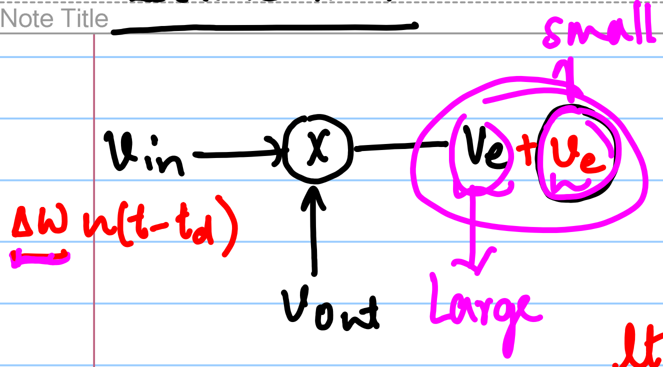
Lecture # 11

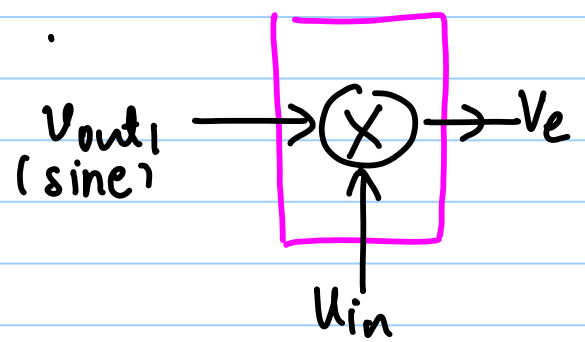


at $t=0$, $\Delta\omega(0) \longrightarrow \Phi_e \neq 0$, $V_e \neq 0$

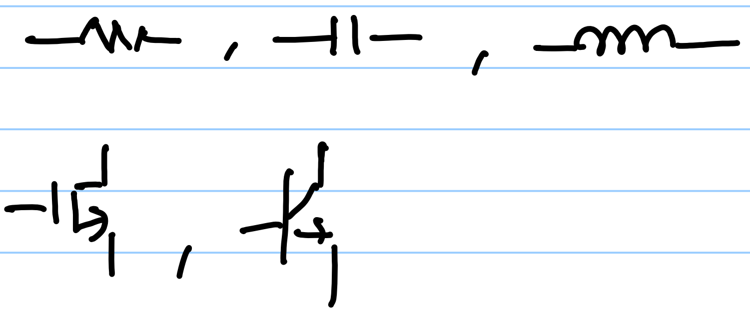
$$V_e = \frac{1}{2}\left[\sin(\Phi_e) + \sin(2\omega_{in}t + \Phi_{os})\right]$$

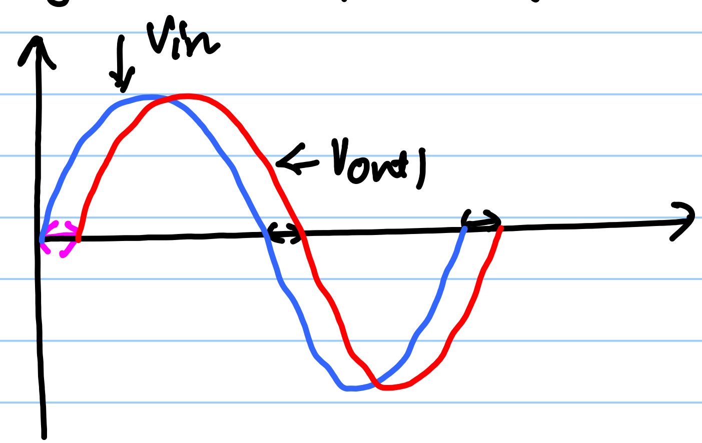
$$\lim_{t\to\infty} V_e = \lim_{t\to\infty} \frac{1}{2}\sin(\Phi_e) \checkmark$$

$$|V_e| \leq \frac{1}{2}$$



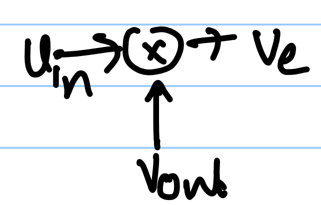
– In PLL, we will have two periodic signals, input (Ref). Output (out)





freq. v_{in}, V_{out} @ 100 MHz

@ 1 GHz.

