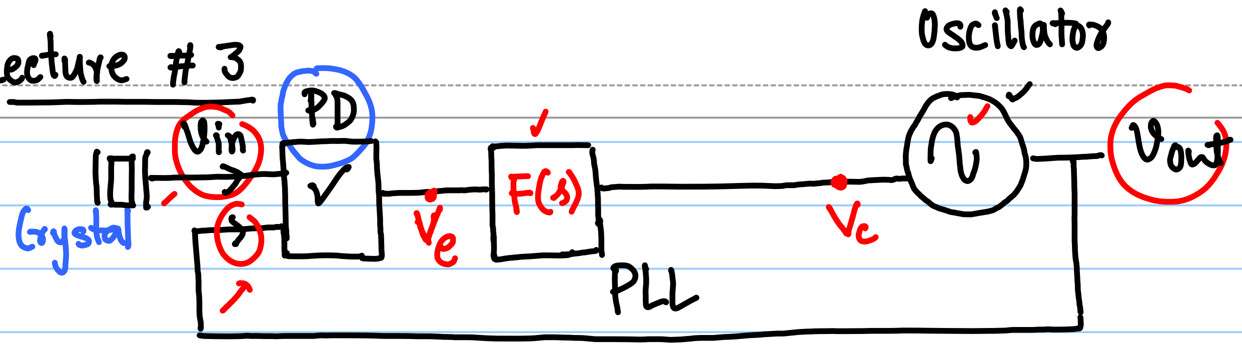
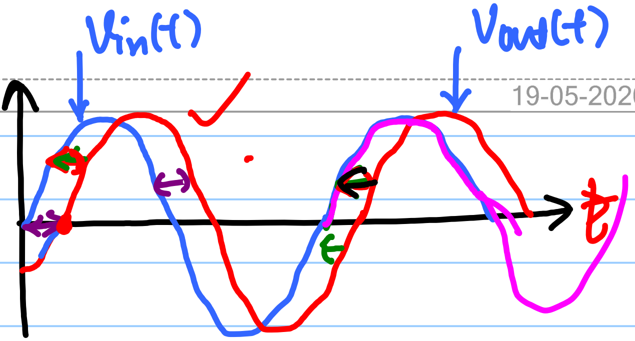
Lecture # 3





$V_{in}(t) = A_{in} \sin(\omega_{in} t)$

$V_{out}(t) = A_{out} \sin(\omega_{out} t)$

$= A_{out} \sin(\underline{\omega_0 t} + \underline{\Phi(t)} \overset{-\pi/4}{})$

✓ $\Phi_{in}(t) = \int \omega_{in}\, dt = \omega_{in} t$

✓ $\Phi_{out}(t) = \int \omega_0\, dt + \Phi(t)$

$= \omega_0 t + \Phi(t)$

$\Phi_{in}(t) - \Phi_{out}(t) = (\omega_{in} - \omega_0)t - \Phi(t)$

$\Phi_{er}(t) = \underline{(\omega_{in} - \omega_0)t - \Phi(t)} = \pi/4$

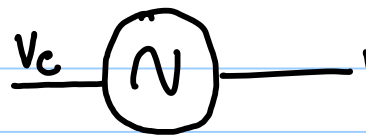
Output of phase detector $\propto \Phi_{er}(t)$

[Gain of PD] = V/rad

Once PLL is locked.

$\omega_{in} = \omega_{out}$

$\frac{d\Phi_{er}(t)}{dt} = 0$



$V_{out} = A_{out} \sin(\underbrace{W_{out}}t)$

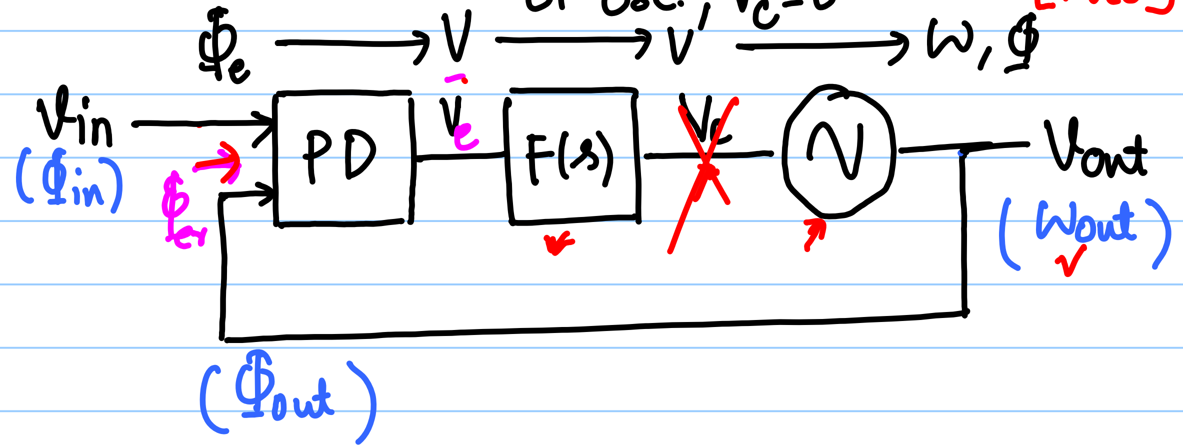
Control voltage, V_c changes the frequecy of osc.

