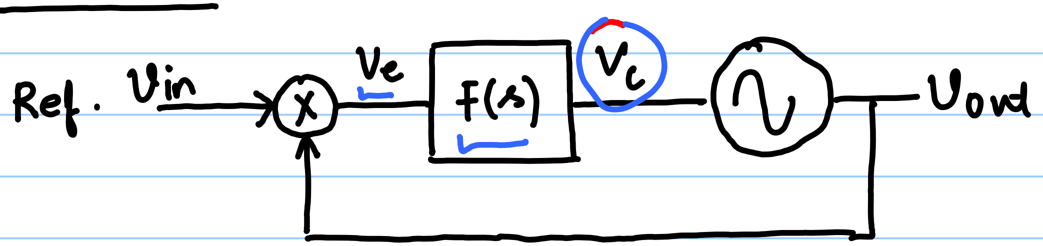
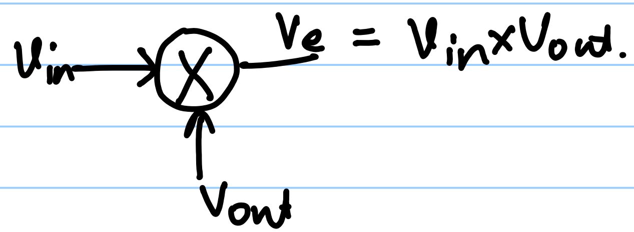
Note Title 21-05-2020

Lecture # 04

Basic PLL





$$V_{in} = A_{in} \sin(w_{in} \cdot t)$$

$$V_{out} = A_{out} \cos(w_{out} t) = A_{out} \cos\left(w_{free} \cdot t + \int K_{vco} \cdot V_c \, dt + \Phi_{out}(0) \right)$$

$$V_e = V_{in} \times V_{out}$$

$$\approx \frac{A_{in} \cdot A_{out}}{2} \sin\left((w_{in} - w_{free})t - \int K_{vco} \cdot V_c \, dt - \Phi_{out}(0) \right)$$

Case 3: $w_{in} - w_{free} = \Delta w$ | Δw is freq. error b/w i/p freq. and free running freq. of oscillation

at $t=0$, $V_c(0) = 0$ so. $w_{in} - w_{out} = w_{in} - w_{free} = \Delta w$

$$V_e = \frac{A_{in} \cdot A_{out}}{2} \times \sin\left(\Delta w \cdot t - K_{vco} \int V_c \, dt \right)$$ ✓ | let $\Phi_{out}(0) = 0$

(or at $t \to \infty$)

If PLL reaches steady state $\Rightarrow$ Finally, $\omega_{out} = \omega_{in}$

$$\omega_{free} + K_{VCO} \cdot V_c = \omega_{in}$$

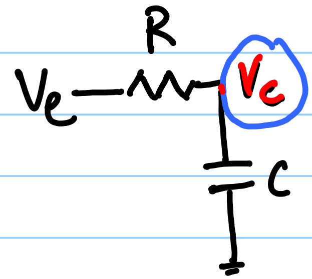
$$\Rightarrow \quad V_c = \frac{\omega_{in} - \omega_{free}}{K_{VCO}}$$

$$\boxed{V_c = \frac{\Delta\omega}{K_{VCO}}} \checkmark$$

$V_c(s) = F(s) \cdot V_e(s)$

$V_c = F(0) \cdot V_e$

$\Rightarrow \quad V_c = V_e$



$$\frac{V_c}{V_e} = \frac{1}{1+sRC}$$

$$\boxed{V_e = V_c}$$

$$\frac{\Delta\omega}{K_{VCO}} = \frac{A_{in} A_{out}}{2} \sin\Big(\overbrace{\Delta\omega \cdot t - \int K_{VCO} \cdot V_c \, dt + \Phi_{es}}^{\Phi_{es}}\Big) \checkmark$$