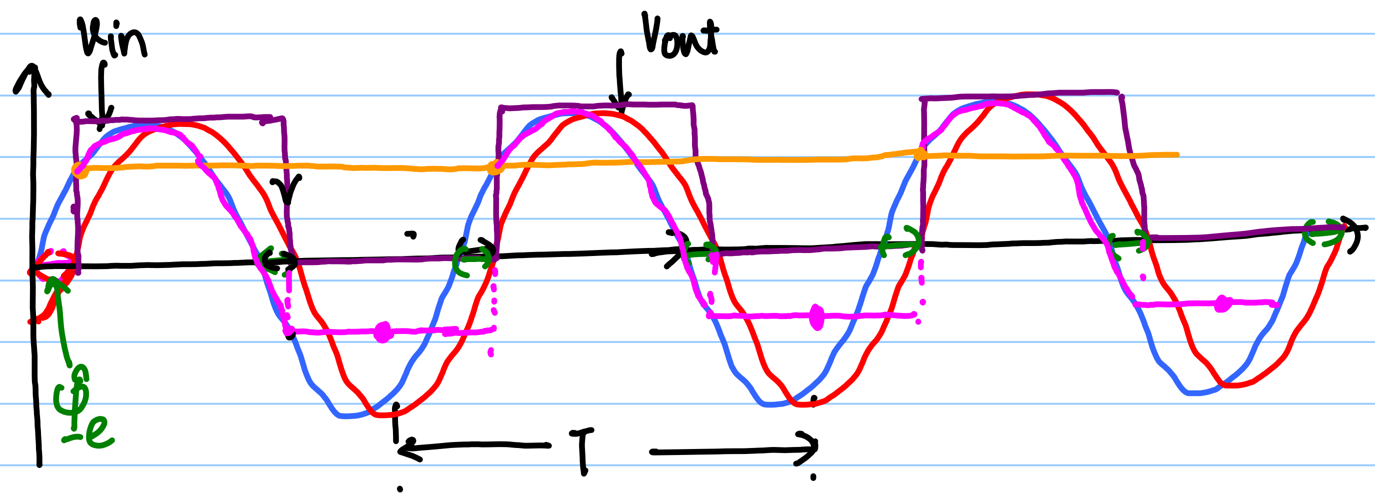


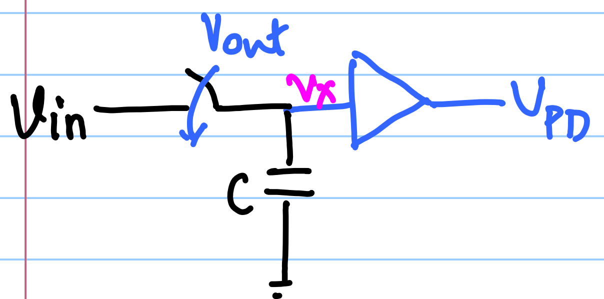
$V_e = \frac{1}{2} \sin(\dot{\phi}_e)$

$\phi_e = 2\pi \cdot \frac{\Delta t}{T}$

$V_e = \frac{1}{2} \sin\left(2\pi \cdot \frac{\Delta t}{T}\right)$

<u>Sample & Hold PD</u>





$V_{PD} = A_{in} \sin\left(\omega_{in} \cdot (nT)\right)$ ✓

$= A_{in} \sin\left(2\pi f_{in} \cdot \left(\frac{n}{f_{out}}\right)\right)$

when $V_{out} = 1$, switch is closed

$V_{out} = 0$ switch is open.

