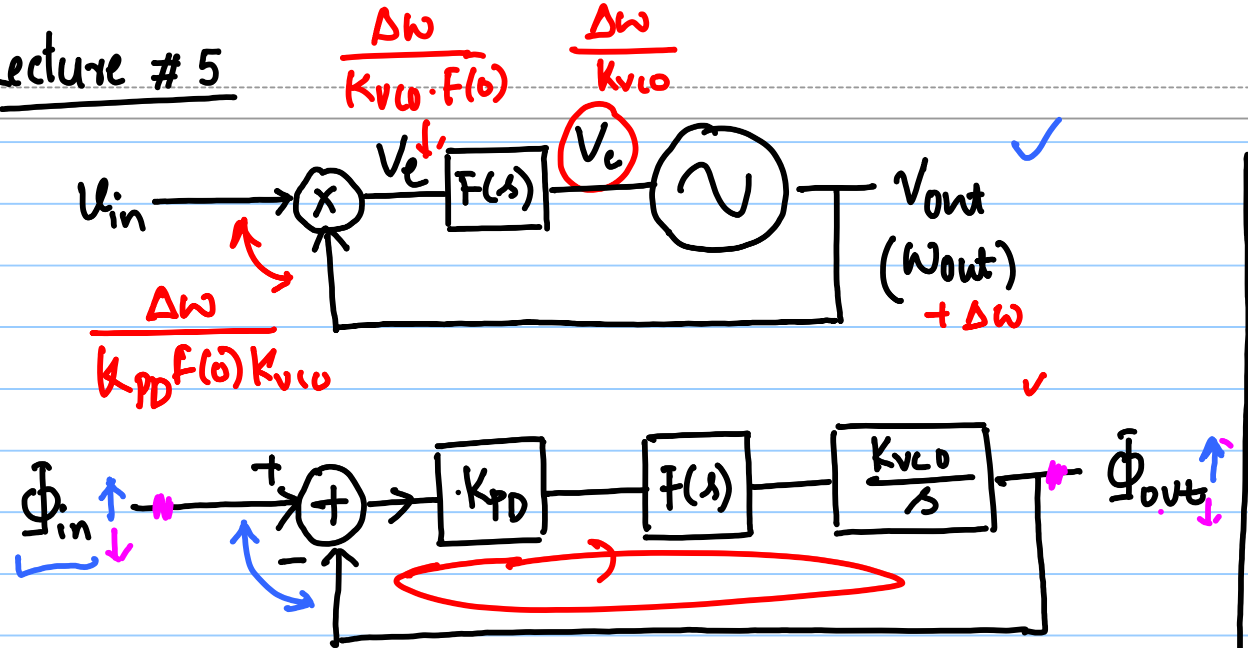
Note Title 22-05-2020

Lecture # 5

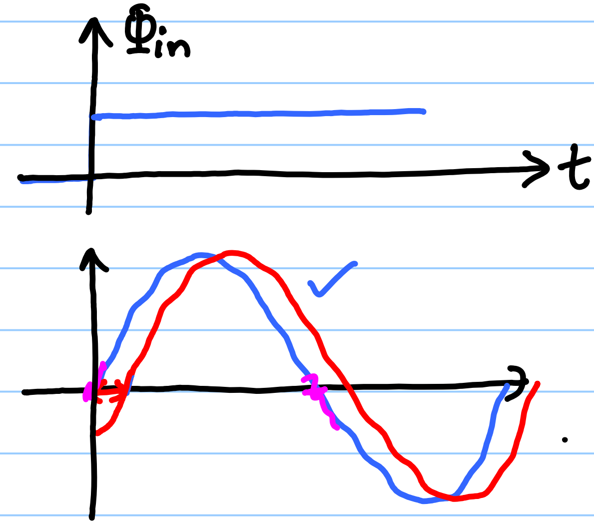


PLL locked (in steadystate)

