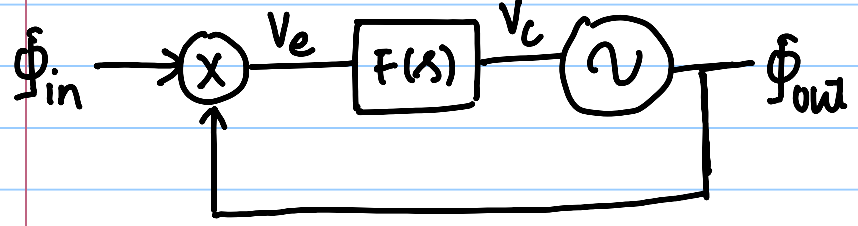
Lecture # 8

Note Title 02-06-2020

Frequency Acquisition in PLLs



Input freq. ω_{in}

Output freq. $\omega_{out} = \omega_{free}$ at $t=0$.

$\underline{\Delta\omega} = \omega_{in} - \omega_{free}$

$$F(s) = \frac{1}{1+s/\omega_p} = \frac{1}{1+sRC}$$

$$\Delta\omega_L \leq K_{VCO} \cdot K_{PD}$$

$$\dot{\Phi}_e = 0 = \Delta\omega - K_{VCO} \cdot K_{PD} \sin(\Phi_e)$$

$$\Delta\omega_p \leq K_{VCO} \cdot K_{PD}$$

1. Lock-in range: $\Delta\omega_L$

   w/o cycle slipping

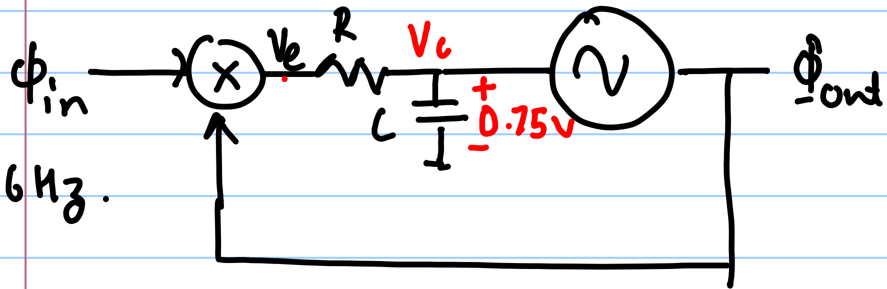
   w/o Φ_e exceeding 2π.

2. Pull-in range: $\Delta\omega_p$

   w/ or w/o cycle slipping.

3. Hold-in range: $\Delta\omega_H$

✓ $\Delta\omega_H$: max. $(\omega_{in} - \omega_{free})$



1GHz.