$$V_{PD} = A_{in} \sin\left(\omega_{in}\left(nT + \frac{T}{2} + \Delta t\right)\right)$$

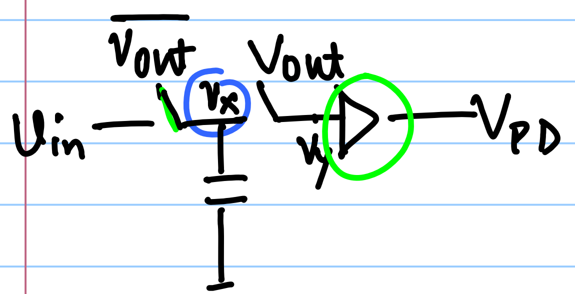
$$= A_{in} \sin\left(\underbrace{2\pi(f_{in}.T)n}_{=2n\pi} + \underbrace{\frac{2\pi f_{in}.T}{2}} + 2\pi.\underline{f_{in}.\Delta t}\right)$$

$$= A_{in} \sin\left(\pi + 2\pi.\frac{\Delta t}{T}\right)$$

$$= A_{in} \sin(\pi + \Phi_e)$$

$$= -A_{in} \sin(\Phi_e)$$

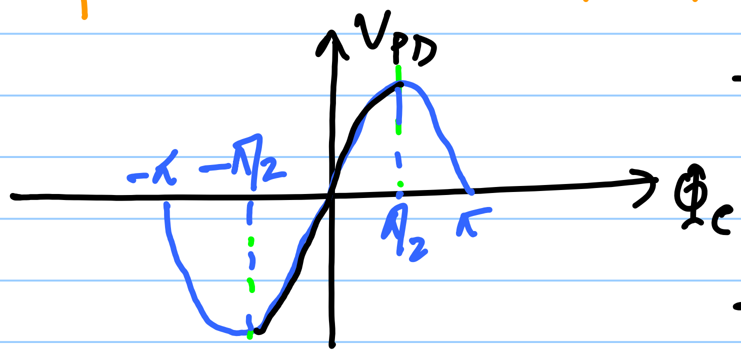
$f_{in}.T = 1$



$$V_{PD} = A_{in} \sin(\omega_{in}.(\Delta t))$$

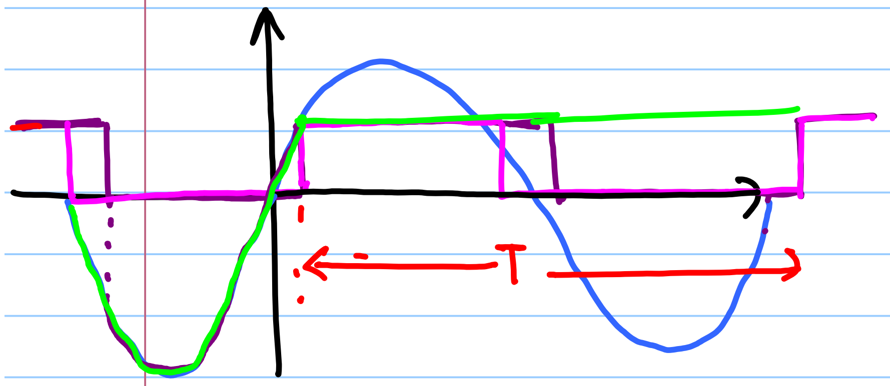
$$= A_{in} \sin\left(2\pi.\frac{\Delta t}{T}\right)$$