$\Rightarrow \omega_{out} = \omega_{in}$

and $\frac{d\Phi_e}{dt} = 0$

$- \ \Phi_{in} = \Phi_{in}(0)\, u(t)$ ✓



$$\lim_{t\to\infty} \Phi_{out}(t) = \lim_{s\to 0} s\, \Phi_{out}(s)$$

$$LG(s) = K_{PD}\, F(s)\, \frac{K_{VCO}}{s}$$

$$\frac{\Phi_{out}(s)}{\Phi_{in}(s)} = \frac{LG}{1+LG} = \frac{1}{1+\frac{1}{LG}}$$

$$\lim_{t\to\infty} \Phi_{out}(t) = \lim_{s\to 0} s \cdot \Phi_{in}(s) \cdot \frac{1}{1+\frac{1}{LG}}$$

$$= \lim_{s\to 0} \cancel{s} \cdot \frac{\hat{\Phi}_{in}(0)}{\cancel{s}} \frac{1}{1+\frac{1}{LG}}$$

$$= \lim_{s\to 0} \hat{\Phi}_{in}(0) \frac{1}{1+\frac{s}{K_{PD}F(s).K_{VCO}}}$$

$$= \lim_{s\to 0} \hat{\Phi}_{in}(0) \frac{1}{1+\frac{s}{K_{PD}F(0)K_{VCO}}}$$

$$= \hat{\Phi}_{in}(0)$$

$$\hat{\Phi}_e = \hat{\Phi}_{in} - \hat{\Phi}_{out} = \hat{\Phi}_{in} - \hat{\Phi}_{in} \cdot \frac{LG(s)}{1+LG(s)}$$

$$\hat{\Phi}_e(s) = \frac{\hat{\Phi}_{in}}{1+LG}$$

$$\lim_{t\to\infty} \hat{\Phi}_e(t) = \lim_{s\to 0} s \cdot \hat{\Phi}_{in}(s) \frac{1}{1+LG}$$

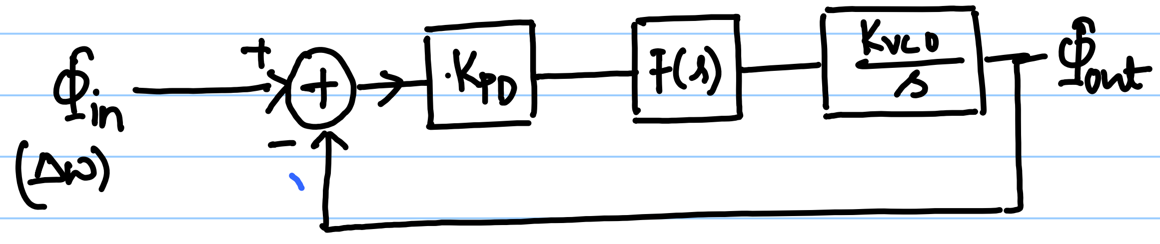
$$= \lim_{s\to 0} \hat{\Phi}_{in}(0) \frac{1}{1+\underbrace{\frac{K_{PD}F(s)K_{VCO}}{s}}}$$

$$= 0$$

if $\quad \omega_{in} \xrightarrow{t=0^+} \omega_{in} + \Delta\omega(0)$

$\omega_{out} \xrightarrow{t\to\infty} \omega_{in} + \Delta\omega_{out}.$

$\Phi_{out} \xrightarrow{t\to\infty}$



$\phi = \int \omega \cdot dt$

$\Phi(s) = \frac{\omega(s)}{s}$

$$\frac{\Phi_{out}(s)}{\Phi_{in}(s)} = \frac{LG}{1+LG}$$

$$\frac{\omega_{out}(s)/s}{\omega_{in}(s)/s} = \frac{LG}{1+LG} \Rightarrow \frac{\omega_{out}(s)}{\omega_{in}(s)} = \frac{LG}{1+LG}$$

$$\lim_{t\to\infty} \omega_{out}(t) = \lim_{s\to 0} s \cdot \omega_{out}(s) = \lim_{s\to 0} s \cdot \frac{\Delta\omega(0)}{s} \frac{1}{1+\frac{1}{LG}}$$

$$= \Delta\omega(0) \times \frac{1}{1+\frac{s}{K_{VCO} K_{PD} F(s)}}.$$

$$\Phi_e(s) = \Phi_{in}(s) - \Phi_{out}(s) = \frac{\Phi_{in}(s)}{1+LG}$$

$$= \frac{\Delta\omega(0)}{s^2} \cdot \frac{1}{1+LG}$$

$$\mathcal{L}\{\Delta\omega(0) \cdot u(t)\} \rightarrow \frac{\Delta\omega(0)}{s}$$