Φ_{es}: Error in steady state

Assume $A_{in} = A_{out} = 1$

$$\frac{\Delta\omega}{K_{VCO}} = \frac{1}{2} \sin\Big(\underbrace{(\Delta\omega - K_{VCO} \cdot V_c)}_{:0} t + \Phi_{es}\Big)$$

$$= \frac{1}{2}\sin(\Phi_{es})$$

$$\sin(\Phi_{es}) = 2\cdot\frac{\Delta\omega}{K_{VCO}}$$

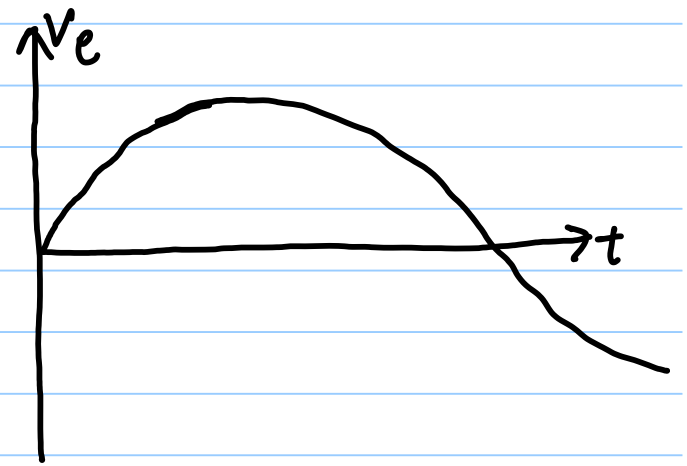
$$\Phi_{es} = \sin^{-1}\left(\frac{2\cdot\Delta\omega}{K_{VCO}}\right)$$

✓

$$V_{out} = \cos(\underset{\omega_{in}}{\underset{\downarrow}{\omega_{out}}}\cdot t - \Phi_{es})$$

at $t=0$

$$V_e = \frac{1}{2}\sin\left(\Delta\omega\cdot t - \int K_{VCO}\underset{V_c=0}{\underset{\downarrow}{V_c}}\cdot dt\right)$$



$$\Downarrow$$

$$V_c \approx V_e$$

$$\Downarrow$$

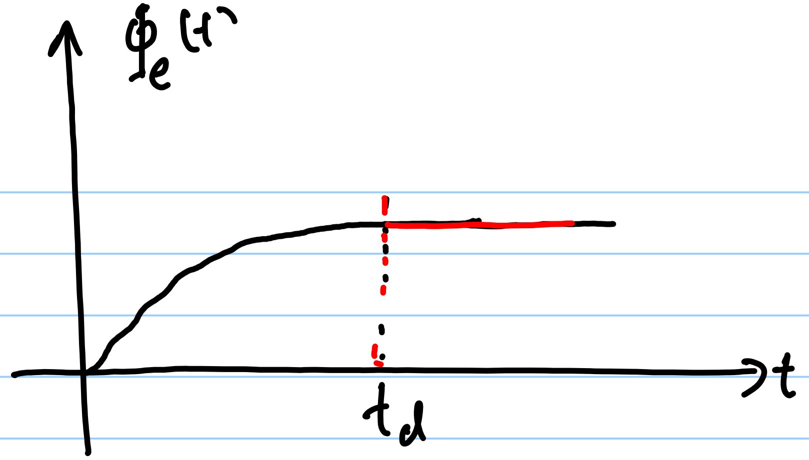
$$\omega_{out} = \omega_{free} + K_{VCO}\cdot V_c$$