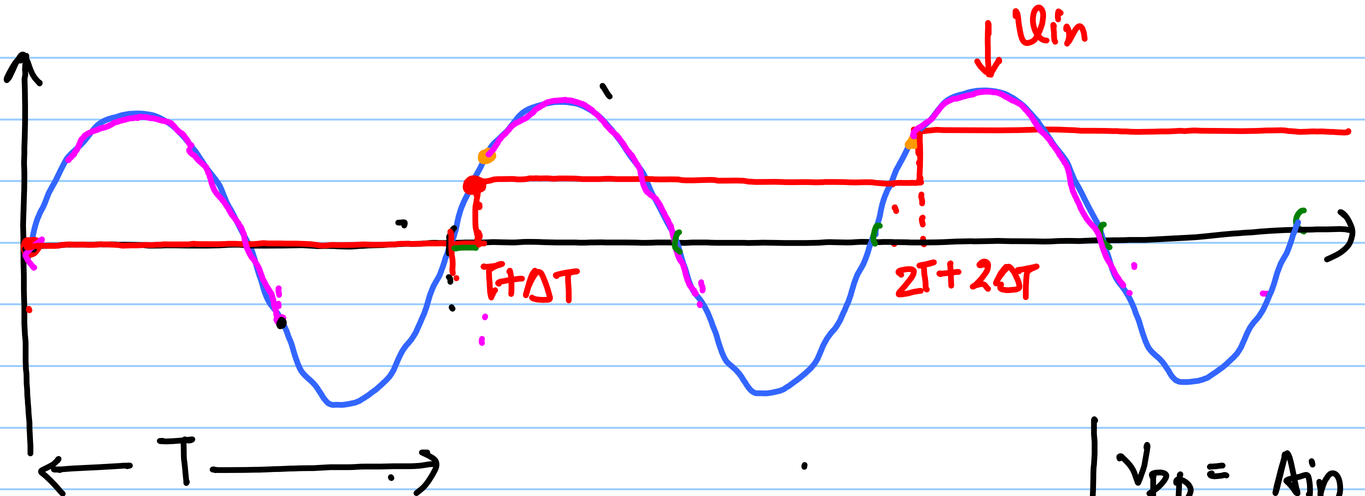
$$= A_{in} \sin(\Phi_e)$$



- Monotonous Range
  $\Phi_e \in \left[-\frac{\pi}{2} \text{ to } \pi/2\right]$

- $K_{PD} = \frac{dV_{PD}}{d\Phi_e} = A_{in}\cos(\Phi_e)$





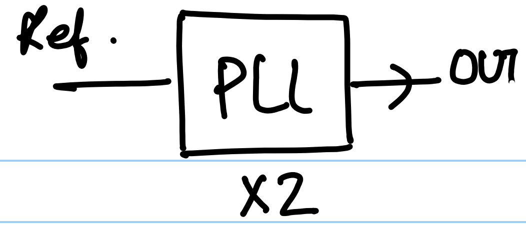
for Vout : time period be T+ΔT

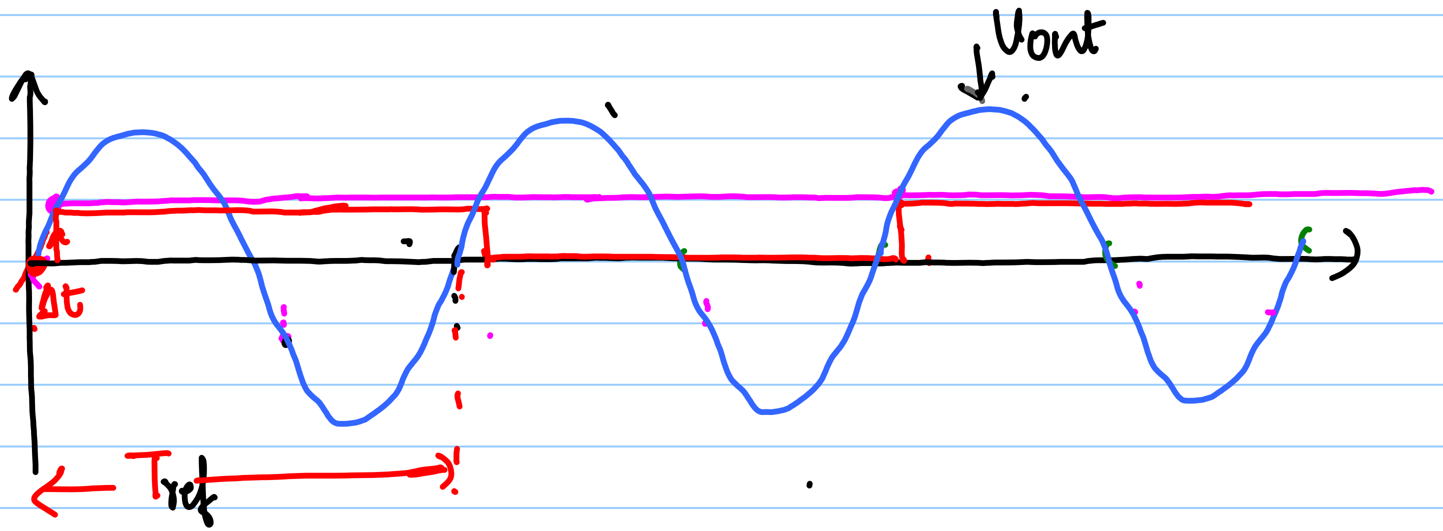
$$V_{PD} = A_{in} \sin\left(\omega_{in}\, n\, (T+\Delta T)\right)$$