$$\Phi_{in} = \int \Delta\omega(0) \cdot u(t) \, dt$$

$$\Phi_{in}(s) = \frac{\Delta\omega(0)}{s^2}$$

$$\lim_{t\to\infty} \Phi_e(t) = \lim_{s\to 0} s \cdot \frac{\Delta\omega(0)}{s^2} \cdot \frac{1}{1+LG}$$

$$= \lim_{s\to 0} \frac{\Delta\omega(0)}{s} \cdot \frac{1}{1+\frac{K_{PD}F(s) \cdot K_{VCO}}{s}}$$

$$= \lim_{s\to 0} \frac{\Delta\omega(0)}{s + K_{PD}F(s)K_{VCO}}$$

$$\boxed{\lim_{t\to\infty} \Phi_e(t) = \frac{\Delta\omega(0)}{K_{PD}F(0)K_{VCO}}}$$

$$V_e = \frac{1}{2}\sin(\Phi_e)$$

$$\Phi_e = \sin^{-1}(V_e)$$

$$-1 \le \sin(\Phi_e) \le 1$$

$$\Delta W_{in}(t) = \Delta W(0)\cdot t\cdot u(t)$$

$$\lim_{t\to\infty} \omega_{out}(t) = \lim_{s\to 0} s\cdot \frac{\Delta\omega(0)}{s^2}\cdot\frac{1}{1+\frac{1}{LG}}$$

$$= \lim_{s\to 0} \frac{\Delta\omega(0)}{s+\frac{s^2}{K_{VCO}\cdot K_{PD}F(s)}}$$

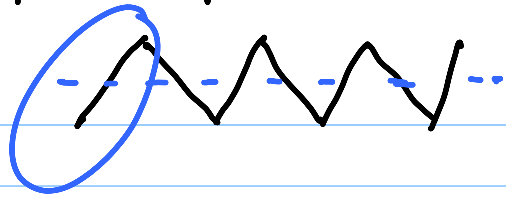
$$\lim_{t\to\infty} \Phi_e(t) = \lim_{s\to 0} s\cdot\frac{\Delta\omega(0)}{s^3}\cdot\frac{1}{1+LG}$$

$$= \lim_{s\to 0} \frac{\Delta\omega(0)}{s^2}\cdot\frac{1}{1+\frac{K_{VCO}\cdot K_{PD}F(s)}{s}}$$

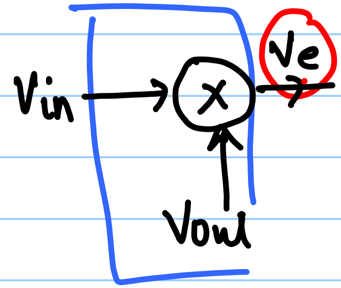
$$= \lim_{s\to 0} \frac{\Delta\omega(0)}{s^2 + K_{VCO}\cdot K_{PD}F(s)\cdot s}$$

$$= \lim_{s\to 0} \frac{\Delta\omega(0)}{s^2 + K_{VCO}\cdot K_{PD}\, s\, \frac{(sK_P+K_I)}{s}}$$

Spread Spectrum Clock

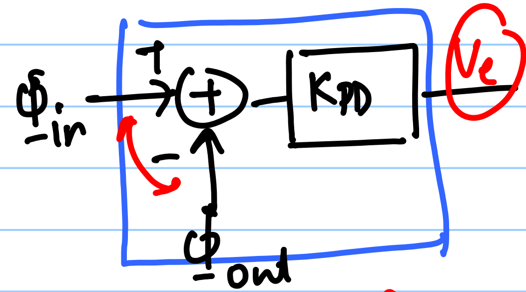


- $\Delta\Phi_{in} = \Delta\Phi_{in}(0)\ u(t)$
- $\Delta W_{in} = \Delta W(0)\cdot u(t)$
  - change is limited by PD.
- $\Delta W_{in} = \Delta W(0)\cdot t\ u(t)$



