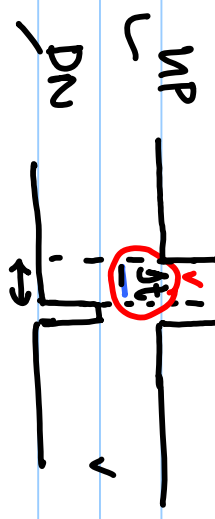
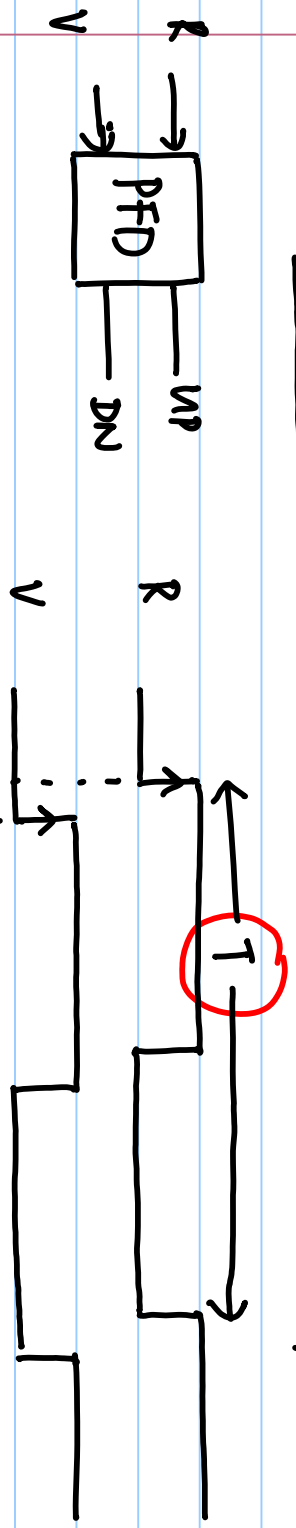
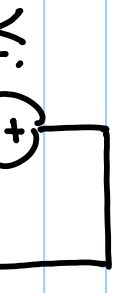
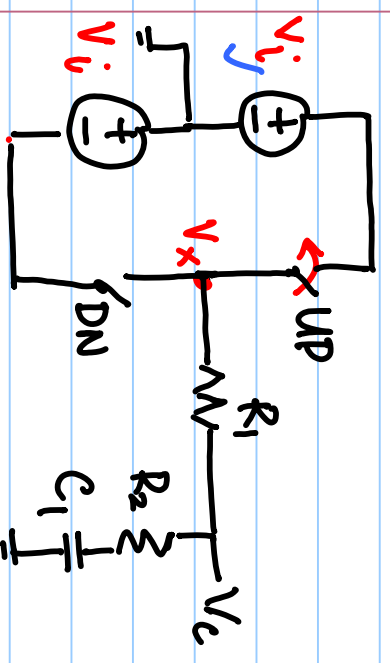