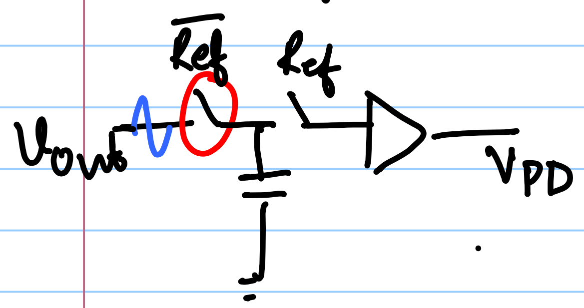
$$= A_{in} \sin\left(\frac{2n\pi}{T}(T+\Delta T)\right)$$

$$= A_{in} \sin\left(2\pi\left(\frac{n\Delta T}{T}\right)\right)$$

Sub-sampled PLLs.







$$V_{PD} = A_{out} \cdot \sin\left(2\pi f_{out} \cdot (nT_{ref} + \Delta t)\right)$$

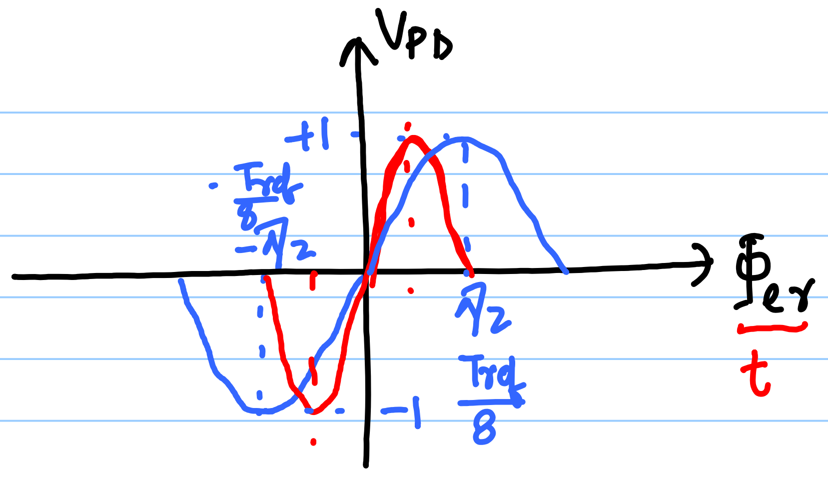
$$= A_{out} \sin\left(2\pi \left(n \cdot \frac{f_{out}}{f_{ref}}\right) + 2\pi \frac{\Delta t}{T_{out}}\right)$$