$V_e = \frac{1}{2}\sin(\Phi_e)$

$-\frac{1}{2} \le V_e \le 1/2$



$\Phi_e = \Phi_{in} - \Phi_{out}$

For frequency step. $\Delta\omega_{in} = \underbrace{\Delta\omega(0)}\, u(t)$

$$\left| K_{PD} \times \lim_{t\to\infty} \phi_e(t) \right| = \left| \frac{\Delta\omega(0)}{\underbrace{K_{PD}\, F(0)\, K_{VCO}}_{+\,V_{es}}} \times K_{PD} \right| < \frac{1}{2}$$

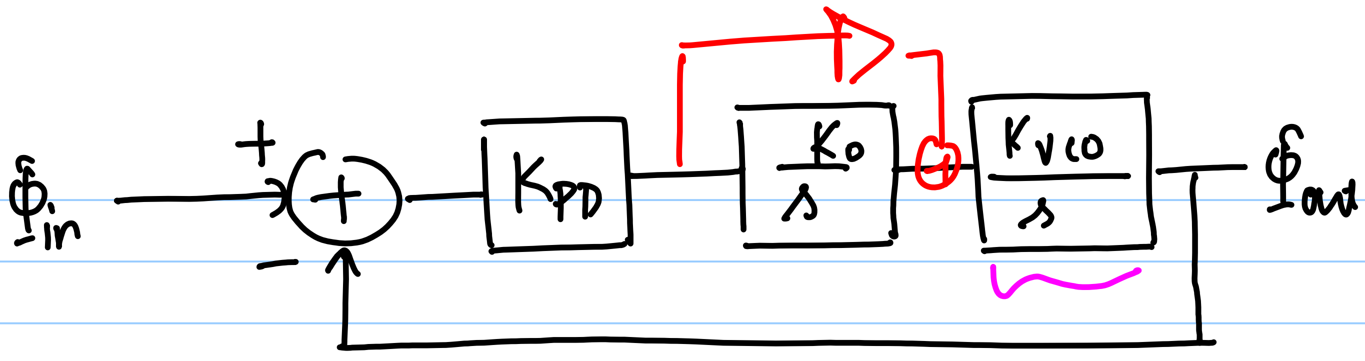
$F(0) \longrightarrow \infty$

$F(s) = \dfrac{1}{s}$ ✓

$$\lim_{t\to\infty} \phi_e(t) = \lim_{s\to 0} \cancel{s} \cdot \frac{\Delta\omega(0)}{s^{\cancel{2}}} \cdot \frac{1}{1 + \dfrac{K_{VCO} K_{PD}}{s} \cdot \dfrac{1}{s}}$$

$$= \lim_{s\to 0} \frac{\Delta\omega(0)}{\underset{0}{s} + \dfrac{K_{VCO} \cdot K_{PD}}{s} \to \infty}$$

$= 0$



$$F(s) = \frac{K_I}{s} + K_p + K_D s$$

$$Lh = \frac{K_{PD} \quad K_{VCO}}{s^2} \left(K_I + s K_p\right)$$

$$F(s) = \frac{K_I}{s} + K_p = \frac{K_I + s K_p}{s}$$

$$\frac{\Delta\omega_{out}}{\Delta\omega_{in}} = \frac{Lh}{1+Lh} = \frac{1}{1 + \frac{1}{Lh}}$$

$$\lim_{t\to\infty} \Delta\omega_{out}(t) : \lim_{s\to 0} s \cdot \frac{\Delta\omega(0)}{s^2} \frac{1}{1 + \frac{s^2}{K_{PD} K_{VCO} (s K_p + K_I)}}$$

$$\lim_{t\to\infty} \Phi_e(t) = \lim_{s\to 0} s \cdot \frac{\Delta\omega(0)}{s^2} \frac{1}{1 + \frac{K_{PD} K_{VCO} \cdot (s K_p + K_I)}{s^2}}$$

$$= \frac{\Delta\omega(0)}{K_{PD} K_{VCO} K_I / s} \downarrow 0$$

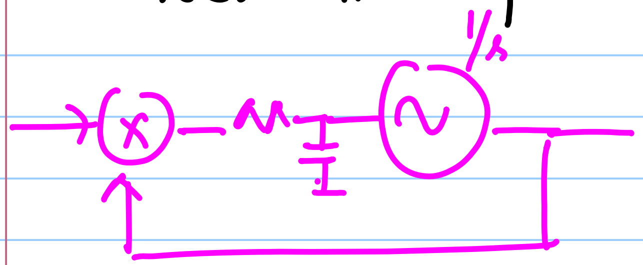
$$= \frac{1}{s^2 + s^2 \cdot K_{PD} K_{VCO} \left(\frac{K_I}{s} + K_p + K_D \cdot s \right)}$$

