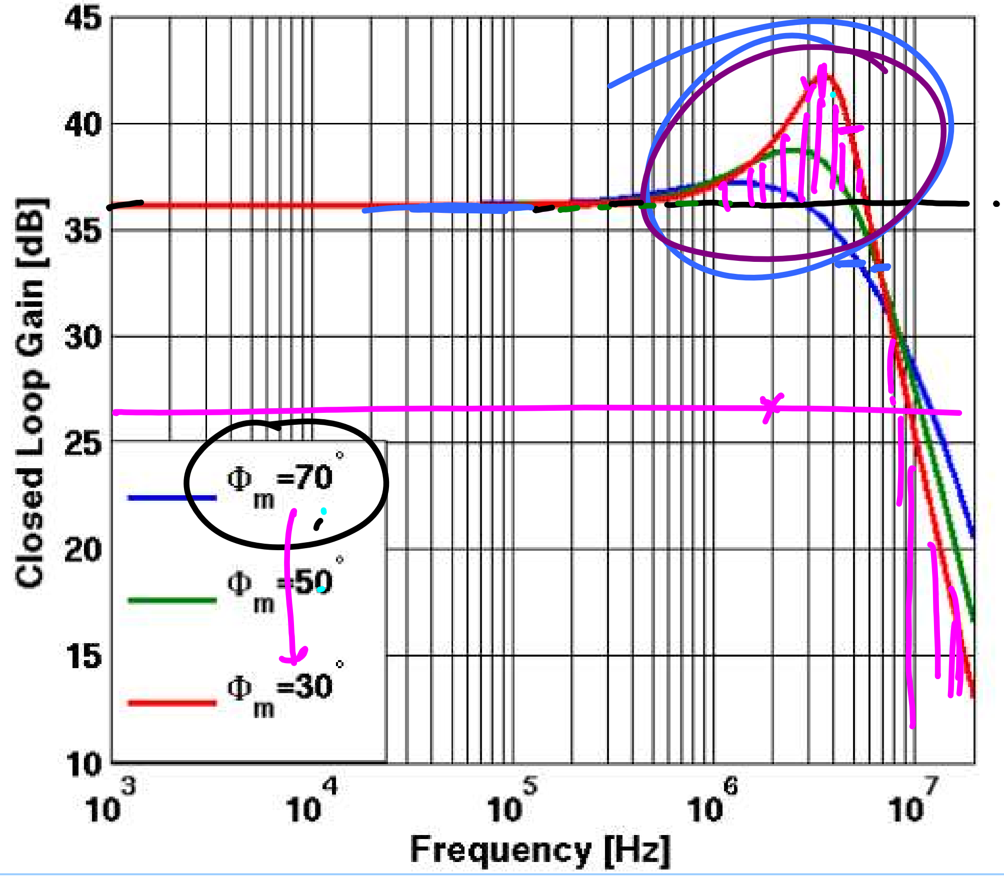
$y = f(x)$



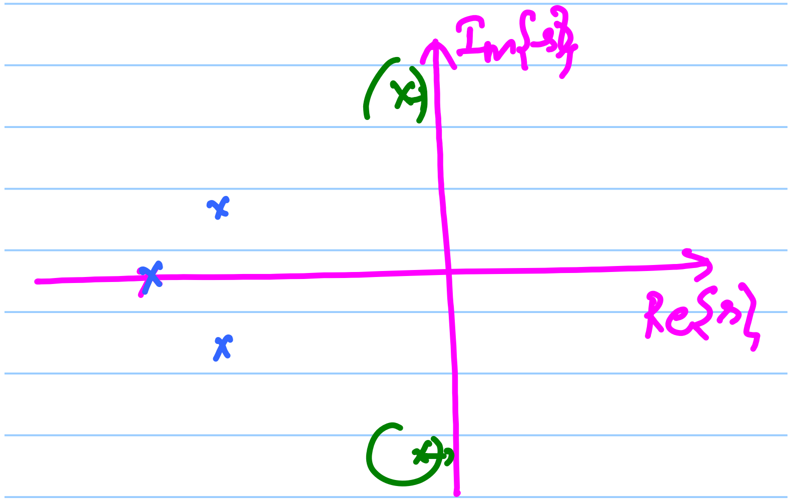
$$\Delta y = \left(\frac{dy}{dx}\right) \cdot \Delta x$$

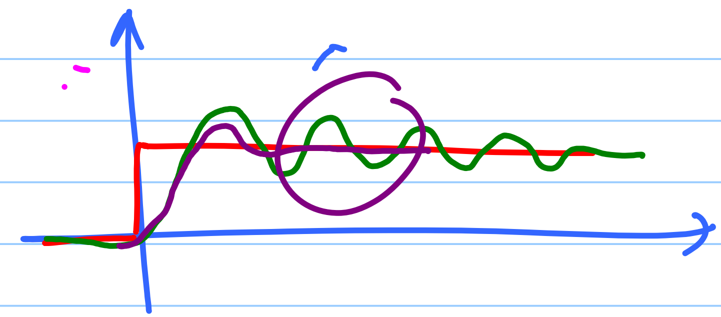
$\omega_n = \omega_{ref}/10$

$\phi_M = 70$



$$\frac{\phi_{out}}{\phi_{in}} = \frac{N \times LG}{1+LG} \checkmark$$





$$LG = \frac{I_{cp} \cdot K_{vco}}{s^2(C_1+C_2)} \frac{(1+s/\omega_z)}{(1+s/\omega_{p3})}$$

$1 + LG = 0$ for closed loop poles.

$$\frac{s^2(C_1+C_2)}{I_{cp} K_{vco}} \left(1+\frac{s}{\omega_{p3}}\right) + (1+s/\omega_z) = 0$$