$W_{out} = \omega_{free} + K_{vco} \cdot V_c$

$1.0GHz = 0.925\ GHz + K_{vco} \cdot V_c$

$V_c = \dfrac{75\ MHz}{100\ MHz/V} = \underline{0.75V}$ ✓

$K_{vco} = 100\ MHz/V$

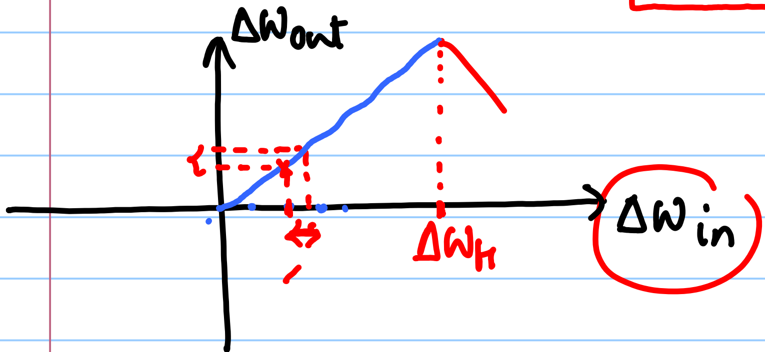
$K_{PD} = 1/2$

$\underline{V_c = V_e} = \underline{\frac{1}{2}\sin(\phi_e)} \le \frac{1}{2}$

$\Delta\omega_L = \Delta\omega_p = \underline{\pm 50}\ MHz. = \Delta\omega_H$

$V_e = 0.75$

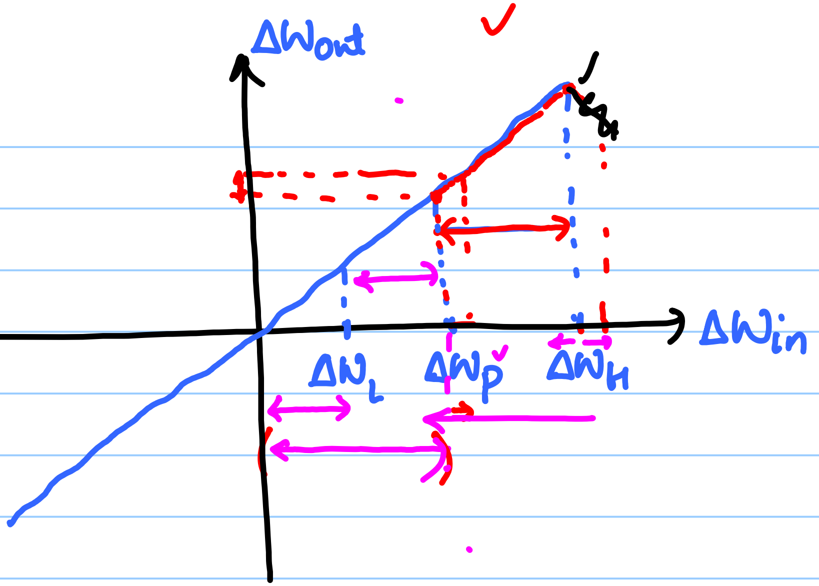
$\omega_{free}: 925\ MHz. \Rightarrow \underline{\Delta\omega = 75\ MHz}.$



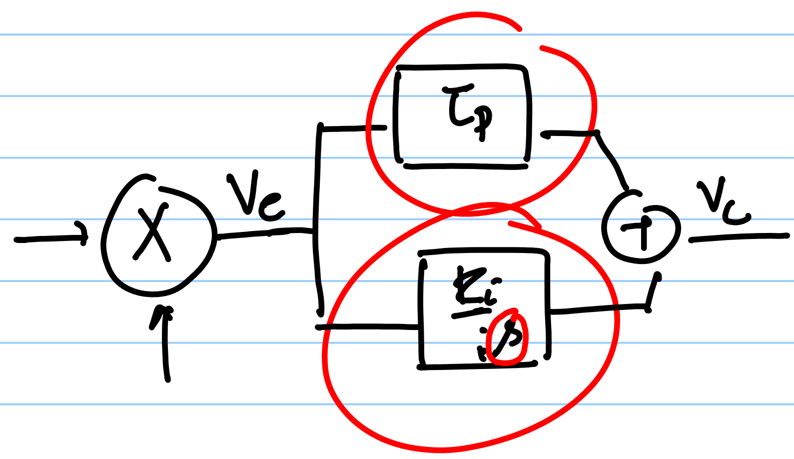
$\Delta\omega_{in} = \omega_{in} - \omega_{in}(0)$