$$V_c = \left(\frac{\tau_p}{\tau_i} + \frac{1}{\tau_i} \right) \Phi_e$$



$$\Phi_e = 2\pi \cdot \frac{\Delta t}{T}$$

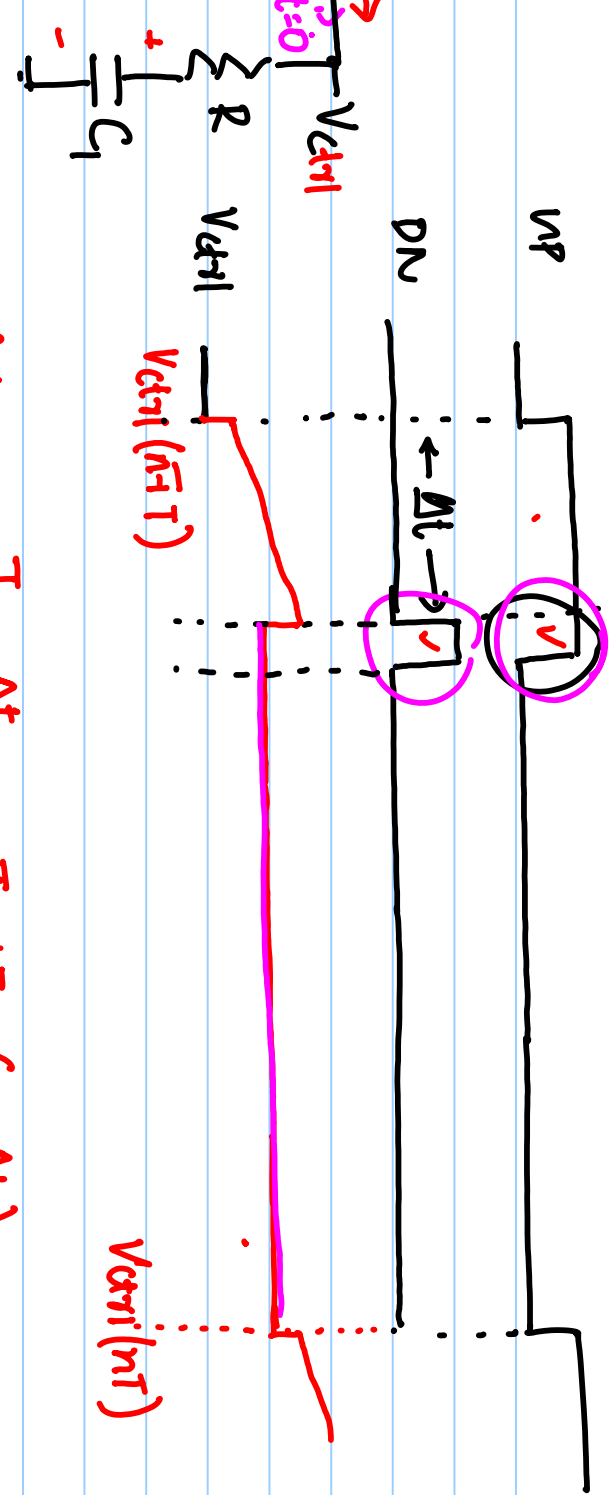
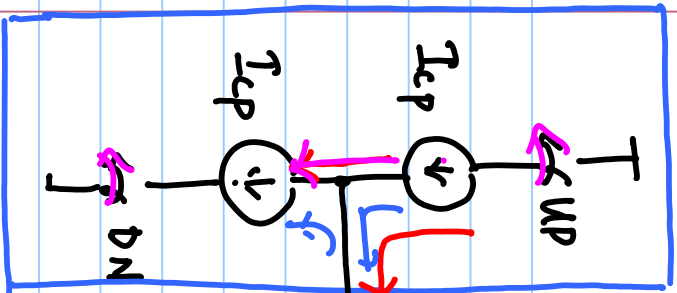


$$i_R = \frac{V_i(t) - V_c(t)}{R_1}$$

$$V_c(t) = i_R \cdot R_2 + \frac{1}{C_1} \int i_R \cdot dt$$

$$V_c(t) = f(\Delta t) = f(\Phi_e)$$

Charge-pump.



$$\Delta V_{ch1} = \frac{I_{cp} \cdot \Delta t}{C_1} = \frac{I_{cp} \cdot T}{C_1 \cdot 2\pi} \left(2\pi \frac{\Delta t}{T} \right)$$