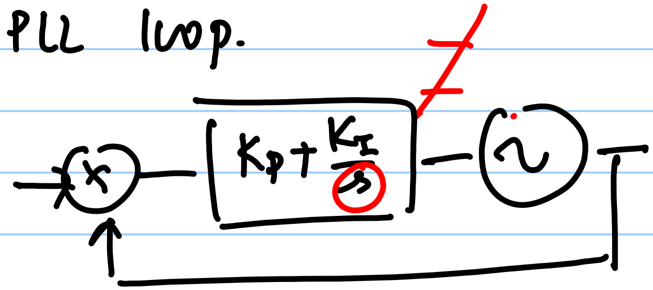
$\Phi_e(t)$

$F(s)$	$\Delta\Phi_0 u(t)$	$\Delta\omega(0)\, u(0)$	$\Delta\omega(0) \cdot t\; u(t)$
No integrators	0	$\frac{\Delta\omega(0)}{K_{PD} K_{VCO} F(0)}$	∞
1 Integrator	0	0	∞
2 Integrators	0	0	0

Type : # of integrators in a loop.

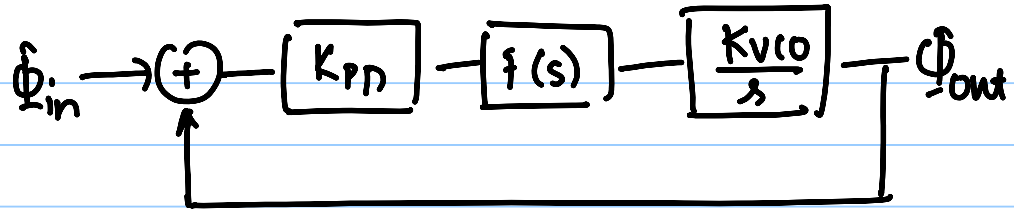
Order : # of poles in PLL loop.





Type - I
Order - 1

Type - II
Order - 2

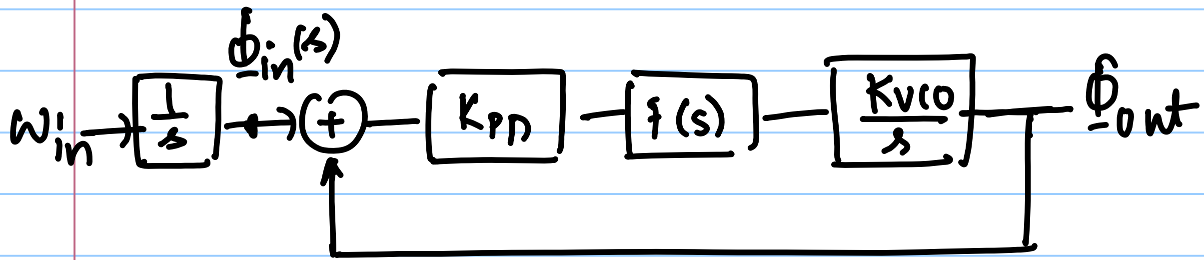


$$\phi = \int \omega \, dt$$

$$\Phi_{in} = \int \omega_{in} \cdot dt$$

$$\Phi_{in}(s) = \frac{\omega_{in}(s)}{s}$$

$$\Phi_{out} = \int \omega_{out} \cdot dt .$$



$$\frac{\Phi_{out}}{\Phi_{in}} = \frac{LG}{1+LG} \checkmark$$

$$\frac{\omega_{out}/s}{\omega_{in}/s}$$

$$\Delta\Phi_{in} = \Delta\Phi(0) \cdot t \quad u(t)$$

$$\Delta\Phi_{in}(s) = \frac{\Delta\Phi(0)}{s^2}$$