$\Delta\omega_{out} = \omega_{out} - \omega_{out}(0)$

$\omega_{in}(0) = \omega_{out}(0)$

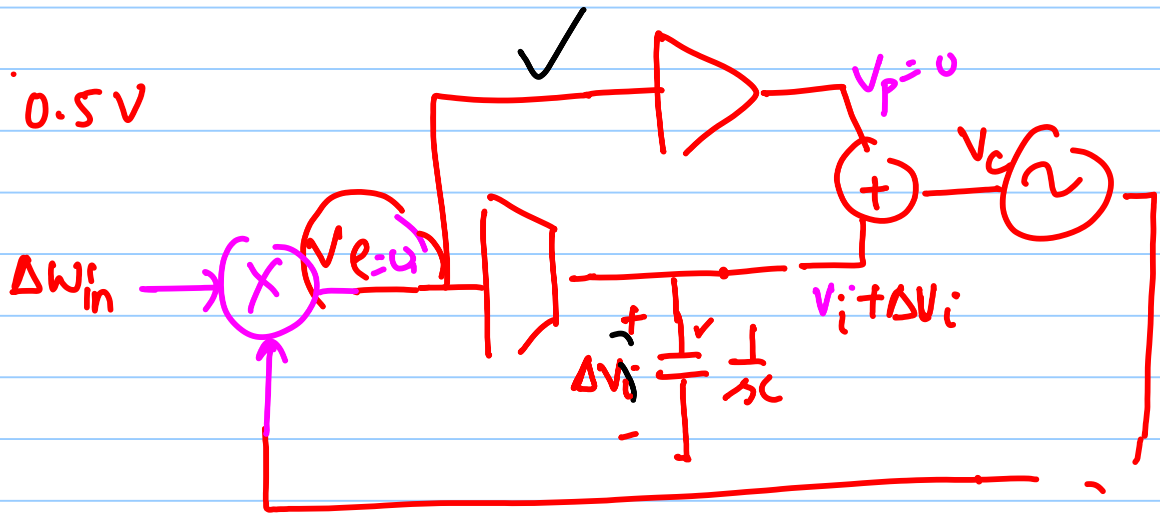


$\Delta\omega_H \geq \Delta\omega_P \geq \Delta\omega_L$



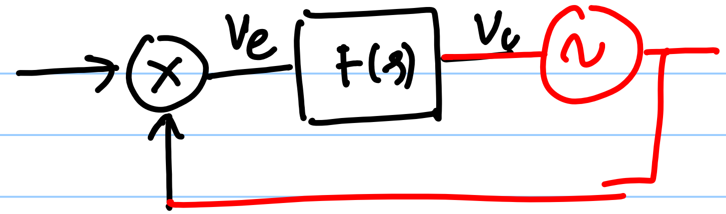
Snapshot #1. ΔW_P , $V_i = 0.5V$

Snapshot #2. $V_i = 0.6V$



Type-II PLL

$$F(s) = \frac{\tau_p}{\tau_i} + \frac{1}{\tau_i s}$$



$$V_c(t) = \frac{\tau_p}{\tau_i} V_e(t) + \frac{1}{\tau_i}\int V_e\,dt$$

$$\omega_{out}(t) = \omega_{free} + K_{vco}\cdot V_c$$

$$= \omega_{free} + K_{vco}\frac{\tau_p}{\tau_i}V_e(t) + \frac{K_{vco}}{\tau_i}\int V_e\,dt$$

$$V_e = \frac{1}{2}\sin(\Phi_e)$$

$$\Phi_e(t) = \omega_{in}\cdot t - \int \omega_{out}\,dt$$

$$= (\omega_{in} - \omega_{free})t - K_{vco}\frac{\tau_p}{\tau_i}\int_0^t \frac{1}{2}\sin(\Phi_e)\cdot dt - \int_0^t \frac{K_{vco}}{\tau_i}\int_0^y \frac{1}{2}\sin(\Phi_e)\,dy\,dt$$

