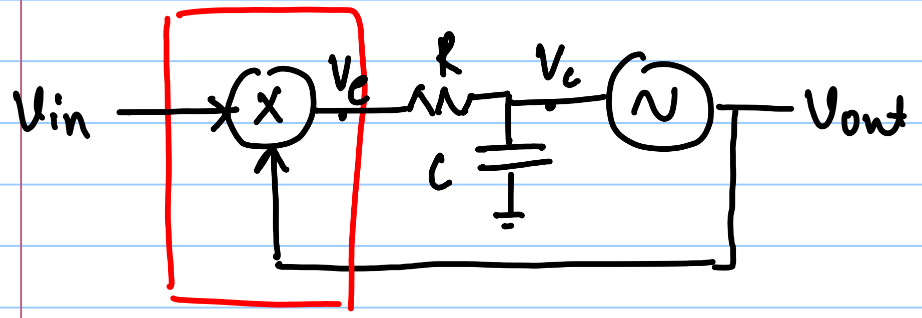
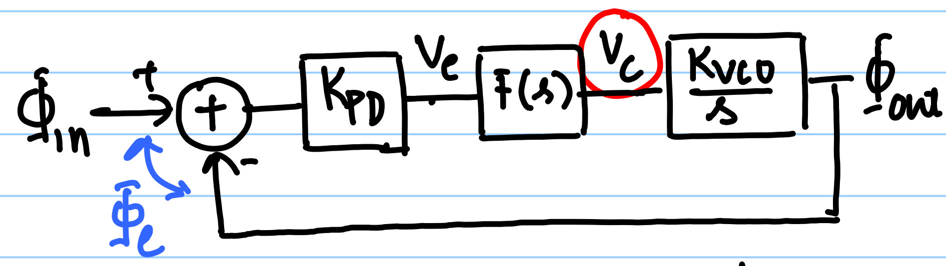
Note Title 26-05-2020

Lecture # 6

Basic PLL





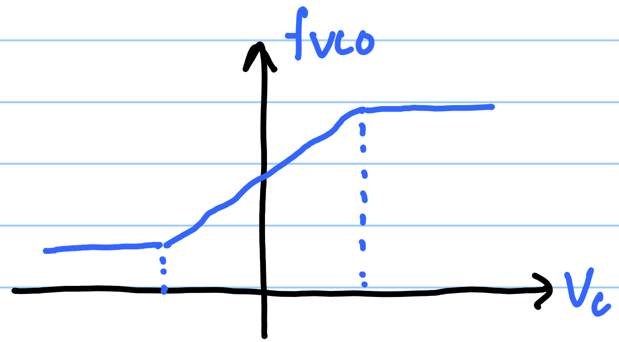
✓Small-signal error.

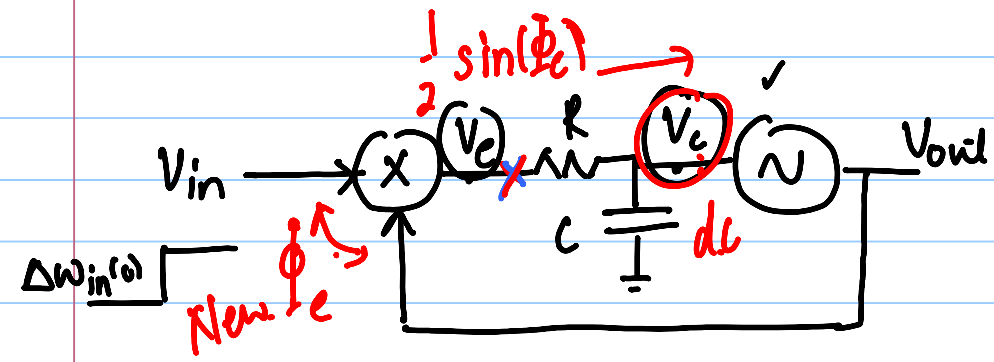
$\lim_{t\to\infty} \Phi_e(t) =$

$\Delta\Phi_{in}(0)\, u(t)$, $\Delta\omega(0)\, u(t)$, $\Delta\omega(0)\cdot t\, u(t)$