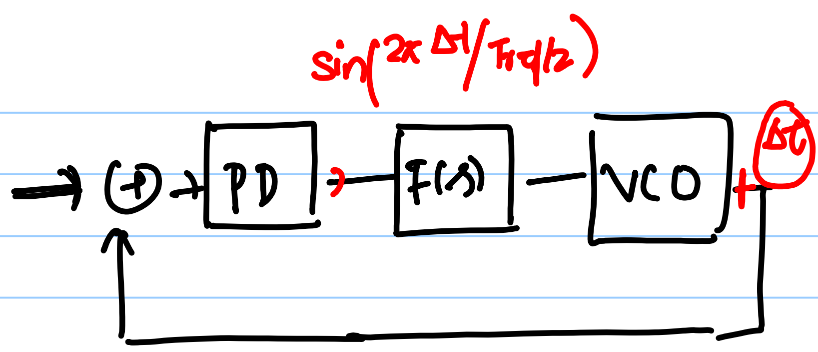
$$= A_{out} \sin\left(2\pi \cdot \frac{\Delta t}{T_{out}}\right)$$

$$\boxed{V_{PD} \doteq A_{out} \sin(\Phi_{er})}$$

($A_{out} = 1$)

Φ_{er} : w.r.t output freq.





if $f_{out} = 2 f_{ref} \Rightarrow \frac{\pi}{2} \rightarrow \frac{T_{out}}{4}$

$\Phi_{er} = \pi/2 = 2\pi \cdot \frac{\Delta t}{T_{out}}$

$\Delta t = \frac{T_{out}}{4} = \frac{T_{ref}/2}{4} = \frac{T_{ref}}{8}$

$f_{out} = 4 f_{ref} \Rightarrow \frac{\pi}{2} \rightarrow \frac{T_{out}}{4} = \frac{T_{ref}/4}{4} = \frac{T_{ref}}{16}$

Case # 1: $f_{out} = 2 f_{ref}$

$V_{PD} = \sin\left(2\pi \cdot \frac{\Delta t}{T_{ref}/2}\right)$

Case # 2: $f_{out} = 4 f_{ref}$

$V_{PD} = \sin\left(2\pi \cdot \frac{\Delta t}{T_{ref}/4}\right)$

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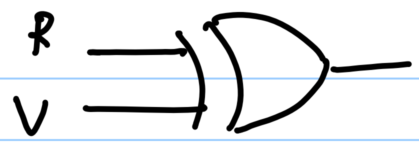
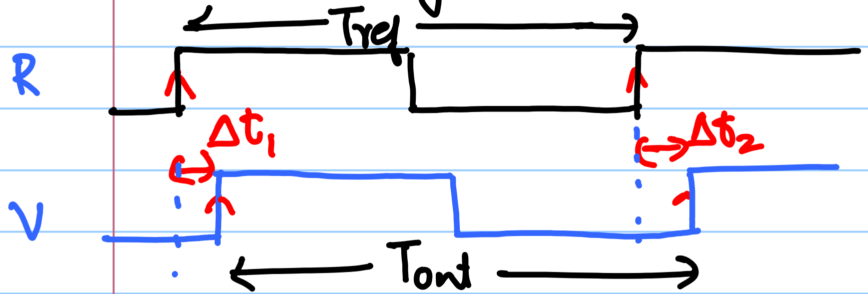
1. Mixer / Multiplier based PD
2. Sample & hold based PD

} Analog PD

Digital PDs.