$$W_{out}(t) = \overset{= W_{in}}{W_{free}} + \underbrace{K_{vco} \cdot V_c(t)} \uparrow$$

$\downarrow$ free running freq. of osc., $V_c = 0$

$\downarrow$ $[K_{vco}] = \frac{rad/s}{V}$

$\underline{\Phi_{out}(t)} = \omega_{free} t + K_{vco} \int \underline{V_c(t)} \cdot dt$



Basic Phase Locked Loop

Gain of PD, $K_{PD} = \frac{dV_e}{d\Phi_{er}}$

Loop filter, $\frac{V_c(s)}{V_e(s)} = F(s)$

Gain of osc, $K_{vco} = \frac{dW_{out}}{dV_c}$. $\left[\frac{rad/s}{V}\right]$

$v_{in} = \sin(w_{in} t)$ ✓

$V_{out} = \sin(w_0 t - \pi/4) = \sin(w_{in} t - \pi/4)$ ✓

Let $t=0.$ $w_{in} = w_0$

$\Phi_{er}(t) = w_{in} t - (w_{in} t - \pi/4)$

$= \underbrace{\pi/4}$

$w_{out}(t) = w_{free} + K_{vco} \cdot V_c(t)$

$w_{out}(t) = w_{in} + K_{vco} \cdot \underbrace{V_c} > w_{in}$

$= w_{in} + K_{vco} \cdot 0 = w_{in}$

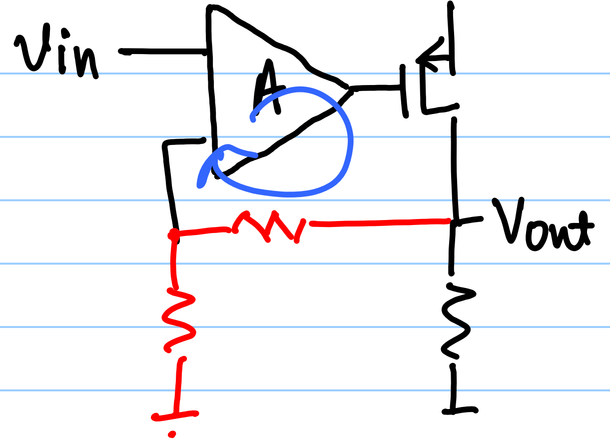
✓ $\frac{d\Phi_{er}(t)}{dt} = 0$ ✗

✓ $w_{out} = w_{in}$

$\underline{\Phi_{er} = 0.}$

Steady state.

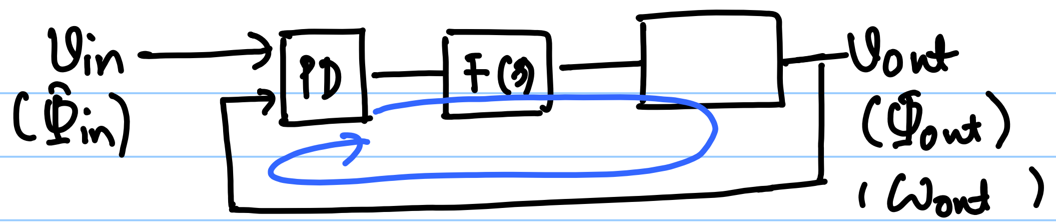
Voltage/Current



$V_{out} = V_{in}$

$H(s) = \frac{V_{out}(s)}{V_{in}(s)} =$

Phase/Frequency.



$\Phi_{out} = \Phi_{in} + K$

$\omega_{out} = \underline{N}\omega_{in},$ Integer-N PLL

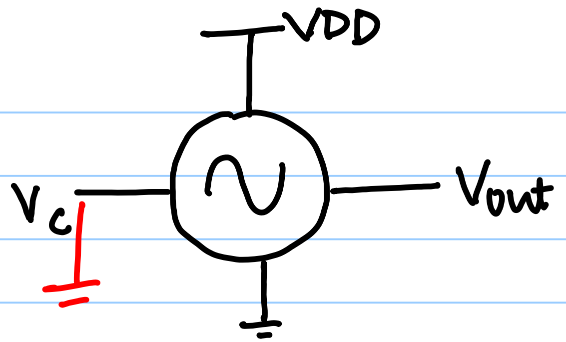
N is integer.