$$= \frac{I_{cp} \cdot T}{2\pi C_1} \cdot \Phi_e$$

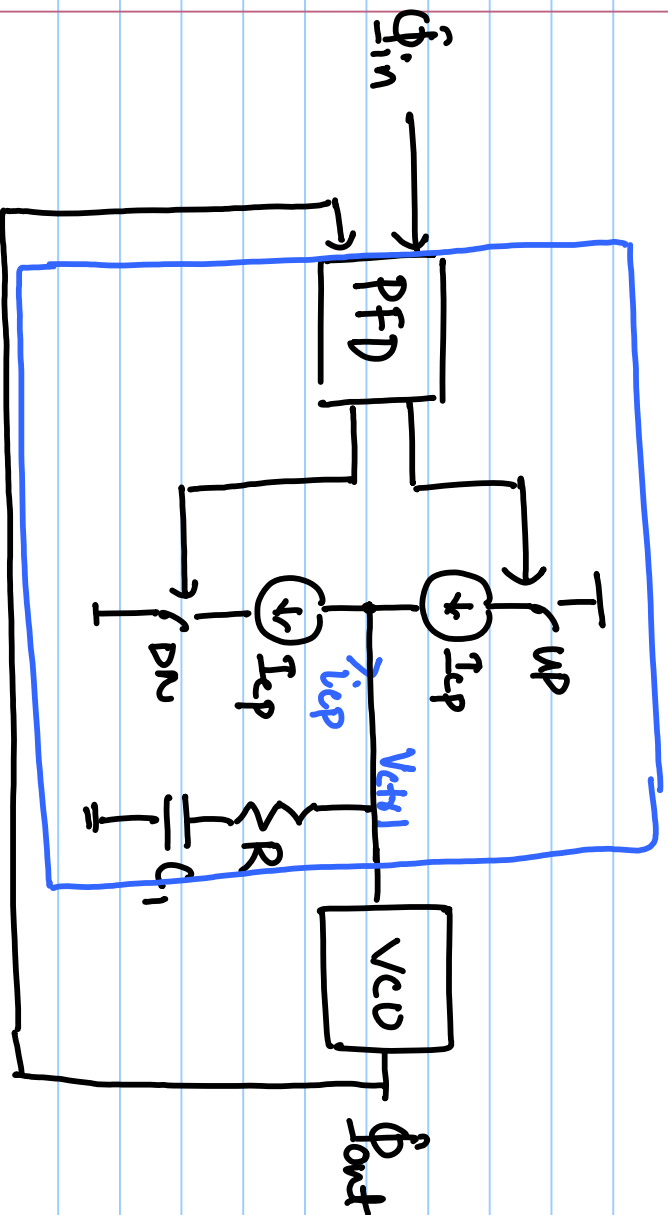
Avg. charge dumped on cap. $Q_{ch1} = \frac{C_1 \cdot \Delta V_{ch1}}{T}$

avg. $I_{avg} = \frac{I_{cp}}{2\pi} \cdot \Phi_e$

$V_{ch1}(s) = I_{avg}(s) \times \left(R + \frac{1}{sC_1} \right) \checkmark$

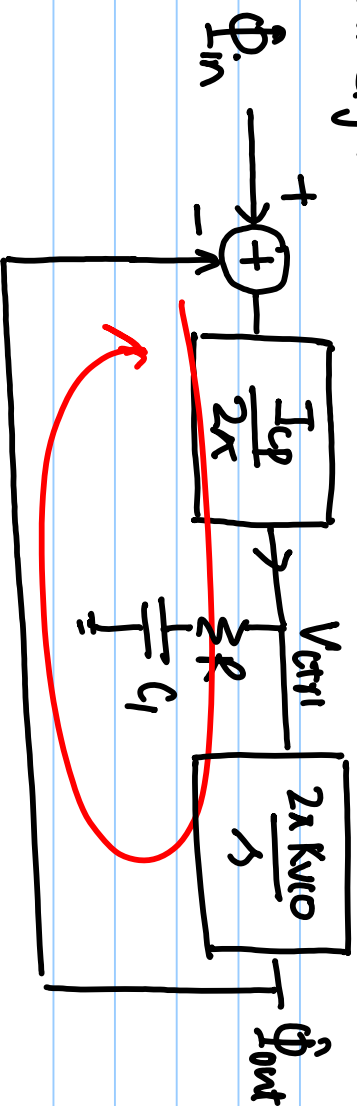
$q = CV$
 $I_{avg} = \frac{q}{T}$

$$V_{ctrl(s)} = \Phi_e \times \frac{I_{cp}}{2\pi} \left(R + \frac{1}{sC_1} \right) \checkmark$$



Charge-pump PLL.