1. EXOR gate PD



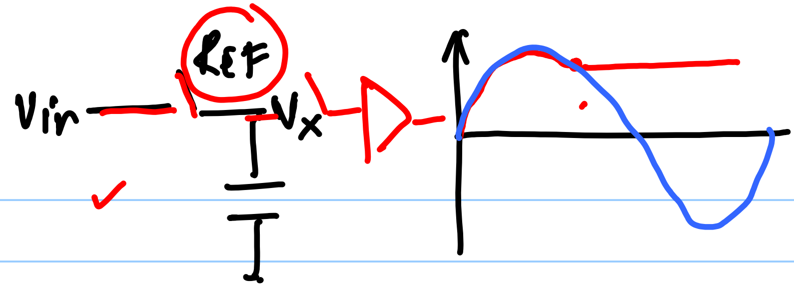
$R = \sin(\omega_{in} t + \Phi_{in}(0))$

$V = \sin(\omega_{out} t + \Phi_{out}(0))$

$\Phi_{er} = (\omega_{in} - \omega_{out})t + \Phi_{in}(0) - \Phi_{out}(0)$

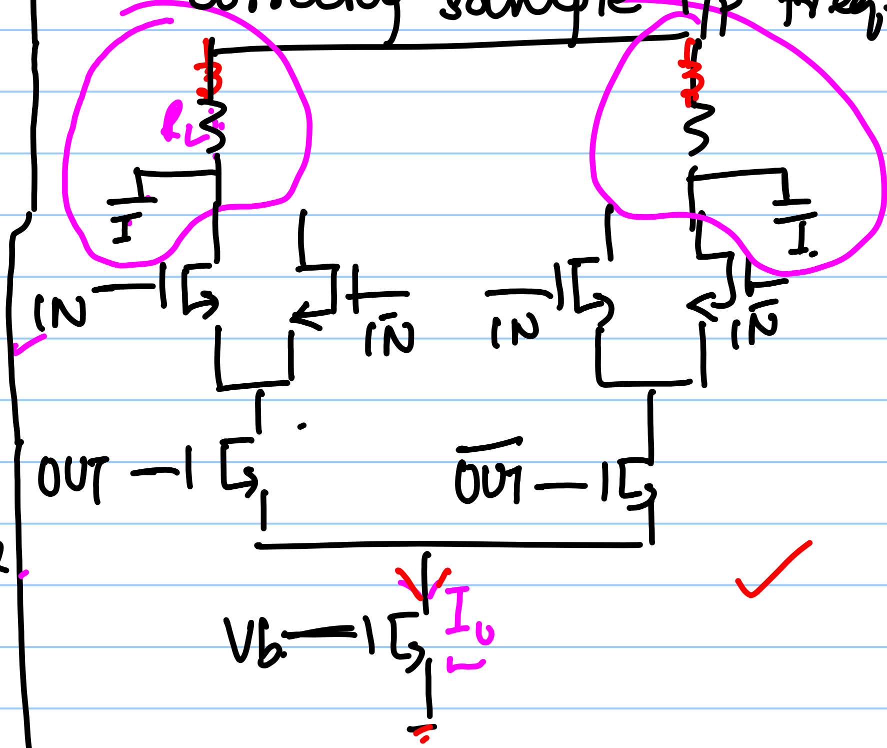
Phase error is defined w.r.t Reference freq.