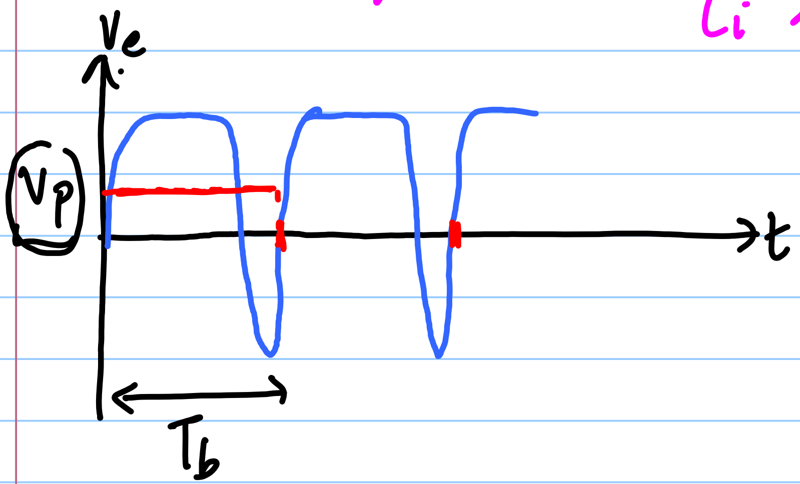
$$= \Delta\omega(0)\cdot t - K_{vco}\cdot\frac{\tau_p}{\tau_i}\frac{1}{2}\int_0^t \sin(\Phi_e)\,dt - \frac{K_{vco}}{\tau_i}\frac{1}{2}\int_0^t\int_0^y \frac{1}{2}\sin(\Phi_e)\,dy\,dt$$

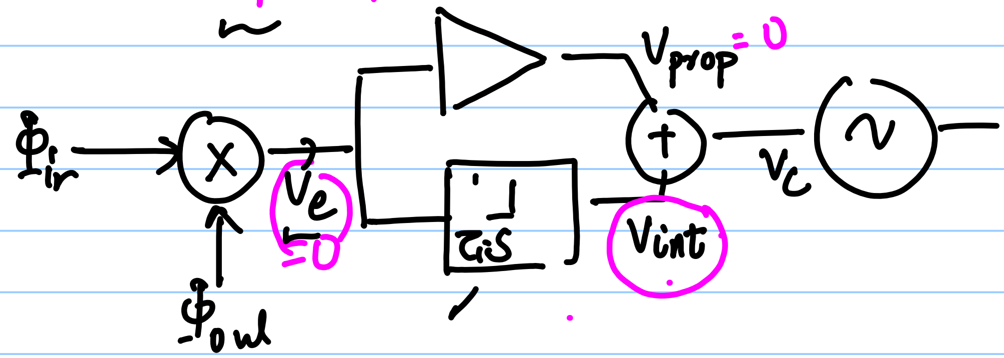
$$\frac{d\Phi_e(t)}{dt} = 0 \Rightarrow \quad \dot{\Phi}_e = \underbrace{\Delta\omega(0) - \left(K_{vco}\cdot\frac{\tau_p}{\tau_i}\right)\frac{1}{2}\sin(\Phi_e)}$$

$$- \frac{K_{vco}}{\tau_i}\frac{1}{2}\int_0^t \sin(\Phi_e)\,dy.$$

$$\dot{\Phi}_e = 0$$

$$\Delta\omega_L \le K_{vco}\cdot\frac{\tau_p}{\tau_i}\frac{1}{2} = K_{vco}\cdot K_p\cdot K_{PD}$$





$$\omega_{out} = \omega_{free} + K_{vco}\cdot V_{int}$$

Assume V_{int} remains constant for 1-beat perio

$$\Delta\Omega = \omega_{in} - \omega_{out}$$

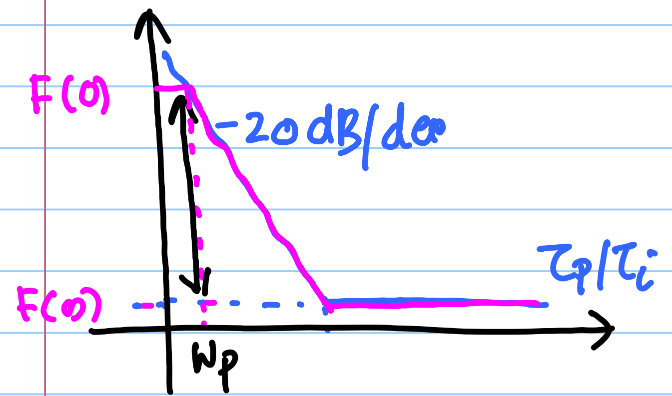
$$= \Delta\omega(0) - K_{vco}\cdot V_{int}$$

$$\Delta\Omega = \Delta\omega(0) - K_{VCO} \cdot \frac{1}{\tau_i} \int_0^t V_e \, d\tau \checkmark$$