$\left| V_e = \frac{1}{2}\sin(\Phi_e) \right| \le \frac{1}{2}$

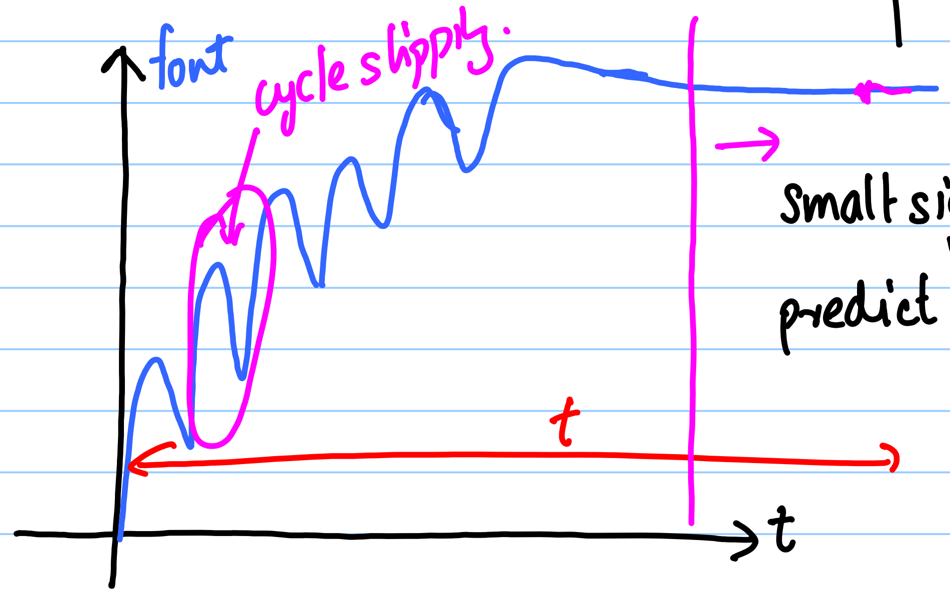
$V_c \lesssim V_e$





$F(s) = \frac{1}{1+sRC}$ ✓

$F(s) = \frac{(1+s\tau_p)}{s\tau_i}$ ✓



→ small signal model of PLL can't predict the large signal settling of PLL.

Ex. $\omega_{in} = 1$ GHz.

$\omega_{out} = \omega_{free} + K_{vco} \cdot V_c$

$K_{vco} = 2\pi \times 100$ Mrad/s.

at $t=0$, $V_c = 0$, $\omega_{out} = \omega_{free} = 0.99\cancel{9}5$ GHz.

at $t=0$, $\Delta\omega = \omega_{in} - \omega_{out} = 5\ MHz$.

$v_{in} = \sin(\omega_{in} \cdot t)$

$v_{out} = \sin(\omega_{out} \cdot t + \hat{\Phi}_{out}(0))$

$\hat{\Phi}_{in}(t) = \omega_{in} \cdot t$

$\hat{\Phi}_{out}(t) = \int \omega_{out}\, dt = \int \omega_{free} + K_{VCO} \cdot V_c \cdot dt$