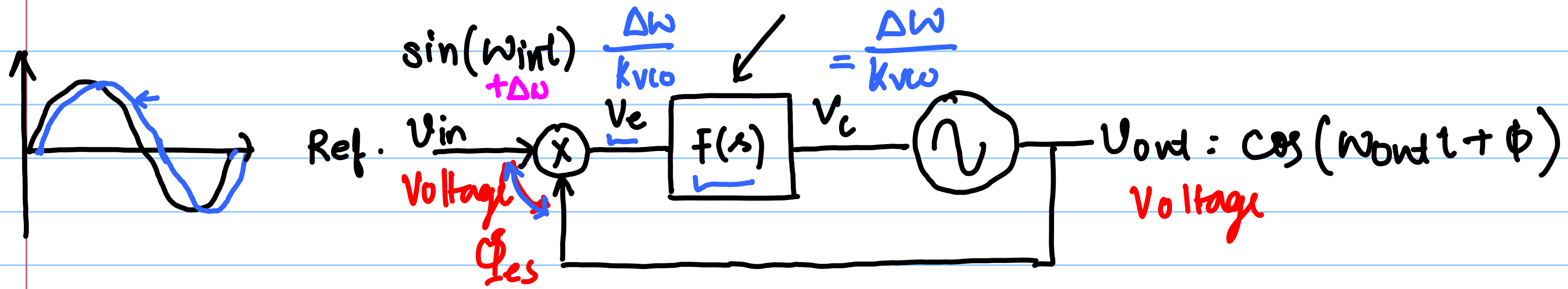
$$= \omega_{free} + K_{VCO}\cdot\times\frac{1}{2}\sin(\Delta\omega\cdot t)$$

$$\Downarrow$$

$$V_{out} = \cos\left(\omega_{free} + \frac{K_{VCO}}{2}\sin(\Delta\omega\cdot t)\right)$$

$$\Phi_e(t) = (\omega_{in} - \omega_{free})t - \int K_{VCO}\, V_c\, dt$$ (integral)





W_{in} : i/p freq.

W_{free} : free running freq. osc.

K_{VCO} : gain of osc

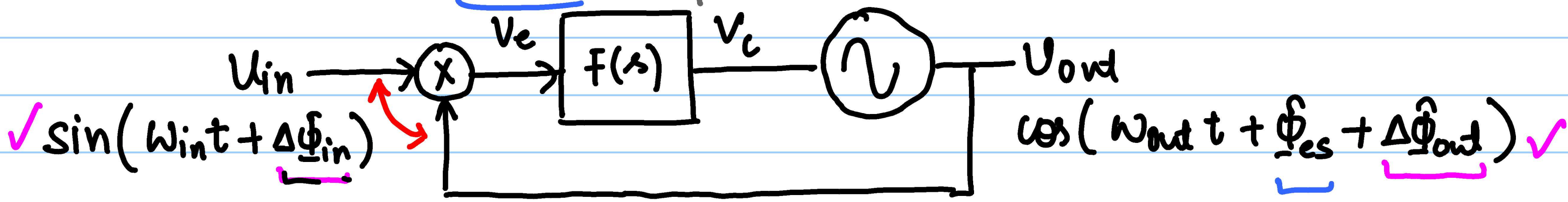
$$F(s) = \frac{1}{1+sRC}$$

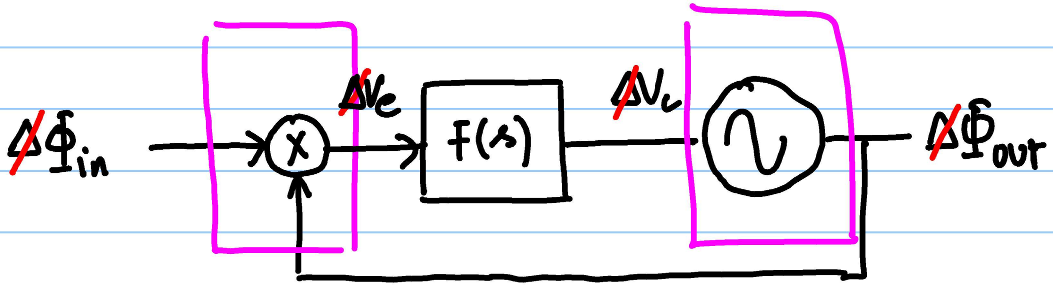
In steady state :