$$\checkmark V_p = K_{PD} \left[\frac{\Delta\Omega}{K} - \sqrt{\left(\frac{\Delta\Omega}{K}\right)^2 - 1} \right]$$

$$K = K_{VCO} \cdot K_{PD} \, F(\infty)$$

$$\checkmark \; \Delta\Omega = \Delta\omega - K_{VCO} \cdot V_{int} = 0$$



$$F(s) = \frac{\tau_p}{\tau_i} + \frac{1}{s\tau_i}$$

Type-I PLL for $\Delta\omega$ freq. error

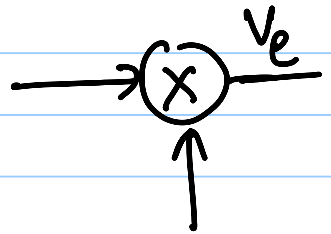
Non zero steady state Φ_e

$(\Phi_e \neq 0)$

Type-II PLL $\Phi_e \neq 0$ Not possible

⇓

Change Vint



V_c

$$V_c(s) = F(\infty)\, V_e(s) + \frac{F(0)}{1 + s/\omega_p}\, V_e(s)$$

Integral path gain : $F(0) - F(\infty)$

$$\Delta\Omega = \Delta\omega - K_{vco}\,(F(0) - F(\infty))\, V_p$$

$$\boxed{\Delta\Omega = \Delta\omega - K_{vco}\,(F(0) - F(\infty))\, K_{PD} \left[\frac{\Delta\Omega}{K} - \sqrt{\left(\frac{\Delta\Omega}{K}\right)^2 - 1} \right]}$$

if PLL locks, above eq. should have complex solution.

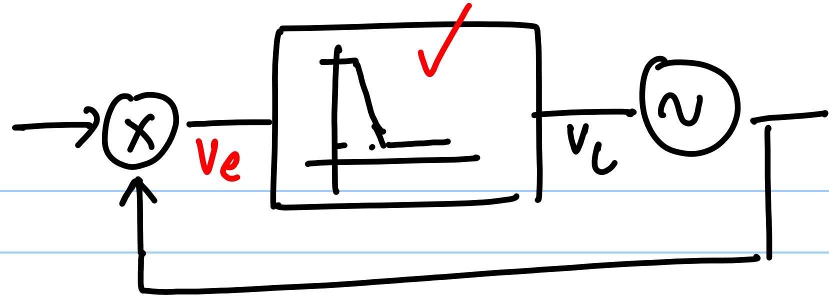
Roots are complex. if $\Delta\omega < K \sqrt{\frac{2K_{dc}}{K} - 1}$

$$K_{dc} = K_{vco}\, K_{PD}\, F(0)$$

$$K = K_{vco} \cdot K_{PD}\, F(\infty)$$

$$\Delta\omega_p < K_{vco} \cdot K_{PD}\, F(\infty) \sqrt{\frac{2F(0)}{F(\infty)} - 1}$$

Hold in range.



$$V_c = \frac{\Delta\omega_H}{K_{vco}}$$

$$V_e = \frac{V_c}{F(0)} = \frac{1}{2}\sin(\Phi_e)$$

$$\sin(\Phi_e) : \frac{2V_c}{F(0)} < 1$$

$$\Rightarrow \quad \frac{2\Delta\omega_H}{K_{vco}\cdot F(0)} < 1$$

$$\Delta\omega_H < K_{vco}\cdot F(0)\cdot\frac{1}{2}$$

$$\Delta\omega_H < K_{vco}\cdot F(0)\, K_{PD}$$

$$\Delta\omega_H \leq K_{vco}\cdot K_{PD}\, F(0)$$

$$\Delta\omega_P < K_{vco} K_{PD} F(\infty)\sqrt{\frac{F(0)}{F(\infty)} - 1}$$

$$\Delta\omega_P < K_{vco} K_{PD}\sqrt{F(0)\, F(\infty)}$$

$$< f(0)$$