$= \omega_{free} \cdot t + K_{VCO} \cdot \int V_c \cdot dt$

$\hat{\Phi}_{er}(t) = \hat{\Phi}_{in}(t) - \hat{\Phi}_{out}(t)$

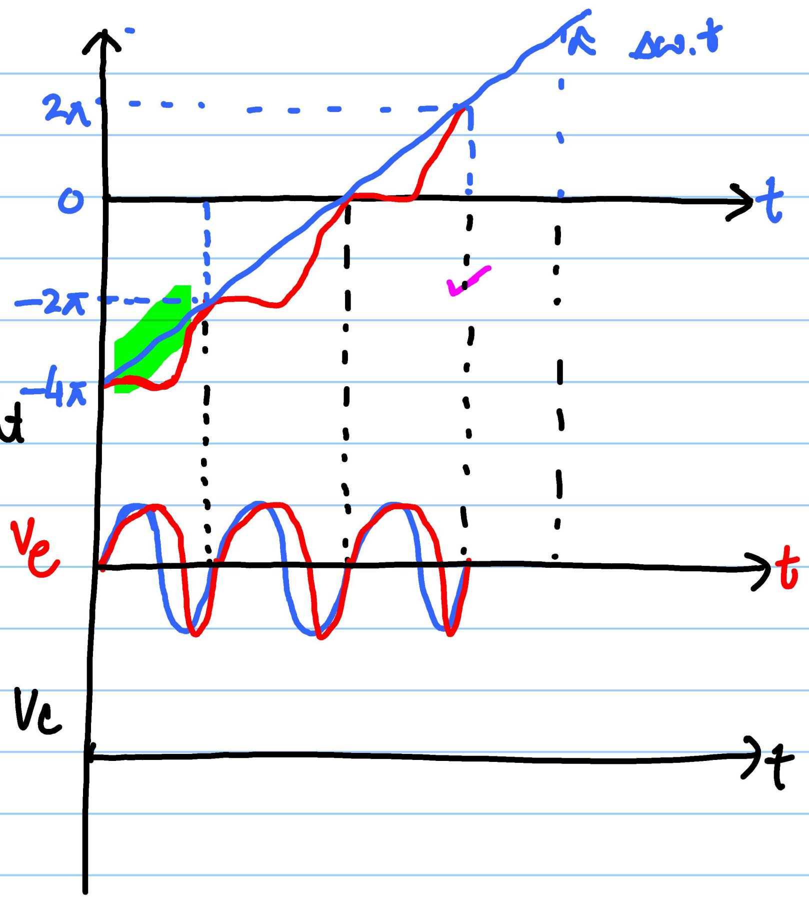
$= \underbrace{\Delta\omega \cdot t}_{\hat{\Phi}_{er}|_{olloop}} - K_{VCO} \int V_c \cdot dt$

$V_e(t) = \underbrace{\frac{1}{2} \sin(\hat{\Phi}_e)} + \cancel{\frac{1}{2} \sin(\overline{\omega_{in} + \omega_{out}} + K_{VCO} \int V_c\, dt)}$