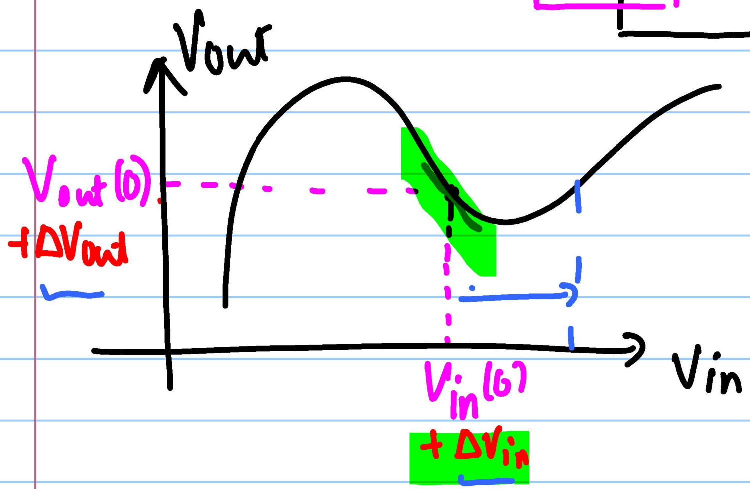
$$V_c = \frac{\Delta\omega}{K_{VCO}}$$

$$\frac{1}{2}\sin(\Phi_{es}) = \frac{\Delta\omega}{K_{VCO}}$$

$\hat{\Phi}_{es} = \sin^{-1}\left(\frac{2 \cdot \Delta\omega}{K_{VCO}}\right) + \Delta\hat{\Phi}_e$ | $V_c = \frac{\Delta\omega}{K_{VCO}} + \Delta V_c$







$V_{out} = f(V_{in})$

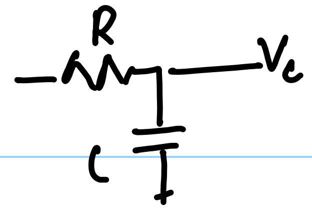
$\Delta V_{out} = f'(V_{in})\Big|_{V_{in} = V_{in}(0)} \times \Delta V_{in}$

$V_{out} = V_{out}(0) + \Delta V_{out}$.

$V_{in} = \sin(\omega_{in}t)$, $V_{out} : \cos((\omega_{free} + K_{VCO} \cdot V_c)t + \hat{\Phi}_{es})$

$V_e = \frac{1}{2}\left[\sin(2\omega_{in}t + \hat{\Phi}_{es}) + \sin(\hat{\Phi}_{es})\right]$

$V_c = \frac{1}{2}\sin(\phi_{es})$ < Changing / Not changing



< Changi / Not chang

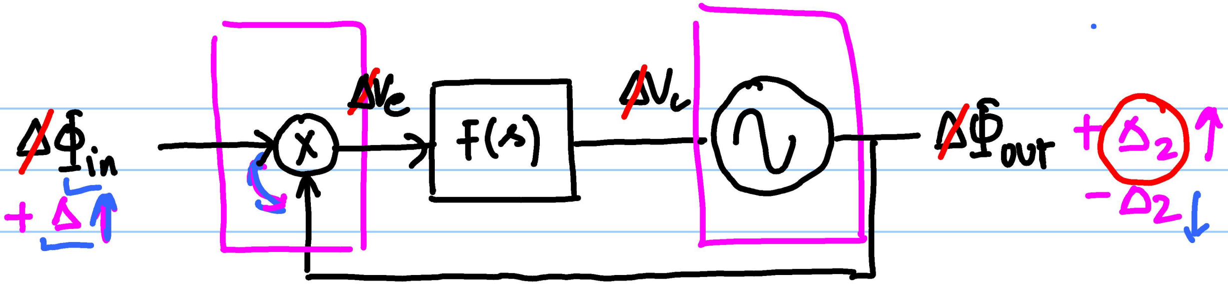
$\omega_{out} = \omega_{free} + K_{vco} \cdot V_c$