Small-signal Model



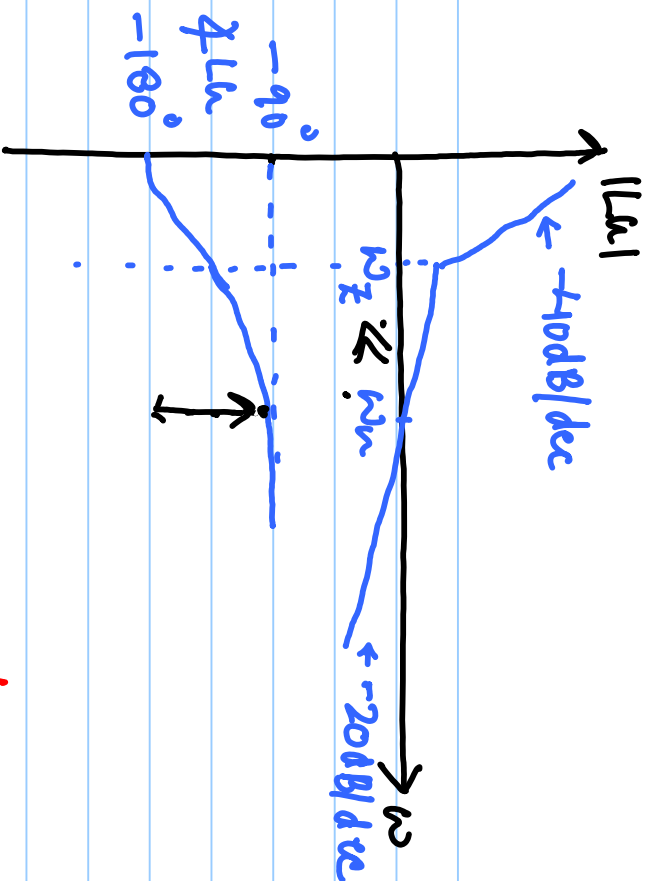
$$K_{vco} = \frac{f_{out}}{V_{ctrl}}$$

$$L_u = \frac{I_{cp}}{2\pi} \left(R + \frac{1}{sC_1} \right) \frac{2K_{vco}}{s}$$

$$L_u = \frac{I_{cp} \cdot K_{vco}}{s^2 C_1} \quad \left(\text{1st/2nd} \right) \quad \left(1 + sRC_1 \right) \gg 1$$

$$\omega_{p1} = \omega_{p2} = 0$$

$$\omega_z = \frac{-1}{RC_1}$$



Unity gain frequency. $\omega_u \mid_{|L_u|=1} \approx I_{cp} \cdot K_{vco} \cdot R$