$\hat{\Phi}_{er}(t) = \Delta\omega \cdot t - K_{VCO} \int V_e \cdot dt$

$= \Delta\omega \cdot t - K_{VCO} \cdot K_{PD} \int \sin(\hat{\Phi}_e)\, dt$

Assume V_e is passed as such to V_c



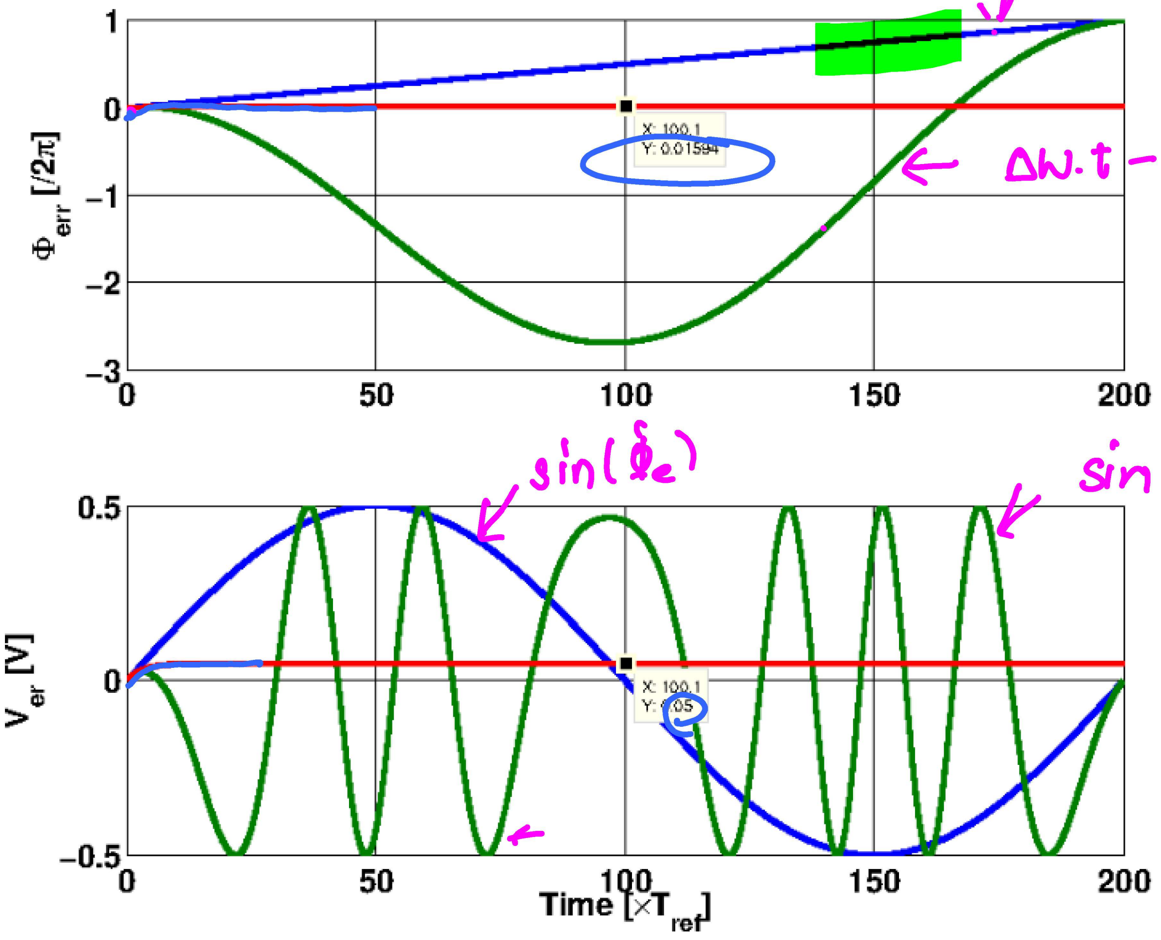
$$\doteq \Delta\omega \cdot t - K_{vco} K_{PD} \int_0^t \sin(\Delta\omega \cdot t)\, dt$$