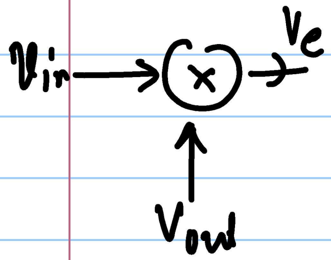
$$\phi = \int \omega \, dt$$

$$= \underbrace{\int_0^{t_1} \omega_1 \, dt + \int_{t_1}^{t_2} \omega_2 \, dt \; - \; \cdot \; + \int_{t_n}^{\infty} \omega_n \, dt}$$

$$= \phi_{os} +$$

$t >> t_n$





$$V_e \approx \frac{A_{in}\cdot A_{out}}{2}\sin\left(\left(\underbrace{\Delta\omega\cdot t - \int K_{VCO}\cdot V_c\,dt}_{=0}\right) + \hat{\Phi}_{es}\right)$$

$$\approx \frac{A_{in}\cdot A_{out}}{2}\sin\left(\hat{\Phi}_{es}\right)$$

$$V_e + \Delta V_e = \frac{A_{in}\cdot A_{out}}{2}\sin\left(\hat{\Phi}_{es} + \Delta\hat{\Phi}_{es}\right)$$

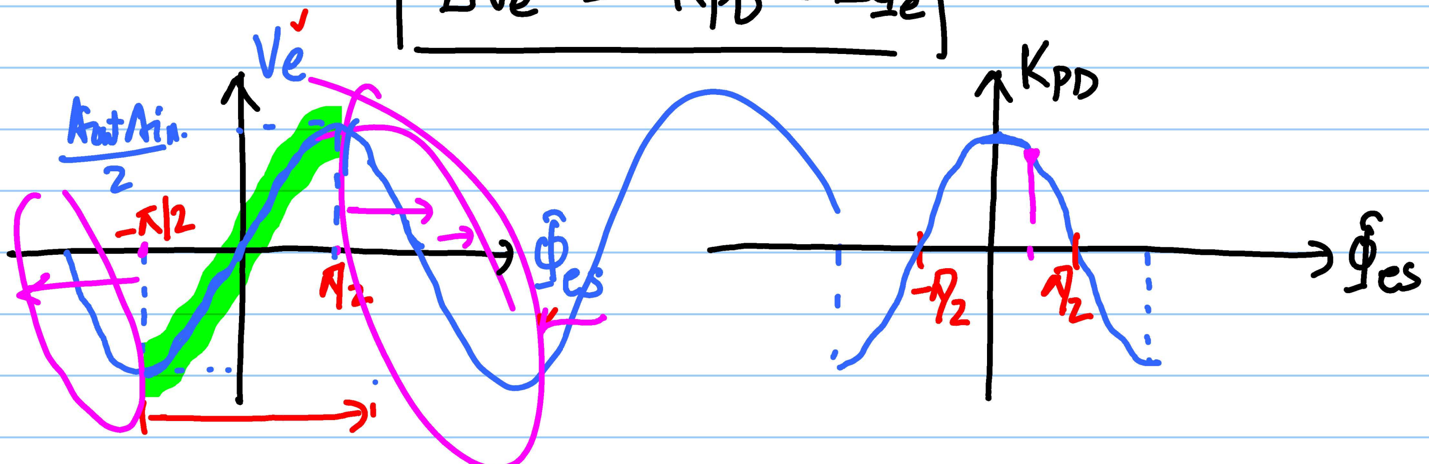
$\hat{\Phi}_{es} = \hat{\Phi}_{in} - \hat{\Phi}_{out}$

$$\boxed{K_{PD} = \frac{dV_e}{d\hat{\Phi}_{es}} = \frac{A_{in}\cdot A_{out}}{2}\cos(\hat{\Phi}_{es})}$$

Gain of Phase Detector.