$$\angle L_u = -180^\circ + \tan^{-1} \left(\frac{\omega}{\omega_z} \right)$$

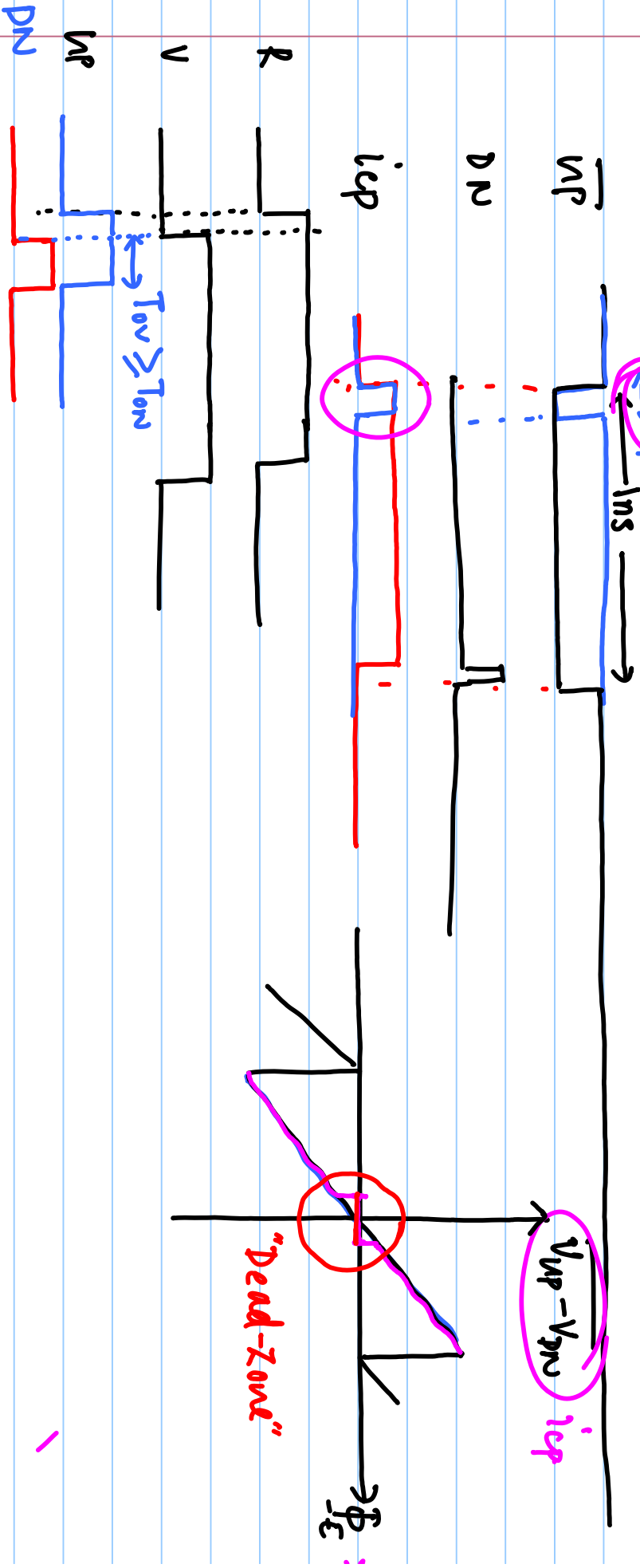
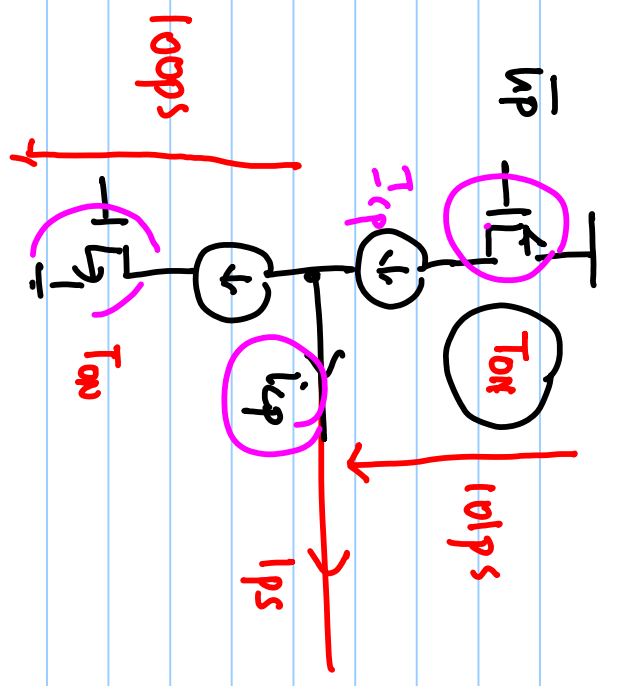
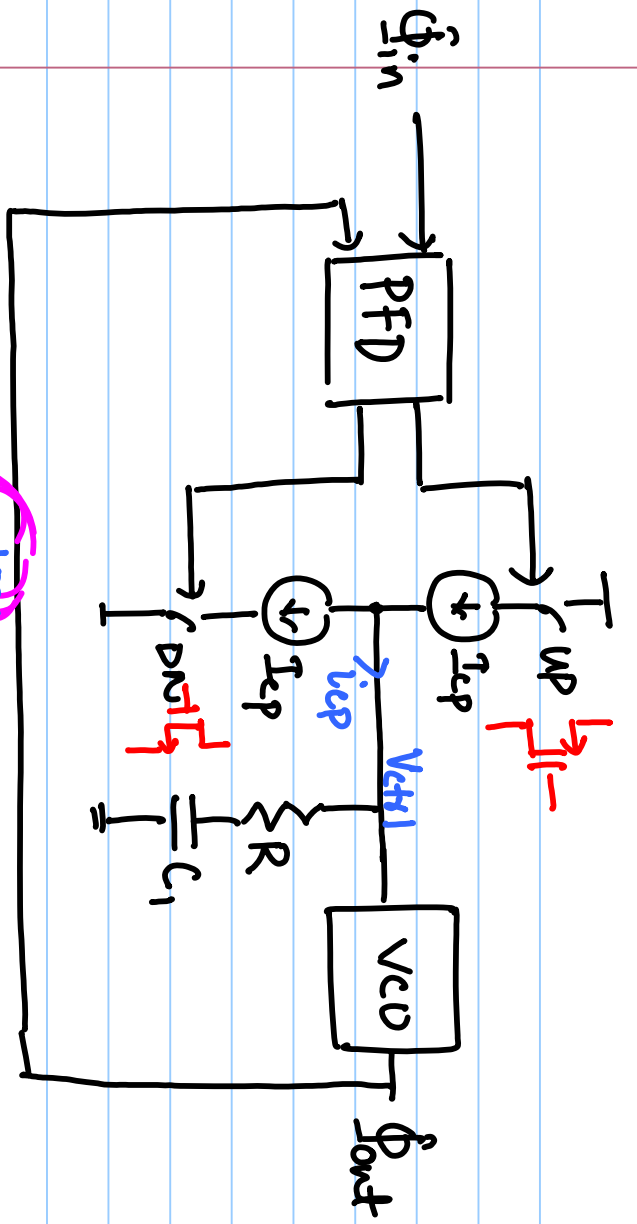
$$\text{Phase Margin} = \tan^{-1} \left(\frac{\omega_u}{\omega_z} \right)$$