$$= \Delta\omega \cdot t - \frac{K_{vco} \cdot K_{PD}}{\Delta\omega} \left(1 - \cos(\Delta\omega \cdot t)\right)$$

$0.01594 \times 2\pi$

$\Delta\omega \cdot t$

$\Delta\omega \cdot t - K_{vco} K_{PD} \int \sin(\Phi_e)\, dt = \Phi_{e,new}$

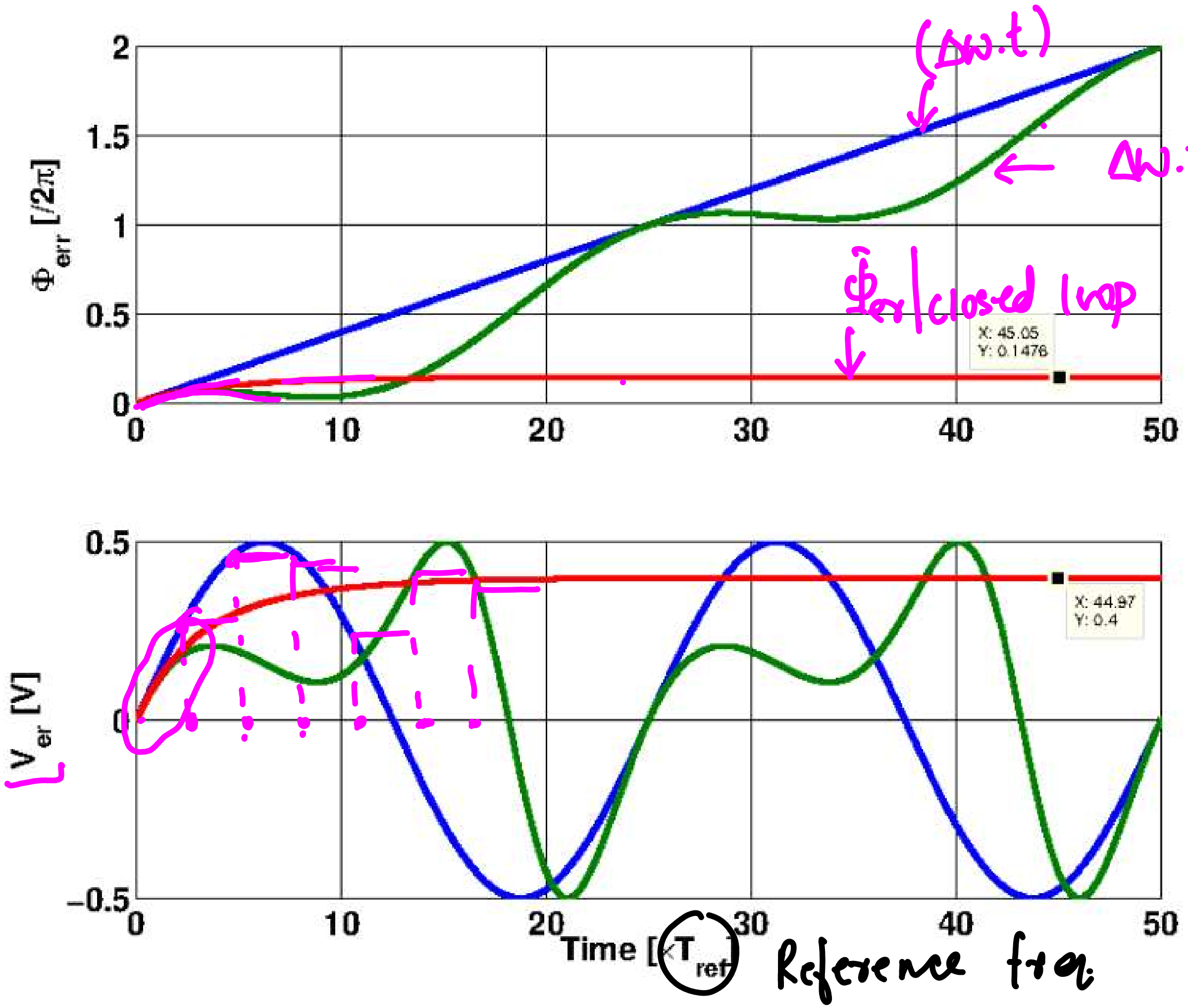


$\sin(\Phi_e)$

$\sin(\Phi_{e,new})$

100 MHz/V × 0.05

= 5 MHz.



Reference freq.

$$\Delta\omega . t - K_{vco} . K_{PD} \int \sin(\Delta\omega . t)\, dt$$

$$\int_0^{T_s} V_c \, dt = V_c(0) . T_s$$

$W_{in} = 1.643$

$W_{free} = 0.9956$

at 0 $V_c = 0$, $V_e = 0$

at $t = T_s$

$$\Phi_c(T_s) = \Delta\omega . T_s - K_{vco} \sum_{k=0} V_c(kT_s) \times T_s$$

$V_e(T_s)$:

$V_c(T_s)$ =

at $t = 2T_s$

$$\Phi_e(2T_s) = \Delta\omega \cdot 2T_s - K_{vco} \sum_{k=0}^{1} V_c(kT_s) \cdot T_s$$

$K_{vco} = 100$ MHz/V

for $\Delta\omega = 5$ MHz $\longrightarrow$ $V_e = 0.05$ V ✓

$\Delta\omega = 40$ MHz $\longrightarrow$ $V_e = 0.4$ V.

$\Delta\omega$

$K_{vco} = 2\pi \times 100$ Mrad/V

$\Rightarrow \dfrac{df_{out}}{dV_c} = 100$ MHz/V

$\dfrac{d\omega_{out}}{dV_c} = 2\pi \times 100$ Mrad/V.