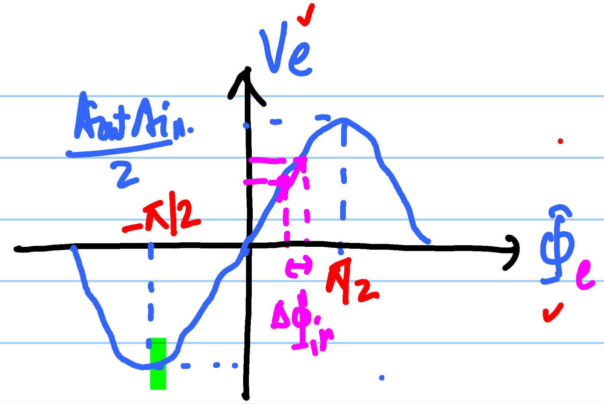
$$\boxed{\Delta V_e = K_{PD}\cdot\Delta\hat{\Phi}_e}$$

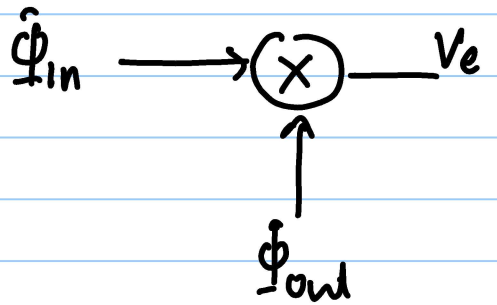
$[K_{PD}] = V/rad.$



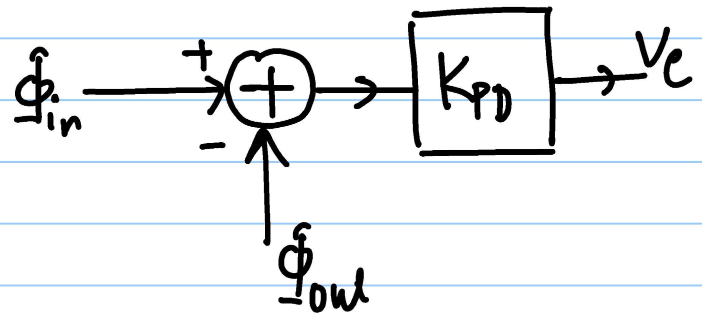
$$\hat{\Phi}_{in} - \hat{\Phi}_{out} = \hat{\Phi}_{es}$$

$$\hat{\Phi}_{e} = (\hat{\Phi}_{in} + \Delta\hat{\Phi}_{in} - \hat{\Phi}_{out}) = \hat{\Phi}_{es} + \Delta\hat{\Phi}_{in}$$



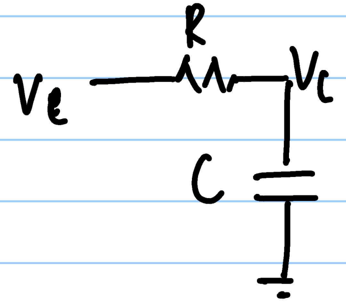


$\Rightarrow$



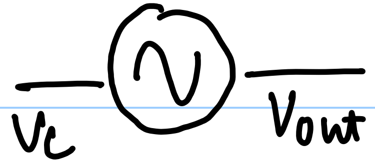
Non-linear

Linear.



$$\frac{V_c}{V_e} = \frac{1}{sRC+1}$$

$\frac{V_c}{V_e}$ = linear transfer function.



$$V_{out} = \cos\left(\omega_{free} \cdot t + \int K_{vco} \cdot V_c \cdot dt\right)$$

$$\hat{\Phi}_{out} = \int \omega_{free} + K_{vco} \cdot V_c \, dt = \int \omega_{out} \cdot dt$$

$$\underbrace{\hat{\Phi}_{out}} + \Delta\hat{\Phi}_{out} = \int \omega_{free} + K_{vco}(V_c + \Delta V_c)\, dt$$

$$= \int \underbrace{\omega_{free} + K_{vco} \cdot V_c \cdot dt} + K_{vco} \int \Delta V_c \cdot dt$$

$$\Delta\hat{\Phi}_{out} = K_{vco} \int \Delta V_c \cdot dt$$

$$\boxed{\frac{\Delta\hat{\Phi}_{out}(s)}{\Delta V_c(s)} = \frac{K_{vco}}{s}}$$