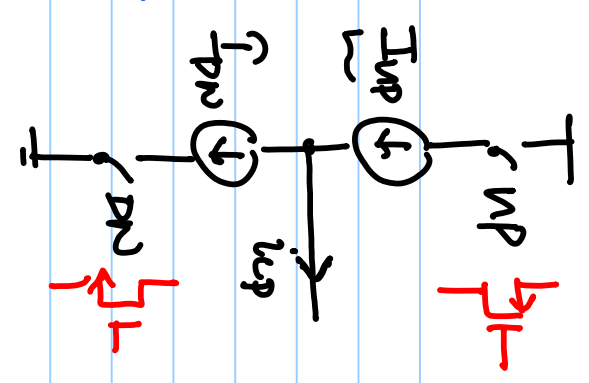
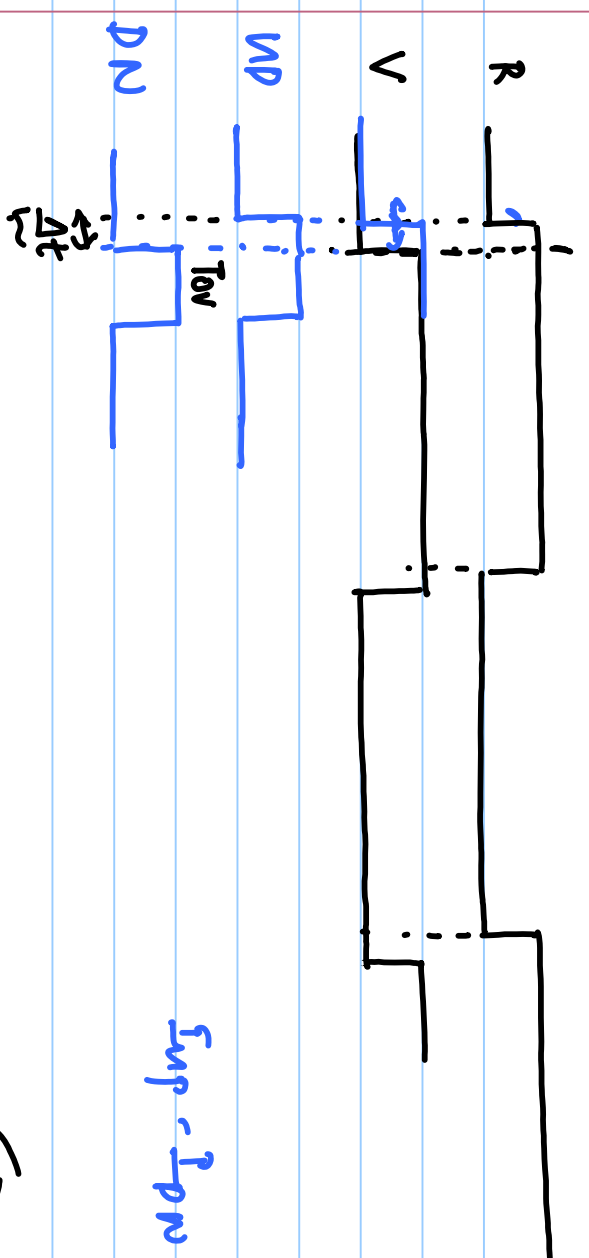
$$L_u = \frac{I_{cp} K_{vco}}{s^2 C_1} \left(1 + sRC_1 \right)$$

$$|L_u| = 1$$

$$\frac{I_{cp} \cdot K_{vco} \cdot \sqrt{1 + \omega_u^2 R^2 / \omega_z^2}}{\omega_u^2 C_1} = 1$$