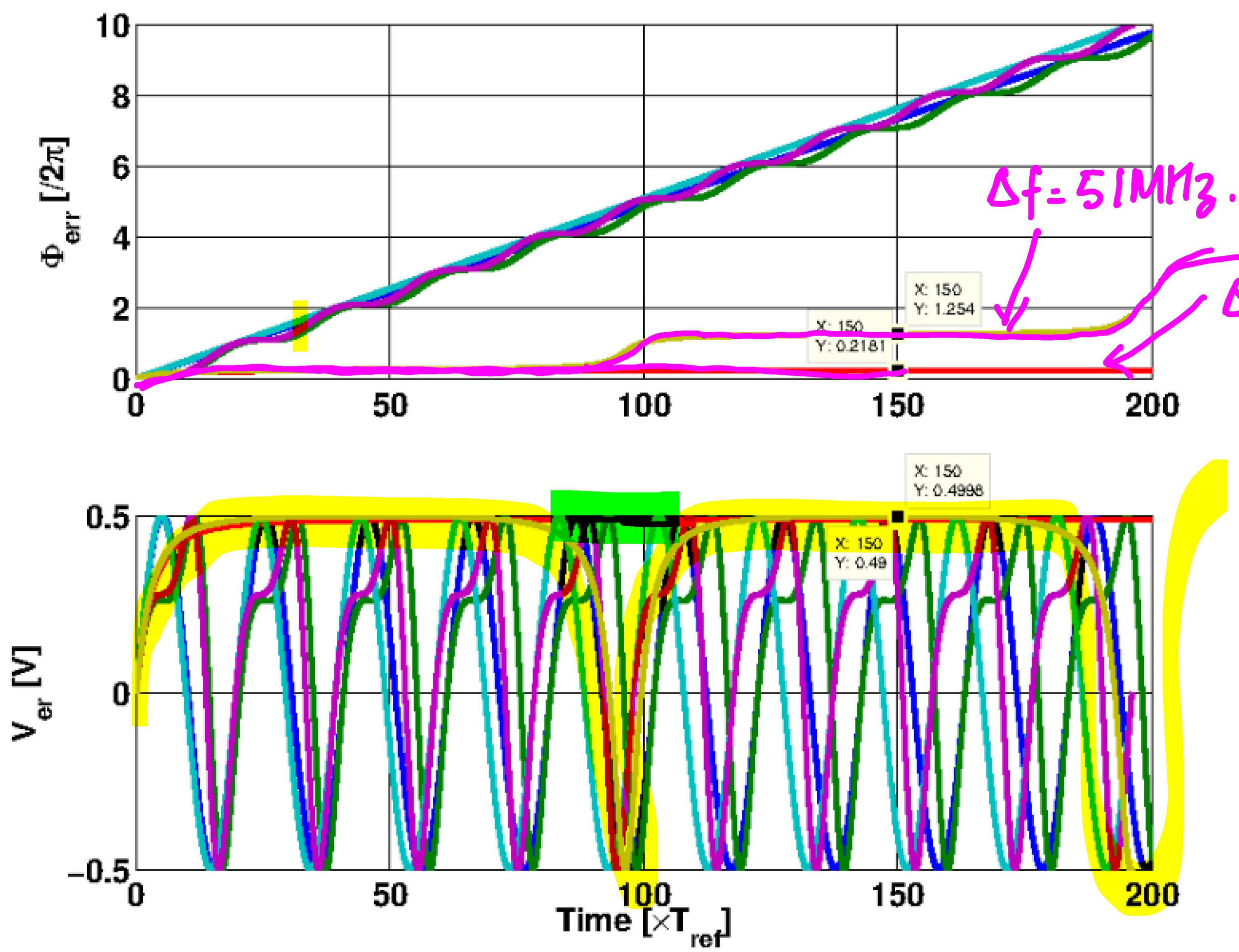
$$|V_e| < \frac{1}{2}$$

$V_c = V_e$

$\Delta\omega = \frac{1}{2} K_{vco} = \underline{50 \text{ MHz}}$.

Above PLL locks to frequency error before Φ_{er} exceeds 2π ✓

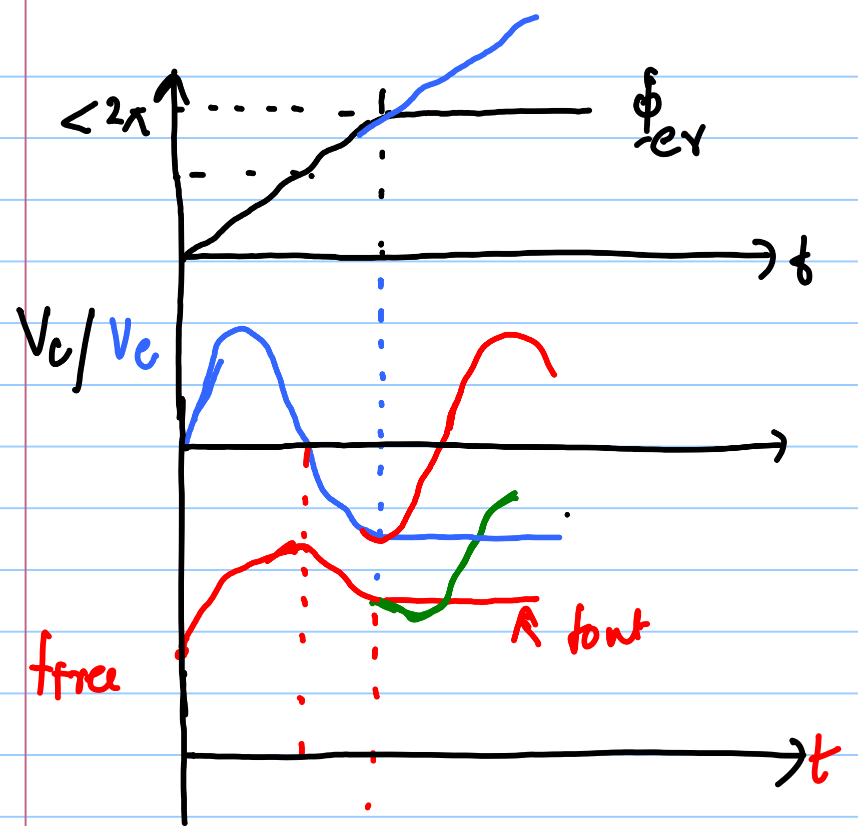
if Φ_{er} doesn't exceed 2π then V_e will <u>not cycle slip</u>.