$= (N+\alpha)\omega_{in}, \quad 0<\alpha<1$

Fractional-N PLL.

$\omega_{out} = \omega_{in}$

$H(s) = \frac{\Phi_{out}(s)}{\underline{\Phi_{in}(s)}}$

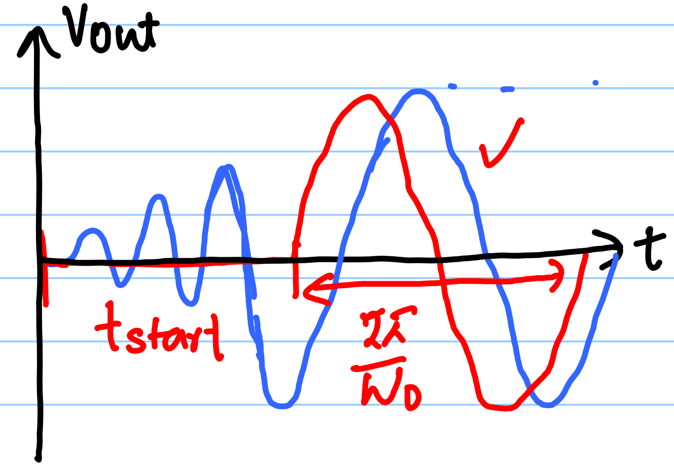


$$V_{out} = \sin(\underline{\omega_{free}} t + \int K_{vco} \cdot V_c \, dt)$$

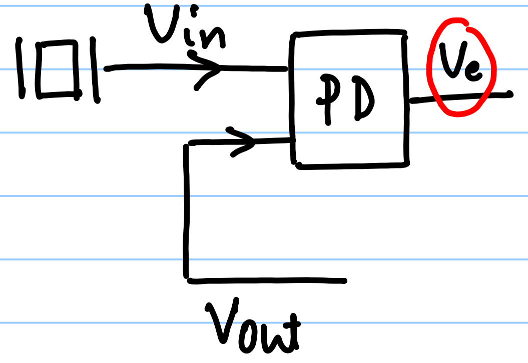
$$\omega_{out}(t) = \omega_{free} + K_{vco} \cdot V_c$$

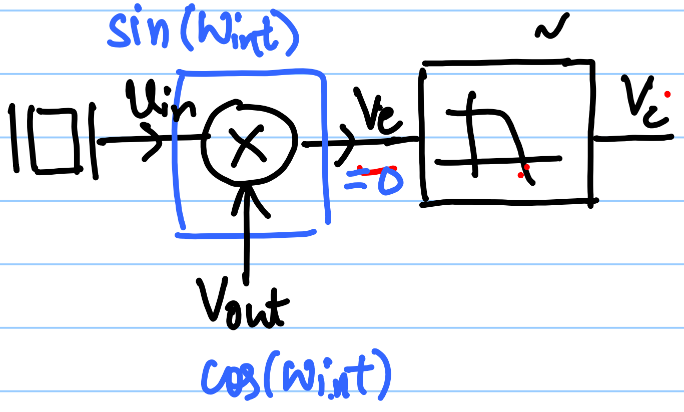
$$\omega_{out}(t)\Big|_{V_c=0} = \omega_{free}$$

$\omega_{free} = \omega_{in}$



Basic PLL





✓ $v_{in}(t) = A_{in}\sin(\omega_{in}t)$ ✓

✓ $v_{out}(t) = A_{out}\cos\left(\omega_{free}t + \int K_{vco}\cdot V_c\,dt + \underset{out}{\Phi_{in}}(0)\right)$ ✓

$\Phi_{in}(t) = \omega_{in}t$ $\qquad \cos(\omega_{free}t)$

$\Phi_{out}(t) = \omega_{free}t + \int K_{vco}\cdot V_c\,dt + \underset{out}{\Phi_{in}}(0)$

$v_{in}\times v_{out} = A_{in}\cdot A_{out}\sin(\quad)\cos(\quad)$

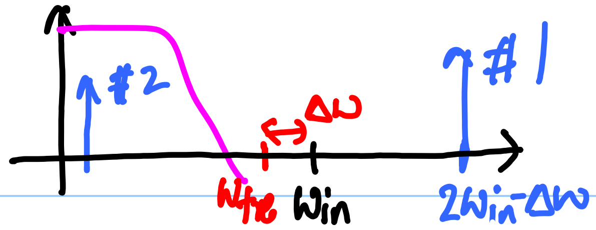
$= \frac{A_{in}A_{out}}{2}\Big[\underset{\cos}{\sin}\left(\overline{\omega_{in}+\omega_{free}}\,t + \int K_{vco}V_c\,dt + \underset{out}{\Phi_{in}}(0)\right)$ ✓ #1