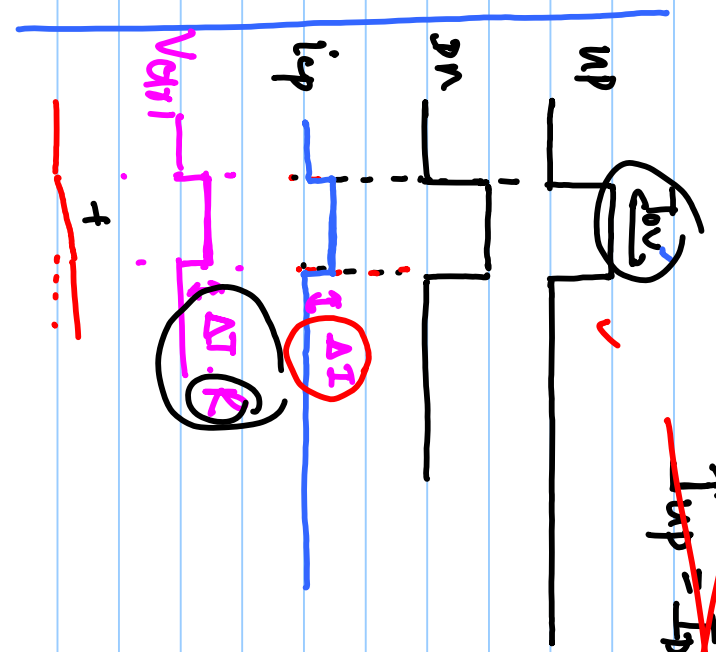
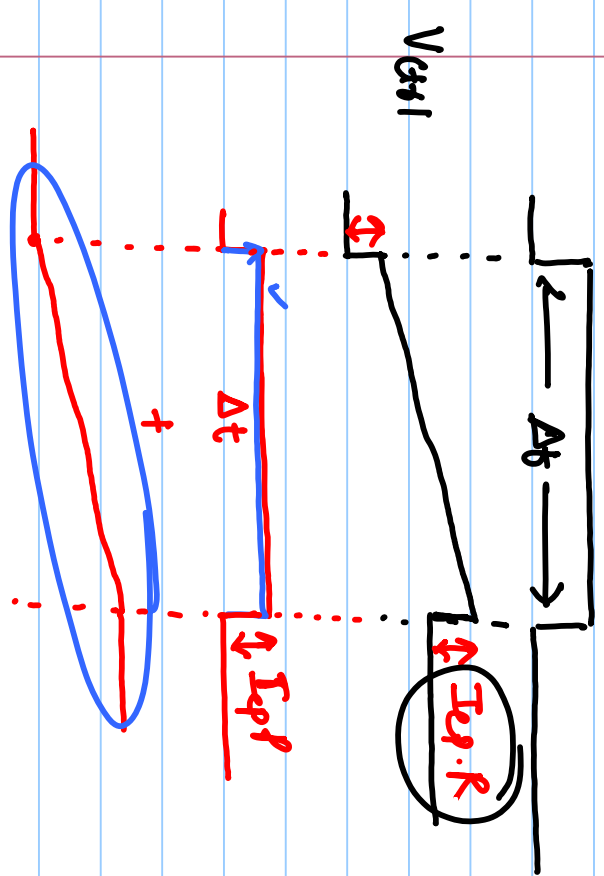
$$\frac{I_{cp} K_{vco} \cdot \omega_u / \omega_z}{\omega_u^2 C_1} = 1$$