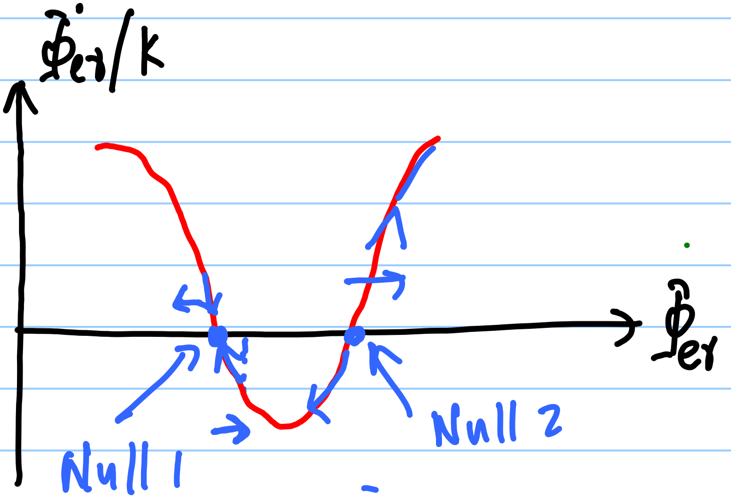


$\Delta f = 51\,MHz$.

$\Delta f = 49\,MHz$

$$\frac{d\phi_{er}}{dt} \neq 0$$

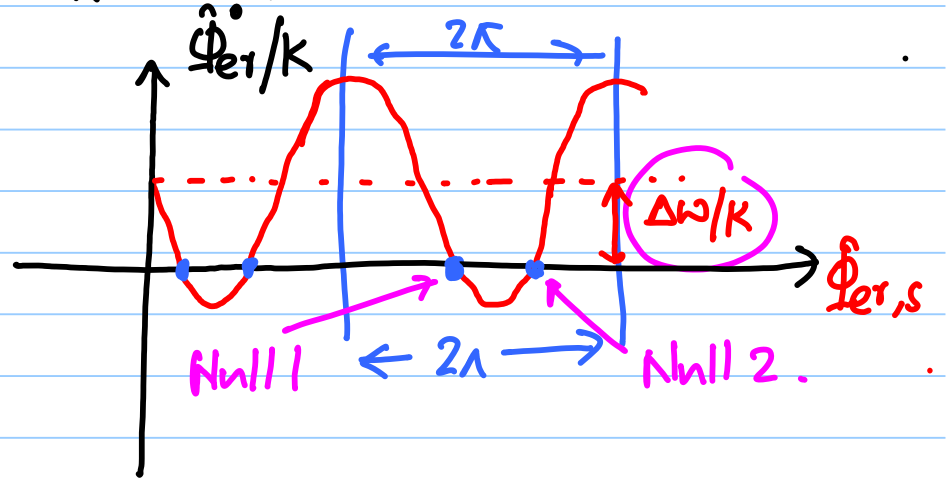




$$\hat{\Phi}_{er}(t) = \Delta\omega . t - K_{VCO} \cdot \int V_c \, dt$$
$$= \Delta\omega . t - K_{VCO} \int K_{PD} \sin(\hat{\Phi}_{er}) dt$$

$$\frac{d\Phi_{er}}{dt} = \Delta\omega - K_{VCO} \cdot K_{PD} \sin(\hat{\Phi}_{er})$$
$$= \Delta\omega - K \sin(\hat{\Phi}_{er}) = 0$$

$$\frac{\dot{\hat{\Phi}}_{er}}{K} = \frac{\Delta\omega}{K} - \sin(\hat{\Phi}_{er,s}) \quad ✓$$



- For $\Delta\hat{\Phi}_{er}$ towards right
  $\dot{\hat{\Phi}}_{er} < 0 \Rightarrow \hat{\Phi}_{er}$ will decrease.