$+ \underset{\cos}{\sin}\left(\overline{\omega_{in}-\omega_{free}}\,t - \int K_{vco}V_c\,dt - \underset{out}{\Phi_{in}}(0)\right)\Big]$ ✓ #2

$2\sin A\cdot\cos(B)$

$= \sin(A+B) + \sin(A-B)$

For. $\underline{\overset{\downarrow}{\omega_{in}} - \overset{\downarrow}{\omega_{free}}} = \underline{\Delta\omega}$.



$$\#1 : \frac{A_{in}A_{out}}{2}\sin\left(\underline{\overline{2\omega_{in}-\Delta\omega}}\, t + \underline{\int K_{vco}\cdot} V_e\, dt + \Phi_{out}(0)\right)^{\checkmark} \quad \text{Reject}$$

$$\#2 : \frac{A_{in}\cdot A_{out}}{2}\sin\left(\underbrace{\underline{\Delta\omega . t} - \int \underline{K_{vco}\cdot V_e} \cdot dt - \Phi_{out}(0)}_{\Phi_{er}(t)}\right) \quad \checkmark \quad \text{Retain}$$

$$\Phi_{in} - \Phi_{out} = \Delta\omega . t - \int K_{vco}\cdot V_e\, dt - \Phi_{out}(0)$$

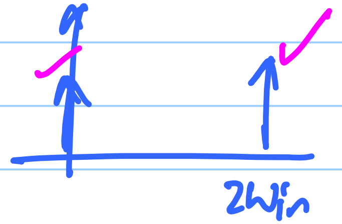
$$V_e = \frac{A_{in}\cdot A_{out}}{2}\left[\sin\left(\overline{2\omega_{in}-\Delta\omega}\, t + \int K_{vco}\cdot V_e\, dt + \Phi_{out}(0)\right) + \sin\left(\Delta\omega . t - \int K_{vco}\cdot V_e \cdot dt - \Phi_{out}(0)\right)\right] \quad \times \text{ (Neglect from analysis)}$$

$$\underline{V_e} \approx \frac{A_{in}A_{out}}{2}\sin\left(\underline{\overset{\Delta\omega \neq 0}{\Delta\omega . t}} - \overset{V_c=0}{\int K_{vco}\cdot V_c\, dt} - \overset{\text{Non-zero}}{\Phi_{out}(0)}\right)$$

$$\downarrow$$

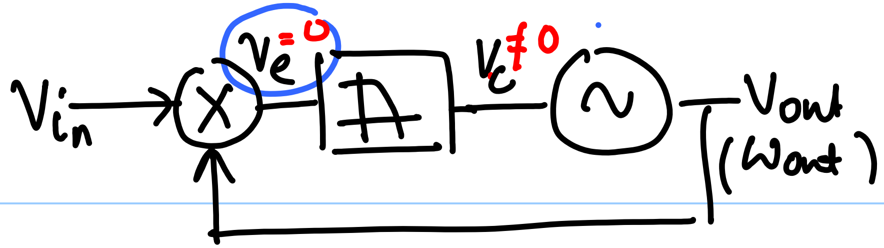
$$\sin\left(\int(\Delta\omega - K_{vco}\cdot V_c)\, dt - \Phi_{in}(0)\right)$$

$$\Phi_{er}^{\checkmark}$$



$$\boxed{V_e = \frac{A_{in}\cdot A_{out}}{2}\underset{\cos}{\sin}\left(\underline{\Phi_{er}(t)}\right)}$$

Ideally, $V_e = K_{PD}\,\Phi_{er}$



$V_e = \frac{A_{in} \cdot A_{out}}{2} \sin(\phi_{er}(t))$

$\omega_{out} = \omega_{free} + K_{vco} \cdot \underset{\sim}{V_c}$

Case 1. at $t=0$ $\Delta\omega = 0$, $\phi_{in}(0) = 0$, $\omega_{free} = \omega_{in}$

In steady state or $t \to \infty$ (dc value)

$\Rightarrow V_c = 0 \Rightarrow V_e = 0 = (\ ) \sin(\phi_{er})$

$\Rightarrow \phi_{er} = 0$

Case 2: at $t=0$, $\underline{\Delta\omega = 0}$, $\phi_{in}(0) = -\pi/4$

$\phi_{er}(0) = \underline{\pi/4}$

In steady state, $V_e = 0 \Rightarrow \sin(\phi_{er}) = 0 \Rightarrow \phi_{er} = 0$

Case 3 at $t=0$, $\underline{\Delta\omega = 1 Mrad/s}$, $\phi_{in}(0) = 0$