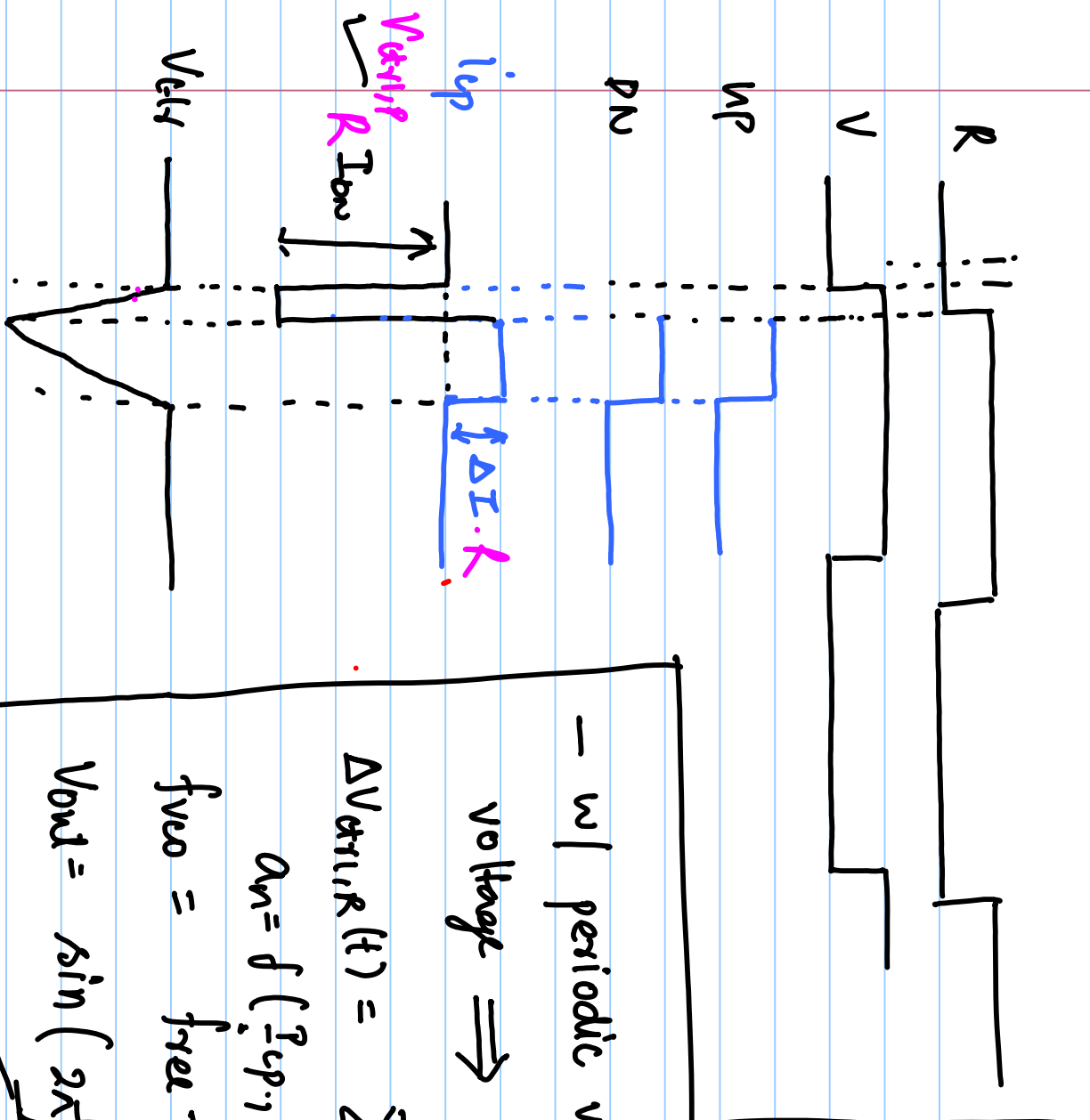




$$I_{MP} - I_{PN} = \Delta I$$

~~$$I_{MP} = I_{PN} = i_{qp}$$~~





- Control voltage will have a ripple periodic w/ reference period.

- w/ periodic voltage ripple at control voltage \Rightarrow periodic frequency ripple.

$$\Delta V_{str1,R}(t) = \sum_{n=1}^{\infty} a_n \cos(n\omega_{ref}t) + b_n \sin(n\omega_{ref}t)$$

$$a_n = f(I_{cp}, R)$$

$$f_{vco} = f_{ref} + K_{vco}(V_{c,ref} + \Delta V_{str1,R}(t))$$

$$V_{out} = \sin(2\pi f_{vco} \omega t)$$

$$\Rightarrow f_{vco, \pm} \approx f_r$$

Oscillator.