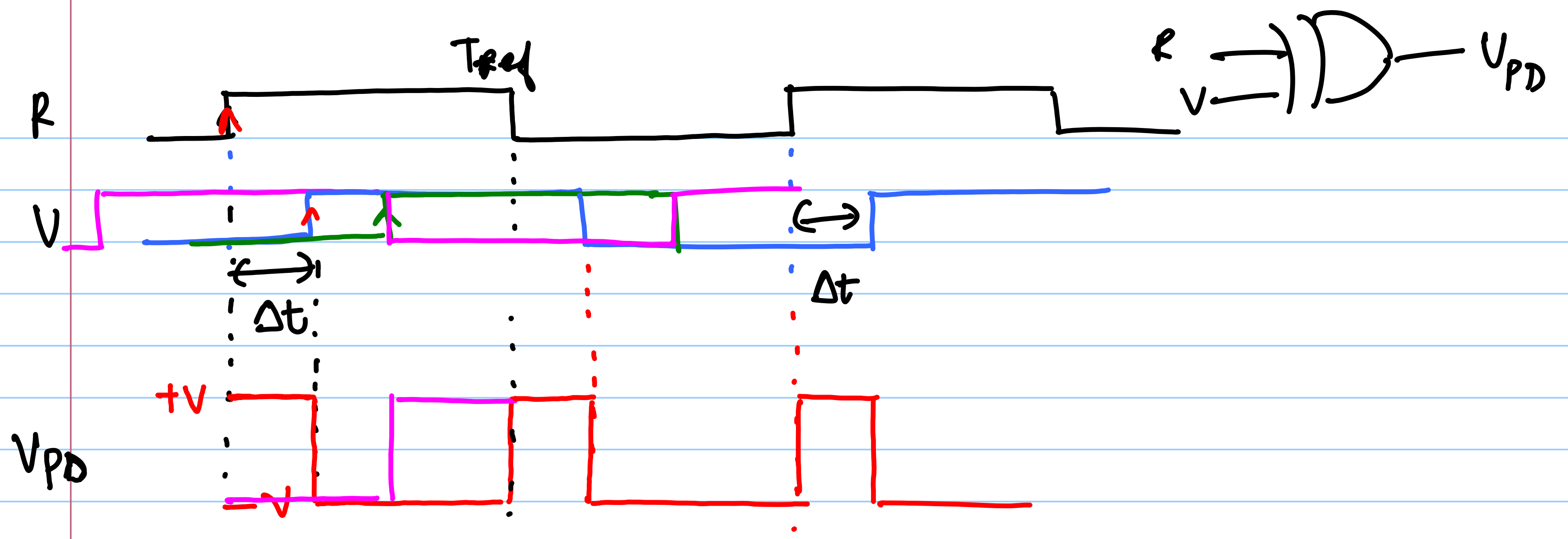
$$\Phi_{er} = 2\pi \cdot \frac{\Delta t}{T_{ref}}$$



In a given technology.

— Upper limit on fref to correctly sample i/p freq.





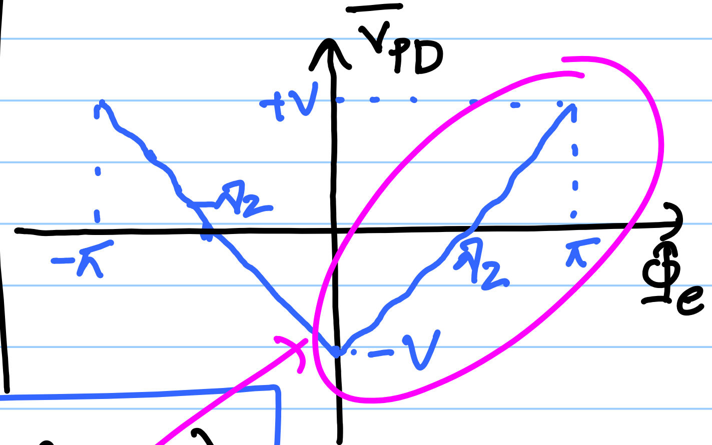
$$\overline{V_{PD}} = \frac{\Delta t \cdot (+V) + \left(\frac{T_{ref}}{2} - \Delta t\right)(-V)}{(T_{ref}/2)}$$

$$= \frac{2V \cdot \Delta t - V \cdot T_{ref}/2}{\frac{T_{ref}}{2}}$$

$$= 2V \cdot 2 \cdot \frac{\Delta t}{T_{ref}} - V$$

$$= 2V \cdot \frac{\Phi_e}{\pi} - V \Rightarrow \boxed{\overline{V_{PD}} = V\left(\frac{2\Phi_e}{\pi} - 1\right)}$$

$$\Phi_e = 2\pi \cdot \frac{\Delta t}{T_{ref}}$$



$$K_{PD} = \frac{2V}{\pi}$$